The Time Evolution for Qubit – Spin Coherent State Interaction System Using Majorana Sphere Representation

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Abstract. We studied the interacting system of qubit with a collection of N qubits or known as spin coherent state. From the interaction of the qubit – spin coherent state system, we can observe many interesting phenomena such as 'collapse and revival' in the time evolution of the qubit state, the qubit of attractor state, the qubit – spin coherent state entanglement, and the occurrence of the Schrödinger's cat state. The event of 'collapse and revival' will be mainly focused in this study. Prior studies had shown the Bloch Sphere and Majorana Sphere representations for the interaction system between qubit and a coherent state. Therefore, in this research, we will analyze the time evolution of the qubit state in the qubit – spin coherent state interacting system for the one qubit case, and represent the dynamics of the qubit state using the Majorana Sphere representation.

Keywords: qubit, spin coherent state, collapse and revival, majorana sphere

INTRODUCTION

Quantum computation is the study of information processing regime of a quantum mechanical system [1]. There are several models which can be used to analyze how quantum information is transferred to a quantum field. One of them is known as spin coherent state model. Developed by S. Dooley and collaborators in 2014, the spin coherent state model was a further continuation from the studies of Jaynes-Cummings model [2,3,4]. This model consists of a single qubit being coupled to N-numbers of qubits or spin-1/2 particles known as spin coherent states [3,5,6]. Even though the Jaynes-Cummings model and the spin coherent state model demonstrate different physical systems, but they are synonymous to one another. It was shown that the spin coherent state model also displays similar interesting observations as in Jaynes-Cummings model such as the 'collapse and revival' of Rabi oscillations, the qubit attractor state, entanglement between qubit and field, and also the presence of the Schrödinger's cat state [2,3,7].

The evolution of a one qubit state probabilities can be graphically represented by simulating the time-dependent spin coherent state models' exact solutions into Majorana sphere. Majorana sphere is a type of sphere in which its plot is the stereographic projection from a complex plane onto the sphere. Compared to the Bloch sphere [8], Majorana representation can fit in multiple qubits as they lie in a symmetric subspace [9]. Prior studies had shown the representation of a one qubit state probabilities in Majorana sphere for the Jaynes-Cummings model [9]. Thus, this research will focus on representing the dynamics of one qubit state probabilities in the spin coherent state model onto the Majorana sphere. By considering the cases of ideal and practical systems where finite detuning is present, the quantum properties of one qubit spin coherent state model will be investigated from the representation.

One Qubit Spin Coherent State Model

Considering the limit $N \rightarrow \infty$, the field mode system of the coherent state is analogous to the N-spin system of the spin coherent state. A state in which all N of the individual spins are in the same pure state is referred to as a spin coherent state [10,11]. The state is parametrized by a complex number $\zeta = \sqrt{\bar{n}}e^{-i\phi}$ and in terms of Dicke state, the spin coherent state can be written as

$$|N,\zeta\rangle_N = \sum_{n=0}^{N} C_n \left|\frac{N}{2}, n - \frac{N}{2}\right\rangle_N \tag{1}$$

where

$$C_n = \frac{1}{(1+|\zeta|^2)^{N/2}} \sqrt{\frac{N!}{(N-n)!\,n!}}\zeta^n \,. \tag{2}$$

$|n\rangle$ are the energy eigenstates while $\bar{n} = |\zeta|^2$ is the average excitation number in the spin coherent state.

S. Dooley and collaborators have studied the interaction system of a single qubit that was coupled to a collection of N-numbers of qubits known as the spin coherent state [3,6,12], hence the name spin coherent state model emerged. This model is also sometimes called as the spin star model or the big spin model [13]. I. Bahari [2] then continued the work to present the two qubit - spin coherent state interaction model. For this research, we will focus on the one qubit case only.

The Hamiltonian for the interaction system between one qubit and spin coherent state is stated as [2]:

$$\hat{H} = \hbar\omega_N\left(\hat{J}_z + \frac{N}{2}\right) + \frac{\hbar\Omega}{2}\hat{\sigma}^z + \frac{\hbar\lambda}{\sqrt{N}}\left(\hat{J}_+\hat{\sigma}_- + \hat{J}_-\hat{\sigma}_+\right). \tag{3}$$

Ω and ω_N is the frequency of the qubit and the spin coherent state respectively. $\hbar$ is the Planck's constant and λ is the dipole-interaction strength between the qubit and the spin coherent state. Operators that act on the spin coherent state are given by $\hat{J}_z = \sum_{i=1}^{N} \hat{\sigma}^z{}_{(i)}$ and $\hat{J}_\pm = \sum_{i=1}^{N} \hat{\sigma}^\pm{}_{(i)}$ while $\hat{\sigma}^z{}_{(i)} = |e_{(i)}\rangle\langle e_{(i)}| - |g_{(i)}\rangle\langle g_{(i)}|$ acts on the individual spins that make up the spin coherent state. The fixed term $\frac{\omega_N N}{2}$ is present in such a way that the ground state eigenvalue of the spin coherent state Hamiltonian $\hat{J}_z$ is zero.

The eigenvalues and eigenvectors for the Hamiltonian of the spin coherent state can be found by solving the eigenvalue equation

$$\hat{H}_N|\psi(t)\rangle_N = E|\psi(t)\rangle_N \,. \tag{4}$$

The wavefunction for the interaction system at time t is written as

$$|\Psi(t)\rangle_N = \sum_{n=0}^{N} a_{e,n}(t)|e,n\rangle_N + a_{g,n}(t)|g,n\rangle_N \tag{5}$$

where $t = 0$ is for initial state. The resulting eigenstates will then further be used to find the exact solution by solving the time-dependent Schrödinger equation

$$|\Psi(t)\rangle = e^{-i\hat{H}_N t/\hbar}|\Psi(0)\rangle_N \,. \tag{6}$$

The exact solution of this one qubit spin coherent state system $|\Psi(t)\rangle$ has been obtained and discussed previously in [2,6]. The density matrix, which is a very valuable tool for dealing with multiparticle systems and helps to determine the purity of the state, can be discovered using this time evolution. The reduced density matrix of the qubit system takes the shape of

$$\hat{\rho}_q(t) = \mathrm{Tr}_f(|\Psi(t)\rangle\langle\Psi(t)|) \,. \tag{7}$$

Collapse and Revival of Qubit State Probabilities

One of the interesting events that can be observed in the one qubit and spin coherent state interaction model is the event of 'collapse and revival' of Rabi oscillation in the interaction system. This event occurs when the oscillation of the fluctuations of the probabilities of the qubit state seems to damp out before completely fade after some time [9]. As later time, the oscillation of the qubit state revives at the same frequency but at different amplitude. These activities of collapse, complete disappearance and revival in the oscillation of the qubit state probability occurs periodically, which at longer times we can notice a sequence of collapse and revival events. The event of collapse and revival is depicted as the blue coloured line in Figure 1 and Figure 2.

