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Irregular Total Labeling On Complete Bipartite Graph And Union Of Complete Bipartite Graphs

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Abstract

Given a graph $G = (V, E)$ with vertex set V and edge set E we define a labeling as a function, where weight of edge xy is written as $w(x, y)$. Total labeling is the sum of xy label and labeled of the vertices that incident to x, thus $w(x, y) = \lambda(x) + \lambda(y)\lambda(xy)$. λ labeling is called as edge irregular total k-labeling of the graph G if for every two different edges e and f of G then $w(e) \neq w(f)$. The smallest k in which graph G can be labeled as edge irregular total k-labeling is called as edge total irregularity strength and is noted as tes (G). In this research we are interested in finding tes (G) of the union of complete bipartite graphs K_1,q and $K_{2,q}$

Keywords: *irregular total labeling, tes (G), bipartite graph*

1. Introduction

Graph is one of mathematical branches that can be used to represent many problem in daily life. After proposing by Euler in 1736, graph emerges as one of important tools that used with others knowledge such as in chemistry, biology, and mostly in mathematics, especially operations research. Some example of graphs applications include: to design of telecommunication networks, to design of energy networks, to represent the molecules structures, to represent the DNA structures, and so on.

One of the topics in graphs is labeling. The labeling of graph was investigated in the first time by Sadlacek in 1963. Some of the labeling techniques that had been investigated such as : Gracefull labeling (Ringel and Liado, 1996), Harmony labeling (Graham and Sloane,1980), Magic labeling (Kotzig and Rosa,1970; Stewart,1996), and anti magic labeling (Baca et al,2000).

In 2001, Baca et al (in Asmiati, 2002), introduced a new type of labeling, named as edge irregular total labeling which consists of edge labeling and vertex labeling.

For a graph $G = (V, E)$ with vertex set V and edge set E we define a labeling as a function. The weight of edge xy is written as $W(x, y)$. Total labeling is the sum of xy label and labeled of the vertices that incident to xy. Thus $w(x, y) = \lambda(x) + \lambda(y)\lambda(xy)$.

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1. Introduction

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λ labeling is called as edge irregular total k-labeling of the graph G if for every two different edges e and f of G then $w(e) \neq w(f)$. The smallest k in which Graph G can be labeled as edge irregular total k-labeling is called as edge total irregularity strength and is noted as tes (G).
One interesting fact about the labeling is that every graph G can be labeled by edge irregular total k-labeling. But, to find the tes (G) is not that easy as finding the labeling. Therefore in this research we try to find the tes (G) of some special graphs which are: the union of complete bipartite graphs $K_{1,q}$ and $K_{2,q}$.

2. Some previous results in literature.

Theorem (Baca et al, in Asmiati (2002)):

If G =(V,E) is a graph, then $\left\lceil \frac{|E(G)+2|}{3} \right\rceil \leq \text{tes}(G) \leq |E(G)|$

Proof:

Since $w(e) = \lambda(a) + \lambda(b) + \lambda(e)$ where a = x, b = y, e = xy, and $w(e) \geq 3$,
then $w(e) \geq |E(G)| + 2$

Notice that tes (G) $\geq$ maks $\{\lambda(a), \lambda(b), \lambda(e)\} \geq \frac{w(e)}{3} \geq \frac{E(G)+2}{3}$

Therefore $\left\lceil \frac{|E(G)+2|}{3} \right\rceil \leq \text{tes}(G)$

To find the upper bound, choose $\lambda(x) = 1$, $\forall x \in V(G)$ and give every edge label start from 1 until $|E(G)|$. This labeling procedure makes every edge has a different weight and therefore $\text{tes}(G) \leq |E(G)|$ Theorem 1 of Baca et al gives the upper bound and lower bound of tes(G) for every graph, and also give procedure on labeling construction.

Theorem ((Baca et al, in Asmiati (2002)):

If G is a complete bipartite graph $K_{1,q}$, then tes $(K_{1,q}) = \left\lceil \frac{q+1}{2} \right\rceil$

Proof:

Complete bipartite graph $K_{1,q}$ has one centre vertex which is vertex with degree q where $|V(k_1, q)| = q + 1$ and $|E(k_1, q)| = q$.

