Assessment of shell theories for cross-ply and angle-ply laminated cylindrical shells under thermo-mechanical loads

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Abstract

- Assessment of 2D shell theories is presented for analysis of simply supported anisotropic cylindrical shells under mechanical and thermal load.
- Classical lamination theory (CST), first order shear deformation theory (FSDT), and a new higher order shear deformation theory (HSDT13).
- Navier type analytical solution are obtained for both angle-ply and cross-ply cylindrical shells problems.
- Findings reflect the limitations of CST and FSDT. Results obtained clearly shows the importance of higher order terms in HSDT13.

Introduction

High speed aerospace vehicles

- Made of laminated composite material
- Exposed to high temperature condition due to aerodynamic heating
- Thus, analysis of such structures under thermal and mechanical loads are required

Introduction – cont.

- 3D elasticity solutions had been proven to provide very accurate results for the analysis. But such solutions are limited only to certain loading cases and boundary conditions. (Ren, 1987 & 1989) (Bhaskar and Varadan, 1993)
- 2D theories are used to overcome the limitations of 3D elasticity solutions.
- CST are based on Love-Kirchhoff assumptions such as neglecting transverse shear strain. Thus, it may be valid for only thin shell laminate.
- FSDT are developed to include transverse shear deformation with assumption of the strain is uniform throughout the thickness. Shear correction factor is required to provide accurate results. (Whitney and Sun, 1974)

Introduction – cont.

- To overcome the FSDT limitations, HSDT have been developed. (Reddy and Liu, 1985)
- HSDT employed parabolic distribution for the displacement field and no shear correction factors are required.
- Later, HSDT with zig zag theories was introduced to improved the accuracy of the in-plane response. (Bhaskar and Varadan, 1991).
- All the above theories was developed for mechanical loading and later been extended to thermal loading.

Introduction – cont.

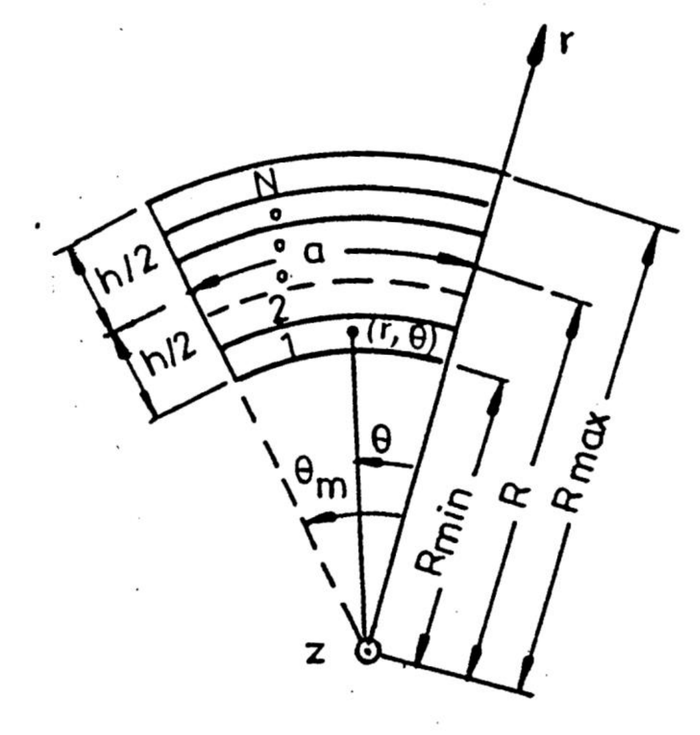
- 2D shell theories accuracy can be assessed by validating its against 3D elasticity benchmark solutions.
- For thermal analysis, such benchmark solution were not available until papers by Saleh et al (2015&2016) were published.
- Hence, all the available theories were not validated against 3D elasticity solutions.
- Furthermore, all the above theories were reported only for cross ply layups.
- In this work, detail assessment of 2D theories were made to determine the range of the applicability of these 2D models.