The initial state for an interaction system between one qubit and spin coherent state is

$$|\Psi(0)\rangle_N = \sum_{n=0}^{N} C_n \left|\frac{\zeta}{\sqrt{N}}\right\rangle_N \left(C_e|e\rangle + C_g|g\rangle\right) \tag{8}$$

where C_n is the coefficient for the spin coherent state which is given in Eq. 2. The probabilities of the qubit being in either ground state or excited state are [2]:

$$P_g(t) = \langle g|\hat{\rho}_q|g\rangle = \frac{N!}{\left(1+\frac{|\zeta|^2}{N}\right)^N} \sum_{n=0}^{N} \frac{1}{(N-n)!\,n!}\left(\frac{\zeta^2}{N}\right)^n \cos^2\left(\lambda\sqrt{(n+1)\left(1-\frac{n}{N}\right)}\,t\right) \tag{9}$$

$$P_e(t) = \langle e|\hat{\rho}_q|e\rangle = \frac{N!}{\left(1+\frac{|\zeta|^2}{N}\right)^N} \sum_{n=0}^{N} \frac{1}{(N-n)!\,n!}\left(\frac{\zeta^2}{N}\right)^n \sin^2\left(\lambda\sqrt{(n+1)\left(1-\frac{n}{N}\right)}\,t\right). \tag{10}$$

The collapse and revival events can be described by the destructive and constructive interference [2]. The collapse and total disappearance happened because of the destructive interference; where there are two different phases of two oscillating terms added together which leads to the cancellation of phase. As the time evolves, the phase difference will become smaller and at later time, the phase of oscillating terms will begin to give a constructive interference. The revival phase has a maximum at revival time t_r, where as simplified in [2] takes the shape of

$$t_r = \frac{2\pi\sqrt{\bar{n}}}{\lambda\left(1-\frac{\bar{n}}{N}\right)} = \frac{2\pi|\zeta|}{\lambda\left(1-\frac{|\zeta|^2}{N}\right)}. \tag{11}$$

Attractor State of Qubit

The appearance of "attractor state" is another intriguing event that can be seen in the one qubit spin coherent state model. Gea-Banachole [8] has shown that at halfway to the revival time t_r, the qubit detaches itself from the field unconditionally, and the qubit state evolves to the attractor state. The formula for this attractor state is

$$\left|\psi_{+,att}\right\rangle_N = \frac{1}{\sqrt{2}}\left(e^{-i\phi}|e\rangle + i|g\rangle\right) \tag{12}$$

where this state depends on the initial phase of the spin coherent state ϕ and independent of the qubit's initial condition. The probability of the qubit being in attractor state is given by

$$P_{+,att}(t) = \left\langle\psi_{+,att}\middle|\hat{\rho}_q(t)\middle|\psi_{+,att}\right\rangle \tag{13}$$

where $\hat{\rho}_q(t)$ is the reduced density matrix stated in Eq. 7. This quantity was plotted as yellow line in Figure 1 and Figure 2.

Attractor state depends on the phase of the initial coherent field state θ as in Eq. 12, but it is independent of the initial qubit state. When the qubit state evolves to attractor state, the information about the initial qubit state is transferred into the spin coherent state, in which now they are in a Schrödinger cat state. It is a coherent superposition of two opposite-phased spin coherent states. [14,15]. This behavior of exchanging information from the qubit to the spin coherent state happens to-and-fro throughout time [9].

Linear Entropy

Erwin Schrödinger had introduced the idea of entanglement [16], and entropy is the tool that is used to quantify it. The results of entropy evaluation include the purity of the quantum state, disruption, or missing information due to insufficient measurement of the system [17], and this formulation is accessible at any moment. The degree of entanglement between a single qubit and a spin coherent state was determined using linear entropy. Linear entropy for this system is described as

$$S_q^L(t) = 1 - \mathrm{Tr}\left(\hat{\rho}_q(t)^2\right) \tag{14}$$

where $\hat{\rho}_q(t)$ is the reduced density matrix stated in Eq. 7. When $S_q^L(t) = 0$, it corresponds to a totally pure state and when $S_q^L(t) = 1$, it refers to a completely mixed state. Non- zero entropy indicates that the reduced density matrix is mixed, or in other words it means that there is some entanglement between the qubit and spin coherent state. Linear entropy was plotted as red line in Figure 1 and Figure 2.