Suppose that $V(k,q) = \{x_1^1, y_i^1, y_i^2, \ldots, y_i^q\}$ and $E(k,q) = \{x_i^1 y_i^1, x_i^1 y_i^2, \ldots, x_i^1 y_i^q\}$
Give label vertices and edges with the following procedure:

$$\lambda(x_1^1) = 1 \,,\ \lambda(y_1^j) = \left\lfloor \frac{j+1}{2} \right\rfloor,\ \lambda(x_1^1 y_1^j) = \left\lceil \frac{j+1}{2} \right\rceil,\ j = 1,2,3,\ldots,q.$$

Then get $w(x_1^1 y_1^j) = 2 + j$ and it can be easily seen that the weight of the edges starts from 3 to q+2.

Therefore tes $(K_{1,q}) = \left\lceil \frac{q+1}{2} \right\rceil$

Theorem (Asmiati, 2002) :

If G is a complete bipartite graph $K_{2,q}$, then tes $(K_{2,q}) = \left\lceil \frac{2q+2}{3} \right\rceil$

3. Tes (G) for complete bipartite graph, and union of complete bipartite graphs

Result 1:

If G is the union of complete bipartite graph $m(K_{1,q})$, $m \in Z^+$, then
tes (G) = $\left\lceil \frac{mq+2}{3} \right\rceil$; $\forall\, m \geq 2, m \in Z^+$.

Proof :
The union of complete bipartite graph $m(K_{1,q})$ is a disconnected graph which consists of m components, where every component is a complete bipartite graph $K_{1,q}$. The $m(K_{1,q})$ graph has the sets of points :

$V = X_1 \cup X_2 \cup \ldots \cup X_m \cup Y_1 \cup Y_2 \cup \ldots \cup Y_m$
$X_i = x_i^1$, $Y_i = \{ y_i^1, y_i^2, \ldots, y_i^q \}$
and the set of edges $E = E_1 \cup E_2 \cup \ldots \cup E_m$,
$E_i = \{ x_i^1 y_i^1, x_i^1 y_i^2, \ldots, x_i^1 y_i^q \}$, i = 1,2,...m.

Give label vertices and edges of the union of complete bipartite graph $m(K_{1,q})$) using the following procedure:

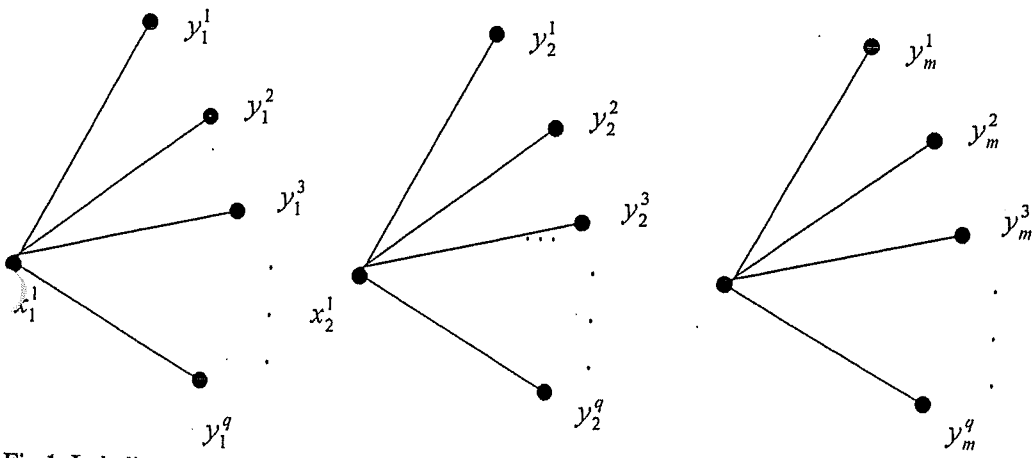

Fig 1. Labeling construction for $m(K_{1,q})$

$$\lambda(x_1^1) = \left\lceil \frac{mq+2}{3} \right\rceil, m > 1, \ i = 1,2,3,\ldots m$$

$$\lambda(y_i^j) = \left\lceil \frac{mq+2}{2} \right\rceil + \left\lceil \frac{j}{2} \right\rceil - \left\lceil \frac{q}{2} \right\rceil, \ j = 1,2,3,\ldots,q, \ m > 1 ; \ i = 1,2,3,\ldots,m$$