Formulation – Geometric definition

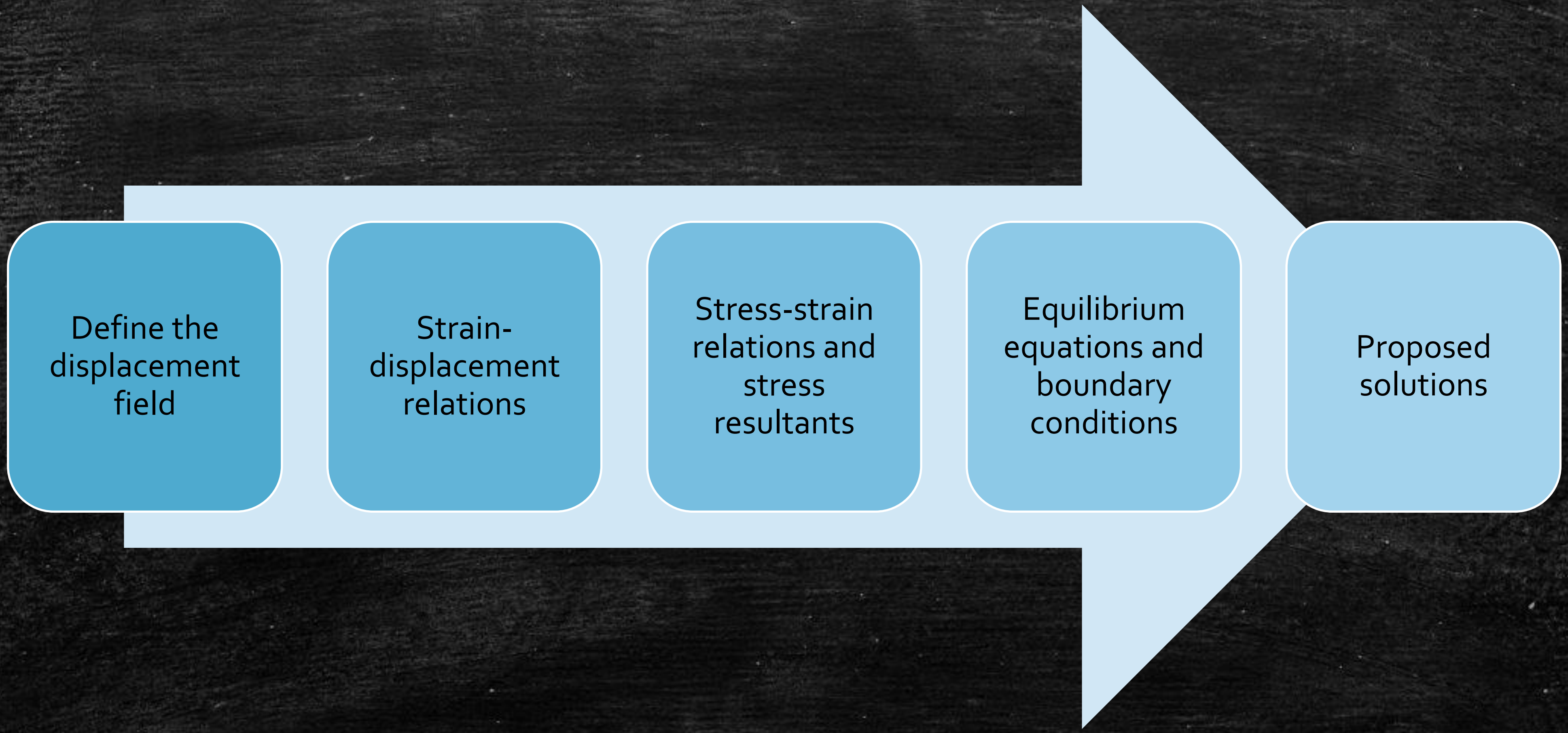
- Consider an infinitely long laminated cylindrical panel as in the figure.
- The coordinate are as follow :

$$0 \leq x \leq \infty \,,\; 0 \leq \theta \leq \theta_m \text{ and} - h/2 \leq z \leq h/2.$$

- The mean radius, R and load are uniform along the x axis; thus, shell undergoes cylindrical bending $(\varepsilon_x = 0)$.