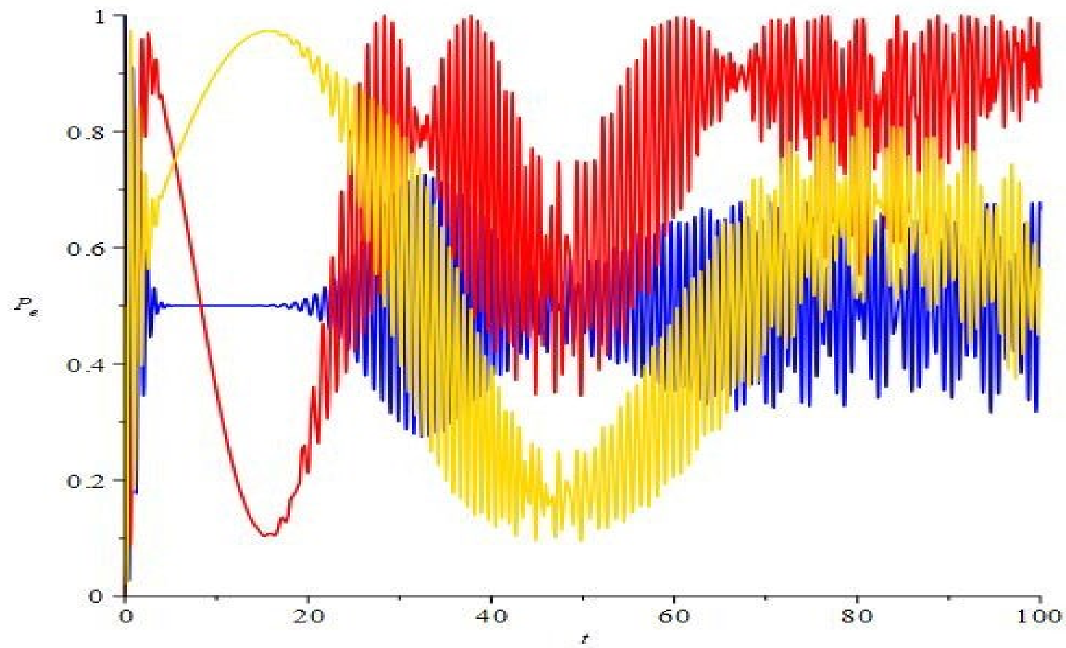


FIGURE 1. One qubit spin coherent state model at zero detuning in which $C_e = 1$, $C_g = 0$, $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$. The blue line indicates the probability of the qubit to be in excited state $|e\rangle$, yellow line depicts the probability of it being in attractor state, and red line shows linear entropy.

Figure 1 depicts the dynamics of one qubit spin coherent state model with zerodetuning. Initially, the qubit is in excited state. On the figure, the yellow line depicts the presence of attractor state. When $t = 0$, the curve is at 0 as well. This tells us that there is an absence of attractor state at the beginning of the interaction. As time evolves, the line slowly evolves towards the maximum value of 1 at $t = 17$ seconds. This exact moment is called as the half revival time $\frac{t_r}{2}$, and the qubit undergoes disentanglement from the spin coherent state, while eventually becomes an attractor state. Afterwards, as there is a coherent superposition of two spin coherent states with opposing phase, the spin coherent state is in Schrödinger's cat state in which it now includes all the information about the initial qubit state. As time goes on, there will be another information swap between the qubit and the spin coherent state, but this time the spin coherent state's information will be sent back to the qubit. This process of transferring information resumes to occur periodically throughout time. The red line represents the linear entropy. The line is getting closer to zero at half revival time $\frac{t_r}{2}$, which shows that there is no entanglement between the qubit and the spin coherent state. The qubit is in a pure state known as attractor state, and it has all information about itself. However, as the system is evolving towards revival time t_r, the curve approaches 1 indicating that there is a presence of entanglement between the qubit and the spin coherent state. The information from the qubit is transferred to the spin coherent state, making the qubit to be in a mixed state.

To simulate the case of practical system where there is always the possibility for an error to occur, we add value to the detuning, δ. Finite detuning occurs when the system interacts with the environment. This interaction causes a finite difference between the frequency of the spin coherent state ω_N and the frequency of the qubit Ω. This difference can be explained by considering two situations in which the qubit is initially in the excited and ground states. When the qubit is initially in excited state, finite detuning occurs when the emitted photon does not release the same energy into the spin coherent state, as compared to the system's lowest possible energy. While when the qubit is initially in ground state, finite detuning happens because the energy of the absorbed photon is not exactly the energy required for the qubit to become excited.

Figure 2 shows the time evolution of one qubit spin coherent state model which begins in the excited state with detuning value, $\delta = 5$. The unique decrease in the entropy for finite detuning is not nearly as noticeable as it was for the prior zero detuning case. The probability of the qubit to be in the attractor state is still close to one, but at the same time decreased again. These distinctions emerge because the system's detuning makes it more challenging to isolate the qubit from the spin coherent state. As a result, the likelihood of the qubit being in an attractor state decreases, and the system remains mixed.

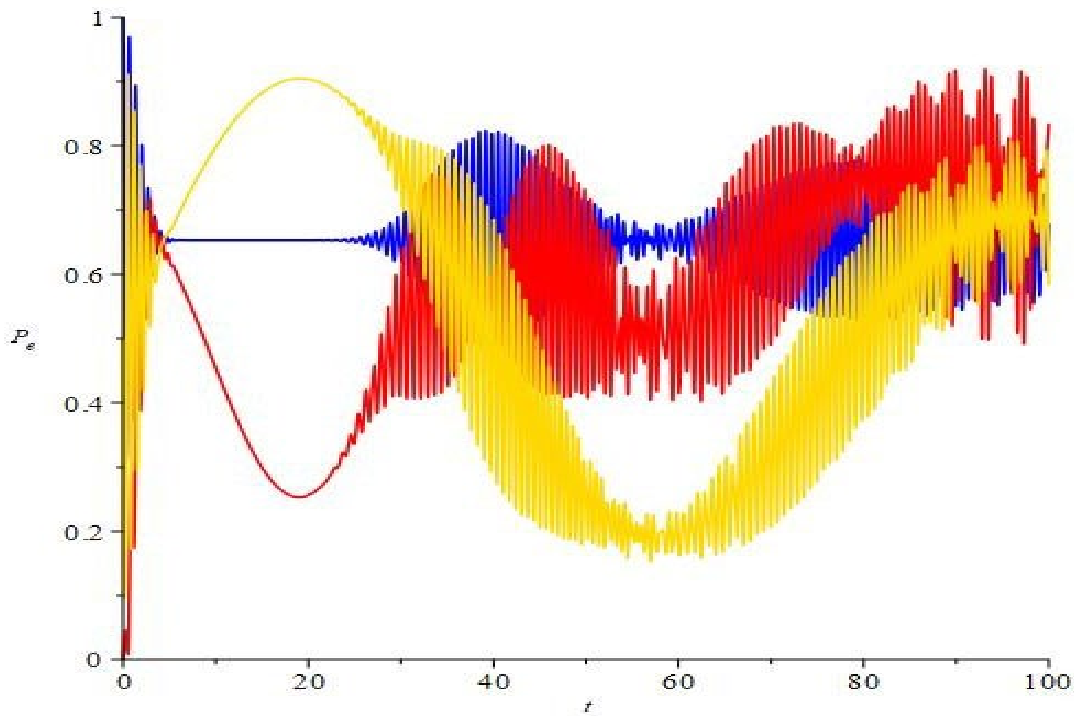


FIGURE 2. One qubit spin coherent state model with value of detuning, $\delta = 5$ in which $C_e = 1$, $C_g = 0$, $N = 100$ and $|\zeta|^2 = 16$. The blue line indicates the probability of the qubit to be in excited state $|e\rangle$, yellow line depicts the probability of it being in attractor state, and red line shows linear entropy.