$$\lambda(x_i^1 y_i^j) = \begin{cases} \left\lceil \frac{mq+2}{3} \right\rceil + \left\lceil \frac{j+1}{2} \right\rceil - \left\lceil \frac{q+1}{2} \right\rceil, j = 1,2,\ldots,q; m > 1; i = 1,2,3,\ldots,m, \text{ m or q are multiplication of 3} \\ \left\lfloor \frac{mq+2}{3} \right\rfloor + \left\lceil \frac{j+1}{2} \right\rceil - \left\lceil \frac{q}{2} \right\rceil, j = 1,2,\ldots,q, m > 1, i = 1,2,3,\ldots,m, \text{ m or q are not multiplication of 3.} \end{cases}$$

Thus, we get $w(x_i^1 y_i^j) = q\,(m-1) + 2 + j$, and it can be easily seen that the weight of the edges star from 3 to mq + 2. That is clear that λ is the irregular total labeling of $m(K_{1,q})$ with tes $m(K_{1,q})$ $\left\lceil \frac{mq+2}{3} \right\rceil$.

Below we give an example of labeling for the union of complete bipartite graph 2 (K_1, 7):

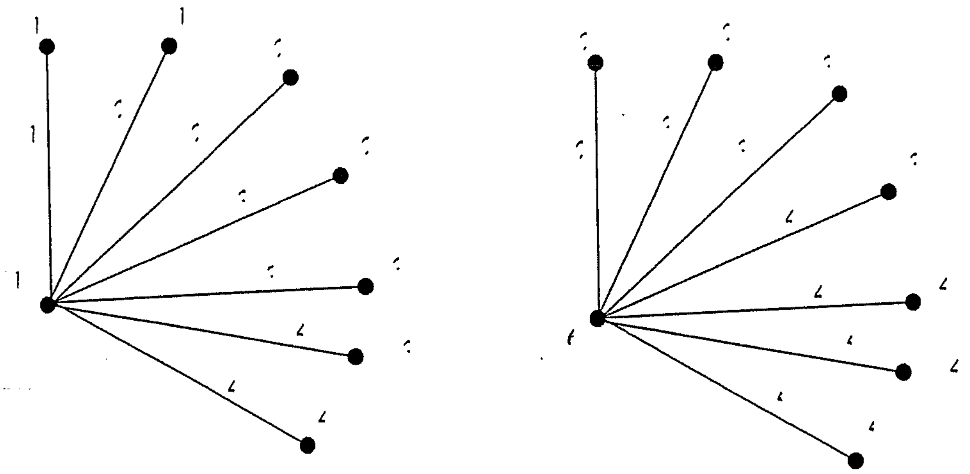

Fig 2. Example for Labeling construction for $2(K_{1,7})$

Result 2 :
If G is the union of complete bipartite graph m ($K_{2,q}$), m ∈ Z^+, then

$$\text{tes}\,(G) = \left\lceil \frac{2mq+2}{3} \right\rceil; \forall\, m \geq 2, m \in Z^+.$$

Proof:

The union of complete bipartite graph m ($K_{2,q}$) is a disconnected graph which consists of m components, where every component is a complete bipartite graph $K_{2,q}$. The $m(K_{2,q})$ graph has the sets of points :

$V = X_1 \cup X_2 \cup \ldots \cup X_m \cup Y_1 \cup Y_2 \cup \ldots \cup Y_m$
$X_i = \{x_i^1, x_i^2\}$, $Y_i = \{y_i^1, y_i^2, \ldots, y_i^q\}$
and the set of edges $E = E_1 \cup E_2 \cup \ldots \cup E_m$,
$E_i = \{x_i^1 y_i^1, x_i^1 y_i^2, \ldots, x_i^1 y_i^q, x_i^2 y_i^1, x_i^2 y_i^2, \ldots, x_i^2 y_i^q\}$, i = 1,2,...m.