Formulation – Flow chart



Formulation – Definition of displacement field

Theories	Displacement field
CST	$u(x,\theta) = u_0(x,\theta) - z\,\partial w/\partial x$ $v(x,\theta) = v_0(x,\theta) - z\,\partial w/\partial\theta$ $w(x,\theta) = w_0(x,\theta)$
HSDT	$u(x,\theta) = u_0(x,\theta) + z\emptyset_x$ $v(x,\theta) = v_0(x,\theta) + z\emptyset_\theta$ $w(x,\theta) = w_0(x,\theta)$
HSDT 9	$u(x,\theta,z) = u_o(x,\theta) + z\emptyset_x(x,\theta) + z^2\,\lambda_x(x,\theta) + z^3\,\zeta_x(x,\theta)$ $v(x,\theta,z) = v_o(x,\theta) + z\emptyset_\theta(x,\theta) + z^2\,\lambda_\theta(x,\theta) + z^3\,\zeta_\theta(x,\theta)$ $w(x,\theta,z) = w_o(x,\theta)$
HSDT 11	$u(x,\theta,z) = u_o(x,\theta) + z\emptyset_x(x,\theta) + z^2\,\lambda_x(x,\theta) + z^3\,\zeta_x(x,\theta) + \psi_k S_x$ $v(x,\theta,z) = v_o(x,\theta) + z\emptyset_\theta(x,\theta) + z^2\,\lambda_\theta(x,\theta) + z^3\,\zeta_\theta(x,\theta) + \psi_k S_\theta$ $w(x,\theta,z) = w_o(x,\theta)$
HSDT 13	$u(x,\theta,z) = u_o(x,\theta) + z\emptyset_x(x,\theta) + z^2\,\lambda_x(x,\theta) + z^3\,\zeta_x(x,\theta) + \psi_k S_x$ $v(x,\theta,z) = v_o(x,\theta) + z\emptyset_\theta(x,\theta) + z^2\,\lambda_\theta(x,\theta) + z^3\,\zeta_\theta(x,\theta) + \psi_k S_\theta$ $w(x,\theta,z) = w_o(x,\theta) + zw_1(x,\theta) + z^2\,\Gamma(x,\theta)$

- $u(x,\theta)$, $v(x,\theta)$ and $w(x,\theta)$ are the displacements at any point.
- ϕ_x and ϕ_θ are the rotations of the normal to the middle plane about *x* and θ respectively.
- $\lambda_x, \lambda_\theta, \zeta_x, \zeta_\theta, w_1$ and Γ are unknown higher-order terms.
- $\psi_k = 2(-1)^k z_k / h_k$

Formulation – Strain-Displacement relations

- $\epsilon_x = (\partial u/\partial x)$
- $\epsilon_\theta = (\partial v/\partial \theta + w_0)/R$
- $\epsilon_z = (\partial w/\partial z)$
- $\gamma_{\theta z} = \partial v/\partial z + (\partial w/\partial \theta - v)/\, R$
- $\gamma_{xz} = \partial u/\partial z + \partial w/\partial x$
- $\gamma_{x\theta} = \frac{1}{R}(\partial u/\partial \theta) + \partial v/\partial x$

Formulation – Stress-Strain relations

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{\theta z} \\ \tau_{xz} \\ \tau_{x\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & 0 & 0 & C_{26} \\ 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & 0 & 0 & C_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_{xx} - \Delta T \alpha_x \\ \varepsilon_{\theta\theta} - \Delta T \alpha_\theta \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} - \Delta T \alpha_{x\theta} \end{Bmatrix}$$

- $C_{11} = \frac{E_1}{(1-v_{12}v_{21})}$, $C_{12} = \frac{v_{21}E_1}{1-v_{12}v_{21}}$
- $C_{22} = \frac{E_2}{1-v_{12}v_{21}}$, $C_{44} = G_{23}$, $C_{55} = G_{13}$, $C_{66} = G_{12}$

Formulation – Equilibrium equations and boundary conditions

- Principle of virtual work :

$$\delta \Pi = \delta(U - W) = 0$$

$$\delta \Pi = \iiint (\sigma_x \delta\varepsilon_x + \sigma_\theta \delta\varepsilon_\theta + \sigma_z \delta\varepsilon_z + \tau_{xz}\delta\gamma_{xz} + \tau_{x\theta}\delta\gamma_{x\theta} + \tau_{\theta z}\delta\gamma_{\theta z}) dz dA - \iint q\, \delta w dA = 0$$

Formulation – Equilibrium equations and boundary conditions

- Boundary conditions are given as, at $\theta =$ constant: one from each of the following bracketed quantities should be specified