Majorana Sphere

The Majorana description of a general spin state was first discovered by Ettore Majorana in 1932 [18], and further studies have been made over time since then [19,20]. Majorana sphere geometrically represents the state of arbitrary spin as the number of spin-1/2 systems. This representation is actually a generalized case of Riemann sphere[21]. Majorana representation takes a product state, projects it into the symmetric subspace and all states that correspond to the symmetric subspace is uniquely generated up to the order of β_j where $j = 1,2, \ldots$ and β_j is equivalent to the number of qubits.

The north pole corresponds to $\beta = 0$ and the south pole represents $\beta = \infty$. We can also say that the north pole indicates the excited state $|e\rangle$ and the ground state $|g\rangle$ is located at its south pole. The right most point indicates $\beta = 1$, providing that $(|e\rangle + |g\rangle)/\sqrt{2}$ and the left most point depicts $\beta = -1$, providing that $(|e\rangle - |g\rangle)/\sqrt{2}$. The general state of a single spin-1/2 particle can be written as the linear combination

$$\begin{aligned} |\psi\rangle &= a|e\rangle + b|g\rangle \\ &= \mathcal{N}(|e\rangle + \beta|g\rangle)\,. \end{aligned} \tag{15}$$

The ratio $\beta = b/a$ uniquely describes the state $|\psi\rangle$ and it can take on any value in the complex plane, including infinity. Here, $\mathcal{N}$ is the normalization factor. A point on the Majorana Sphere has the following coordinates

$$x = \frac{2\Re\mathrm{e}(\beta)}{1 + \Re\mathrm{e}(\beta)^2 + \Im\mathrm{m}(\beta)^2} \tag{16}$$

$$y = \frac{2\Im\mathrm{m}(\beta)}{1 + \Re\mathrm{e}(\beta)^2 + \Im\mathrm{m}(\beta)^2} \tag{17}$$

$$z = \frac{1 - \Re\mathrm{e}(\beta)^2 - \Im\mathrm{m}(\beta)^2}{1 + \Re\mathrm{e}(\beta)^2 + \Im\mathrm{m}(\beta)^2}. \tag{18}$$

Plotting a single point might be effortless, but it is not the case if there are multiple points. For this case, a density matrix must be used. The density matrix for a pure one qubit on Majorana sphere is given by

$$\hat{\rho} = \frac{1}{1 + |\beta|^2}\begin{pmatrix} 1 & \beta^* \\ \beta & |\beta|^2 \end{pmatrix} \tag{19}$$

while several combinations of various pure states can be used to represent a single mixed state [9]

$$\hat{\rho}_\beta = \sum_i \frac{P_i}{1 + |\beta_i|^2}\begin{pmatrix} 1 & {\beta_i}^* \\ \beta_i & |\beta_i|^2 \end{pmatrix} \tag{20}$$

where P_i is a probability which means that the same density matrix $\hat{\rho}$ can be represented in many different bases. In this study, the exact solution of the one qubit – spin coherent state model is used to find the associate density matrix to the respective Majorana Sphere plotting. Then the density matrix will be used to plot the time evolution of the qubit state probabilities of the system in which after setting the basis in terms of $|e, n\rangle_N$ and $|g, n\rangle_N$, the β value is given as below:

$$\beta = \sum_{n=0}^{N-1}\left(C_g C_n \cos\left(\frac{t}{2}\mu_n(\delta)\right) + i\sin\left(\frac{t}{2}\mu_n(\delta)\right) + i\sin\left(\frac{t}{2}\mu_n(\delta)\right)\right)$$

$$\left(C_g C_n(\cos^2\phi_n - \sin^2\phi_n) - 2C_e C_{n-1}\cos\phi_n \sin\phi_n\right)\Big) \;/$$

$$\left(C_e C_n \cos\left(\frac{t}{2}\mu_n(\delta)\right) + i\sin\left(\frac{t}{2}\mu_n(\delta)\right)\right.$$

$$\left.\left(C_e C_n(\sin^2\phi_n - \cos^2\phi_n) - 2C_g C_{n+1}\cos\phi_n \sin\phi_n\right)\right) \tag{21}$$

where

$$\mu_n(\delta) = \sqrt{\delta^2 + 4\lambda^2(n+1)\left(1 - \frac{n}{N}\right)} \tag{22}$$