Give label every edge of $K_{2,q}$ as done by Asmiati (2002). Next, the vertices and the edges of the union of complete bipartite graph $m(K_{2,q})$ can be labeled by the following procedure :

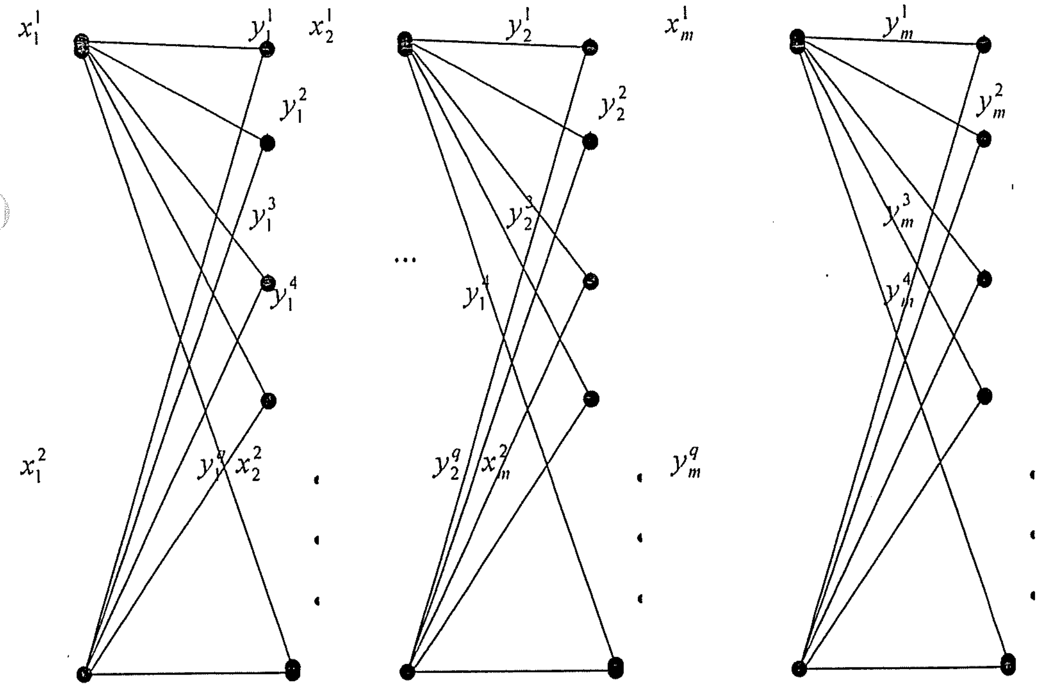

Fig 3. Labeling construction for $m(K_{2,q})$

$$\lambda(x_i^1) = \left\lceil \frac{2mq+2}{3} \right\rceil - 1, m > 1, \ i = 2,3,\ldots m$$

$$\lambda(x_i^2) = \left\lceil \frac{2mq+2}{3} \right\rceil$$

$$\lambda(y_i^j) = \begin{cases} \left\lceil \frac{2mq+2}{3} \right\rceil + j - q; \ 1 \le j \le q, \ m > 1; i = 2,3,\ldots, \text{ m, m or q are multiplica tion of 3} \\ \left\lfloor \frac{2mq+2}{3} \right\rfloor + j - q, 1 \le j \le q, \ m > 1; i = 2,3,\ldots, \text{ m, m or q are not multiplica tion of 3.} \end{cases}$$

$$\lambda(x_i^1 y_i^j) = \left\lfloor \frac{2mq+2}{3} \right\rfloor - q + 1; m > 1, i = 2,3,\ldots m$$

$$\lambda(x_i^2 y_i^j) = \left\lfloor \frac{2mq+2}{3} \right\rfloor ; m > 1, i = 2,3,\ldots m$$

Thus we get $w(x_i^1 y_i^j) = 2q(m-1) + 2 + j$ and $w(x_i^2 y_i^j) = q(2m-1) + 2 + j$

It can be easily seen that the weight of the edges starts from 3 to $2mq + 2$, thus that is clear that λ is the irregular total labeling of $m(K_{2,q})$ with tes $m(K_{12,q})$ is $\left\lceil \frac{2mq+2}{3} \right\rceil$.

Conclusion

The irregular total edge labeling for union bipartite graphs $m(K_{1,q})$ is $\left\lceil \frac{mq+2}{3} \right\rceil$; and for $m(K_{2,q})$ is $\left\lceil \frac{2mq+2}{3} \right\rceil$, $\forall\ m \geq 2$, where $m \in Z^{+}$.

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