$(N_{x\theta}, u_0), (N_\theta, v_0), (M_{x\theta}, \phi_x), (M_\theta, \phi_\theta), (K_{x\theta}, \lambda_x), (K_\theta, \lambda_\theta), (J_{x\theta}, \zeta_x), (J_\theta, \zeta_\theta)$

$(H_{x\theta}, S_x)(H_\theta, S_\theta), (N_{\theta z}, w_0), (M_{\theta z}, w_1)$ *and* $(K_{\theta z}, \Gamma)$

- For laminated cylindrical shell with simply supported boundary conditions such that at $\theta =$ constant

$N_{x\theta} = N_\theta = M_{x\theta} = M_\theta = K_{x\theta} = K_\theta = J_{x\theta} = J_\theta = H_{x\theta} = H_\theta = w_0 = w_1 = \Gamma = 0$

Formulation – Solution

$$[L]\{\delta\} = \{f\}$$

- Solution that satisfies the given boundary condition are as follow:
- $(u_0, \emptyset_x, \lambda_x, \zeta_x, S_x) = (P_1, P_3, P_5, P_7, P_9)\, cos(\pi\theta/\theta_m)$
- $(v_0, \emptyset_\theta, \lambda_\theta, \zeta_\theta, S_\theta) = (P_2, P_4, P_6, P_8, P_{10})\, cos(\pi\theta/\theta_m)$
- $(w_0, w_1, \Gamma) = (P_{11}, P_{12}, P_{13})\, sin(\pi\theta/\theta_m)$

Results and Discussion

- Case 1 : Mechanical Loading

$$q = \bar{q}_0 \, sin(m\pi\theta/\theta_m)$$

- Case 2 : Thermal Loading

$$T = 2\bar{T}_0\left(z/h\right) sin\left(m\pi\theta/\theta_m\right)$$

- Layers are made up of same thickness and same materials
- Graphite-epoxy laminate properties are as follow

$$\frac{E_L}{E_T} = 25; \frac{G_{LT}}{E_T} = 0.5\,; \frac{G_{TT}}{E_T} = 0.2;\ v_{LT} = v_{TT} = 0.25\ and\ \frac{\alpha_T}{\alpha_L} = 1125$$

Results and Discussion – cont.

- Results are normalized using the following dimensionless expression :

➢ Mechanical load problem

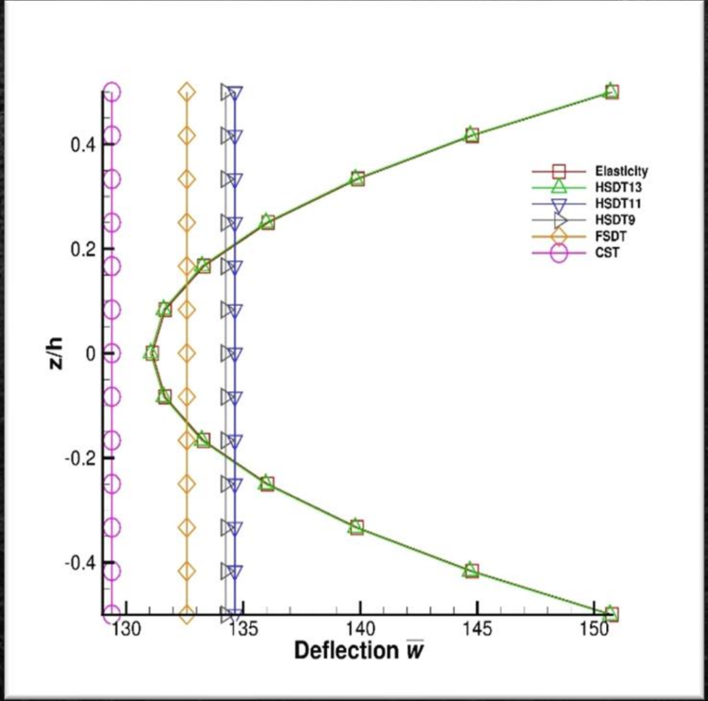
$$\bar{w} = \frac{100E_T w}{q_0 h S^4}, \qquad (\bar{u}, \bar{v}) = \frac{100E_T(u, v)}{(q_0 h S^3)}, \qquad (\bar{\sigma}_x, \bar{\sigma}_\theta, \bar{\tau}_{x\theta}) = \frac{(\sigma_x, \sigma_\theta, \tau_{x\theta})}{(q_0 S^2)},$$

$$(\bar{\tau}_{xz}, \bar{\tau}_{\theta z}) = \frac{\tau_{xz}, \tau_{\theta z}}{q_0 S}$$