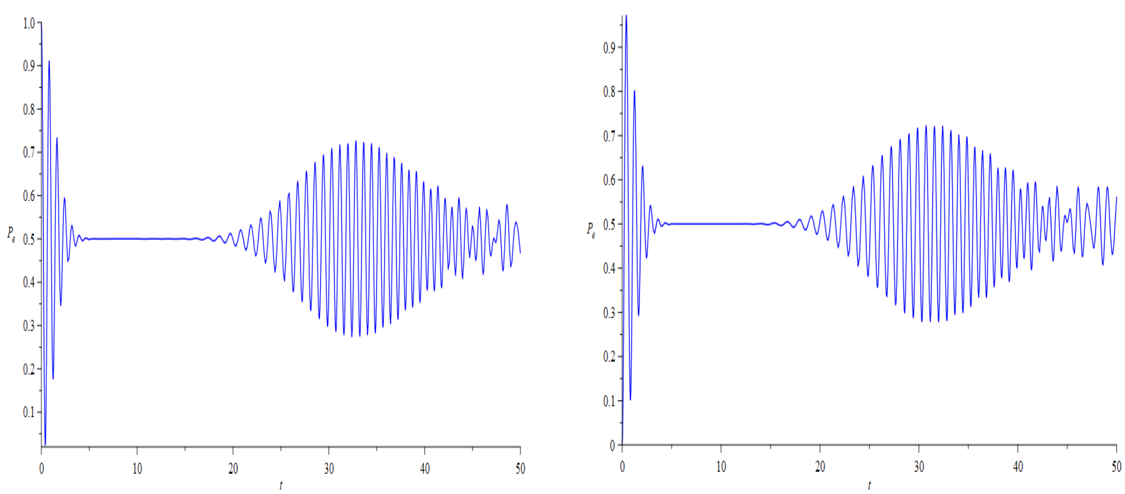
and $\delta = \Omega - \omega_N$ is the detuning between the frequency of the qubit and the frequency of the spin coherent state.

RESULTS AND DISCUSSION

One Qubit Spin Coherent State Model in Majorana Sphere

Case of Zero Detuning

In an ideal case of zero detuning, the frequencies of a system with one qubit Ω and spin coherent state ω_N are equal. Figure 3 depicts the event of 'collapse and revival' of Rabi oscillation for the one qubit spin coherent state model at time $t = 0$ until $t = 50$ seconds for states that are initially in excited and ground states. The y-axis represents the probability of the qubit being in the excited state while the x-axis represents time in seconds.



$(\boldsymbol{a})\ \boldsymbol{C_e = 1,\ C_g = 0}$ $(\boldsymbol{b})\ \boldsymbol{C_e = 0,\ C_g = 1}$

FIGURE 3. The probability of the qubit being in initially (a) excited state, (b) ground state with $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$.

The dynamics of the qubit state probabilities in Figure 3(a) was chosen and transformed into the Majorana sphere. The Majorana sphere representation of the one qubit spin coherent state model that is initially in the excited state at zero detuning is shown in Figure 4 for one complete oscillation. The locations of the points define the purity of the qubit states, and their evolution throughout time indicate the dynamics of qubit state probabilities. The time evolution of the qubit state probabilities is divided into four periods for better analyzation.

Figure 4(a) shows the first period which is in between $t = 0$ and $t = 5$ seconds. The point starts at the top of Majorana sphere at $(0,0,1)$, indicating that the qubit contains complete details about its initial excited state. As the system evolves, the qubit slowly entangles with the spin coherent state and this is indicated by a swirl that regressively moves towards the centre of the sphere. There is an exchange of information between the qubit and spin coherent state, causing the qubit to gradually become a mixed state. After some time, the qubit state probabilities collapse and this can be viewed from the swirl ending and becoming a constant at $z = 0$.

Figure 4(b) depicts a later period of $t = 5$ seconds until $t = 15$ seconds, whereby the previous constant points are now getting closer the surface of the sphere at $(0,1,0)$. This signifies that the qubit is evolving into a pure state and it is regaining its complete information about its initial state. This phenomenon is known as the disentanglement of the qubit from the spin coherent state, and the pure state that is associated with the qubit at this period is known as the attractor state.

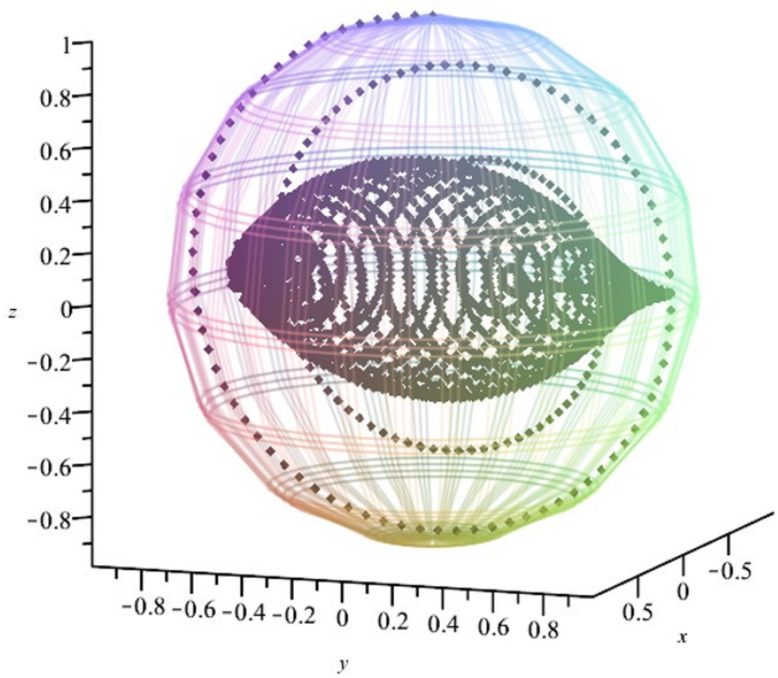
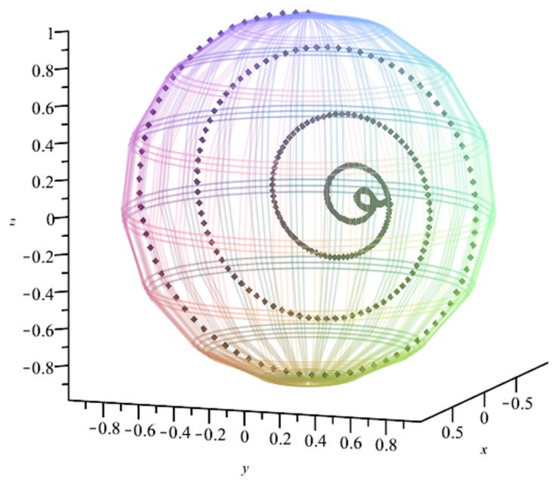
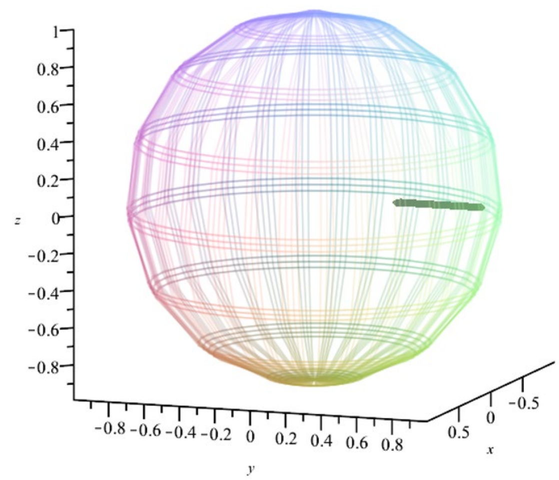


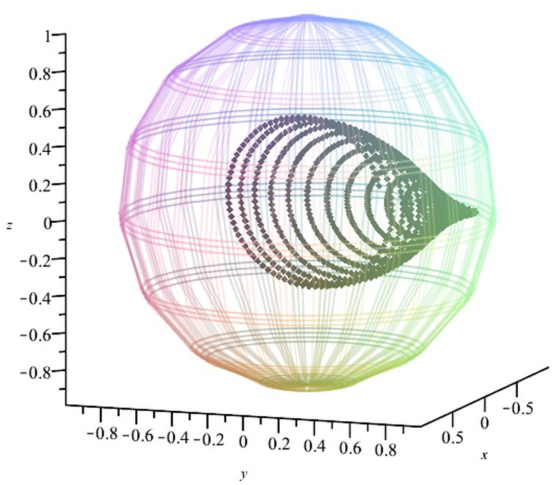
FIGURE 4. Majorana sphere of one qubit spin coherent state model for the case of zero detuning in which $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$. The qubit is originally in excited state and the period is from $0 \leq t \leq 60$ seconds. This complete oscillation is divided into four periods (a), (b), (c), and (d) as below.



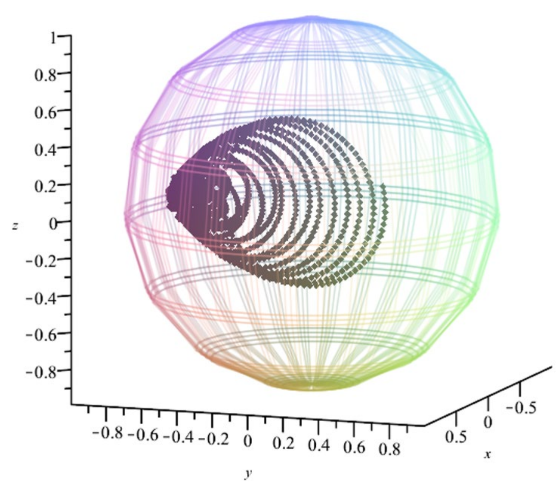
(a) Period $0 \leq t \leq 5$ seconds.



(b) Period $5 \leq t \leq 15$ seconds.



(c) Period $15 \leq t \leq 32$ seconds.



(d) Period $32 \leq t \leq 50$ seconds.

Figure 4(c) shows another period of time in between $t = 15$ seconds and $t = 32$ seconds. There is a progressive swirl from the point on the surface of the sphere towards its centre, which indicates that the qubit is once again in an entanglement with the spin coherent state. Comparing with Figure 4(a), there are now more oscillations or swirls in this period. This distinction illustrates that the qubit system is getting more mixed than the previous entanglement. The dynamics in this period is known as the revival in qubit state probabilities.

The second collapse occurs afterwards during period $t = 32$ seconds to $t = 50$ seconds and it is displayed by Figure 4(d). However, the collapse this time shows that the points regressively swirl to the locationof the second attractor state which is at $(0, -1, 0)$. Periodically, the 'collapse and revival' of qubit state probabilities continue to occur throughout time, and the number of each revival oscillations increases for each time of successive revival.

Case of Finite Detuning

The probabilities of a single qubit spin coherent state model being initially in the excited state for various detuning value δ is shown in Figure 5. This setting can be attained by fixing $C_e = 1$ and

$C_g = 0$ for different values of δ from Eq. 19. The figure shows that the revival peaks and the average for the qubit state probabilities to be in the excited state increase together with the increment in the value of detuning δ. This behavior displays the high likeliness of the qubit to stay in its initial state for the case of high detuning. Also, it is evident that the revival time for high detuning system such as $\delta = 25$ and $\delta = 50$ take way longer time to achieve in comparison with the rest.

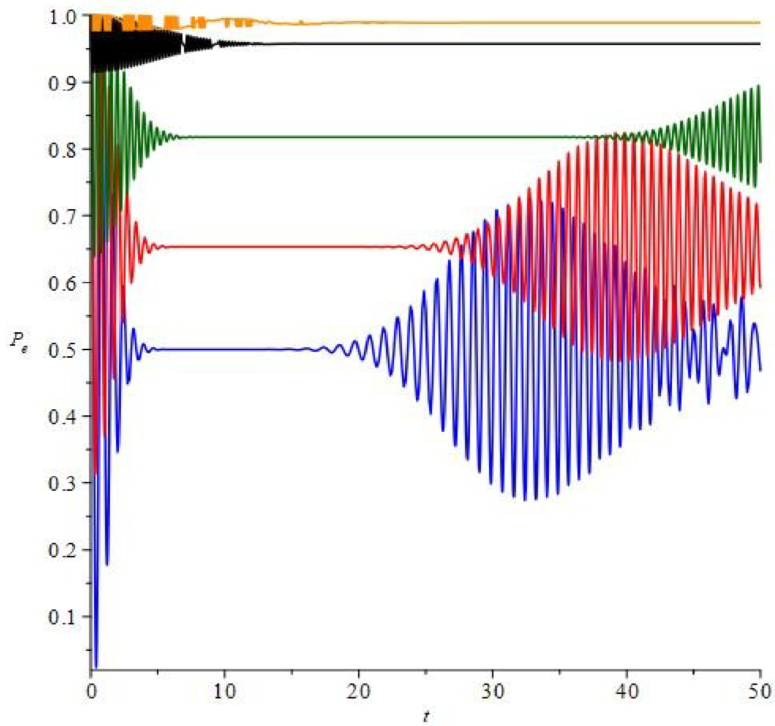


FIGURE 5. Probabilities of one qubit spin coherent state model being in excited state in which $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$ with different value of detuning δ. The blue line shows for $\delta = 0$, red $\delta = 5$, green $\delta = 10$, black $\delta = 25$ and indigo $\delta = 50$.

Figure 6 shows Majorana sphere representation for the one qubit spin coherent state model with $\delta = 5$. The qubit is initially in the excited state and the system evolves from $t = 0$ until $t = 60$ seconds to let the system evolves for one complete oscillation. The point starts off at the north pole of the Majorana sphere at (0,0,1) which signifies that the qubit is initially in its excited state. Then, the points swirl towards the negative side of y-axis and continue to make a spiral until it eventually disappears at $z = 0.3$. This phenomenon is known as the collapse in the qubit state probabilities. Then, a constant horizontal swirl can be seen as in Figure 6(a) and this indicates that the qubit is in a mixed state. Afterwards, a spiral is developed and this shows that the qubit state probabilities is reviving. At later time, the qubit state probabilities collapse once again and this phenomena of 'collapse and revival' occur regularly as time evolves.

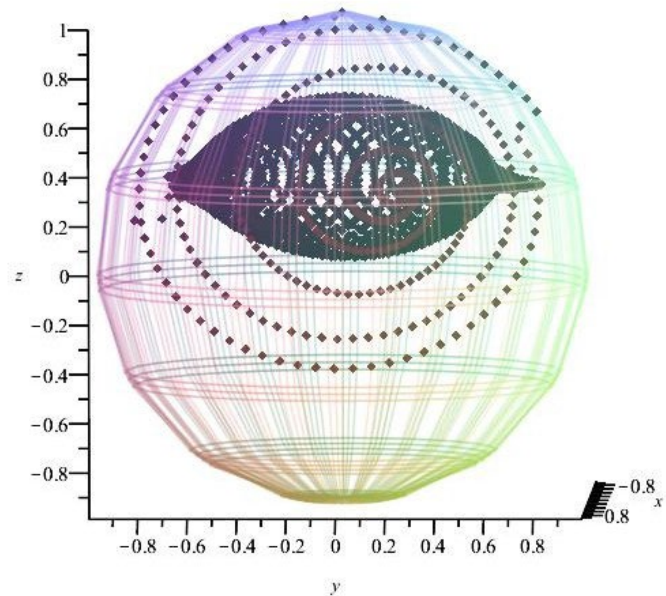
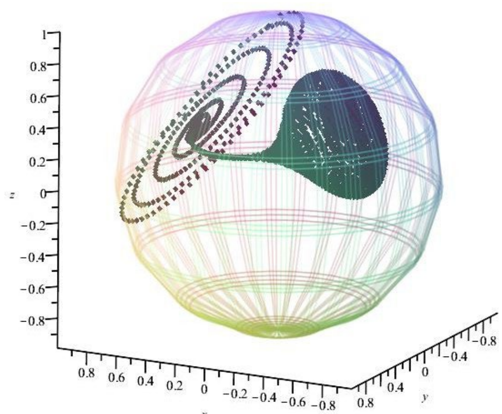
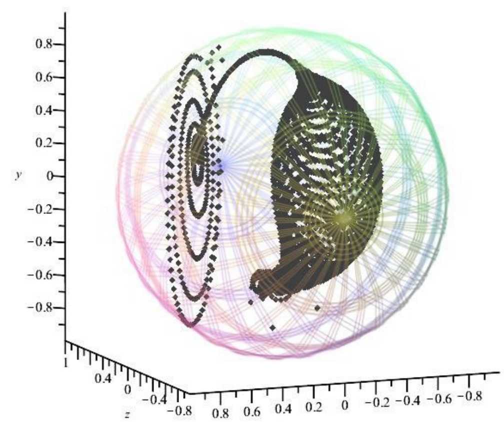


FIGURE 6. Majorana sphere representation for one qubit spin coherent state model, $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$ with value of detuning, $\delta = 5$ from $0 \leq t \leq 60$ seconds. The qubit system is initially in excited state and shown in two different angles (a) and (b) as below.



(a) Rotated horizontally.



(b) Rotated vertically.

The shape made by the trajectory closely resembles Figure 4, but for this case it is somehow more squeezed towards the excited level of the sphere, indicating that it is hard for the qubit to be in its ground state. This happens because the qubit that is initially in the excited state does not liberate enough energy to the spin coherent state, making it more likely to stay in its excited state over time. As viewed in Figure 6(a) and Figure 6(b), the trajectory also does not lie flat vertically as it now evolves through x-axis and y-axis. Figure 6 shows that the points are in a good distance inside the sphere, indicating that the qubit never reaches the attractor state. This tells us that the qubit and the spin coherent state hardly undergo maximal disentanglement, causing the qubit to always stay in a mixed state after $t = 0$. For the earlier zero detuning case however, the single qubit achieves attractor state twice in the span of 50 seconds.

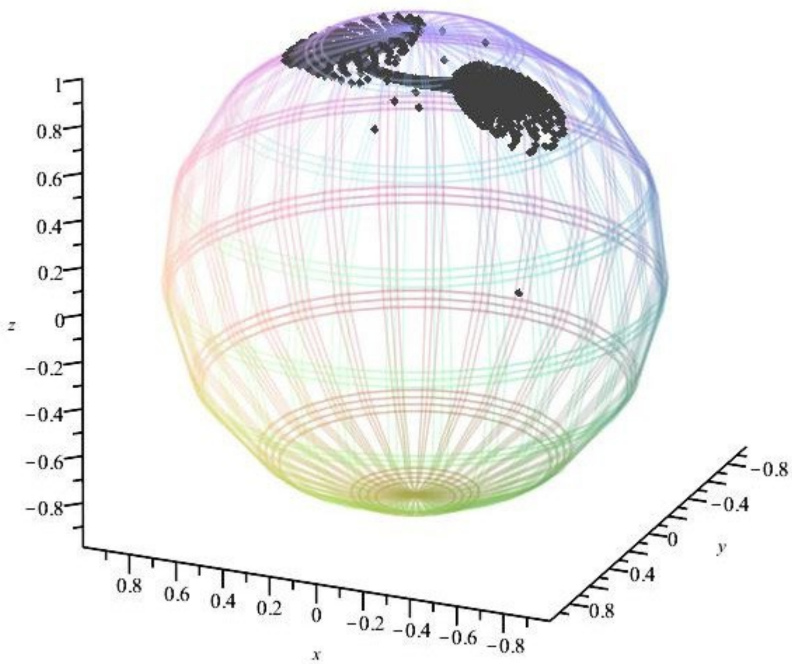
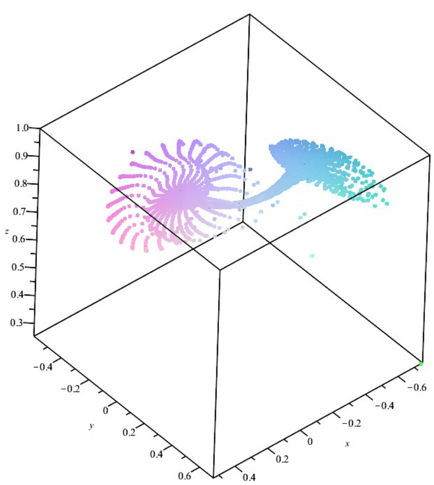
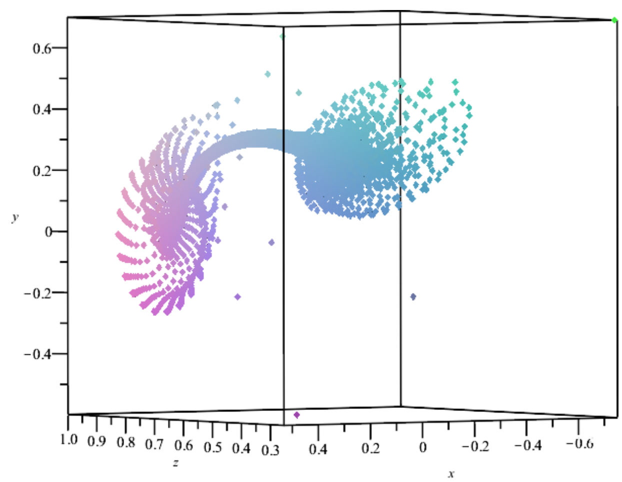


FIGURE 7. Majorana sphere representation for one qubit spin coherent state model, $\lambda = 1$, $N = 100$ and $|\zeta|^2 = 16$ with value of detuning, $\delta = 25$ from $0 \leq t \leq 130$ seconds. The qubit system is initially in excited state and shown in two different angles (a) and (b) in lowered scale as below.



(a) Rotated horizontally.



(b) Rotated vertically.

Figure 7 displays the time evolution of one qubit – spin coherent state model that is set to be initially in the excited state for a higher value of detuning $\delta = 25$, from period $t = 0$ until $t = 130$ seconds. As the revival time is now prolonged, the period is also chosen to be longer so that the dynamics of the qubit state probabilities can have one complete oscillation. Figure 7 displays even stranger pattern in comparison with the earlier detuning cases. The qubit state probabilities do not occupy the lower half of the Majorana sphere, indicating that the qubit never reaches its ground state as time evolves.This unusual behavior happens because the large detuning value prevents the energy released due to photon emission into the spin coherent state from bringing the qubit down to its ground state. Hence, it is more likely for the qubit to always stay in its excited state.

The explanation on the evolution of qubit state probabilities for high detuning cases is more obvious compared to the earlier detuning values. Figure 7(a) shows that the point starts at (0,0,1) and this indicates that the qubit is originally in its excited state. The points then turn into a spiral, gradually fade away and remain in a horizontal trajectory. During this time, the phenomenon described by these dynamics is called as the collapse in the qubit state probabilities and the constant horizontal line indicates that the qubit is in a mixed state. After some time, the qubit state probabilities start to revive and this is illustrated by the development of a new spiral that is located opposite to the initial spiral. A longer time is needed to observe the next 'collapse and revival' event as higher detuning increases the revival time in the qubit state probabilities.

CONCLUSION

The extension from the Jaynes-Cummings model leads to the discovery of the spin coherent state model, which allows us to study the fundamentals of quantum information processing. We have learnt the interaction system between one qubit and spin coherent state. In this research, we emphasized the event of 'collapse and revival' of the Rabi oscillation because its transformation into Majorana sphere alone had displayed interesting quantum phenomena, noticeably the presence of attractor state and Schrödinger's cat states as well as entanglement between the qubit and the spin coherent state.

As for the case of zero detuning, the dynamics of the qubit state probabilities start off at the north pole of Majorana sphere, which agrees with our initialization of the single qubit to be in excited state. Initially, the qubit is in pure state as the point is located at the surface of the sphere, but after $t = 0$ it gradually moves towards the centre of the sphere. As a result of entanglement with the spin coherent state, the qubit is now in a mixed state. This process is known as the collapse in qubit state probabilities. Then, the point moves from inside to the surface of the sphere and this indicates that the qubit is disentangling from the spin coherent state. As the point is moving towards the surface of the sphere, it is becoming a pure state that is known as attractor state. This event is called as the revival in qubit state probabilities and this series of 'collapse and revival' in Rabi oscillation occurs periodically at later times.

In the case of finite detuning, the qubit is again initially in excited state and this setting is depicted by the point that is positioned at the coordinate (0,0,1) on the Majorana sphere. Different than zero detuning case, presence of detuning causes the qubit state probabilities to likely remain in its initial excited state. This is displayed on Majorana sphere in which the trajectory for finite detuning is more squeezed towards the top compared to the zero detuning case. As the value of detuning gets higher, the trajectory only resides on the excited level of the sphere. Furthermore, the qubit is now hardly becoming attractor state as the point never reaches the surface of the sphere after $t = 0$. This means that it remains mixed and entangled with the spin coherent state. Also, adding detuning to the interaction system cause thepoints to make an additional dimension, as they now span on the x, y and z axes.

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