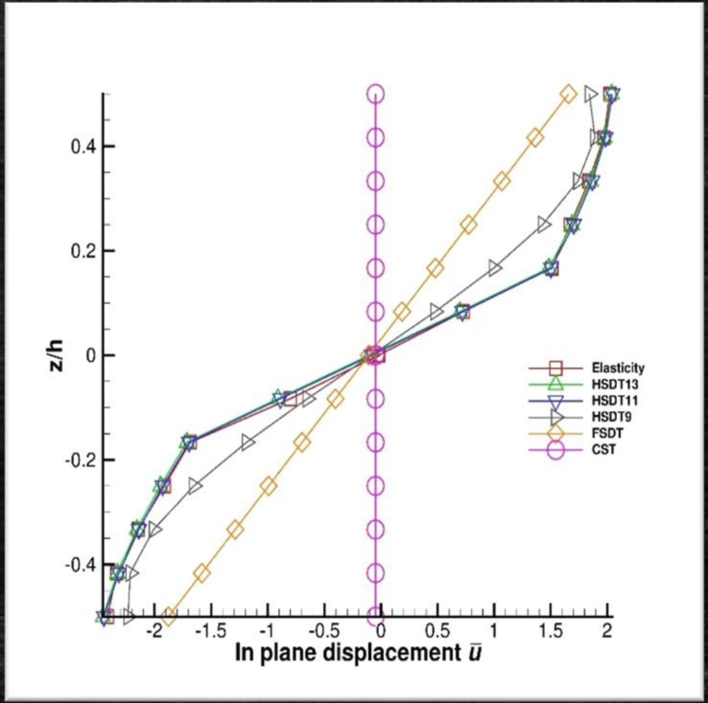
➢ Thermal load problem

$$\bar{w} = \frac{w}{h\alpha_L \bar{T}_0 S^2}, \qquad (\bar{v}) = \frac{(v)}{(h\alpha_L \bar{T}_0 S)}, \qquad (\bar{\sigma_i}, \bar{\tau_{ij}}) = \frac{\sigma_i, \tau_{ij}}{(E_T \alpha_L \bar{T}_0)}$$

Results and Discussion – cont.

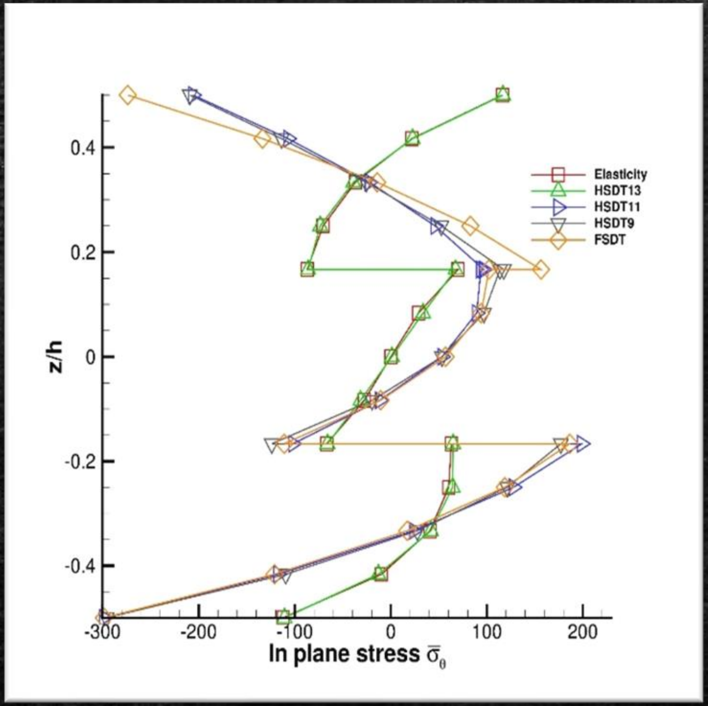


Normalized deflection $\overline{w}$ for (45/-45/45) cylindrical shell under thermal loading.

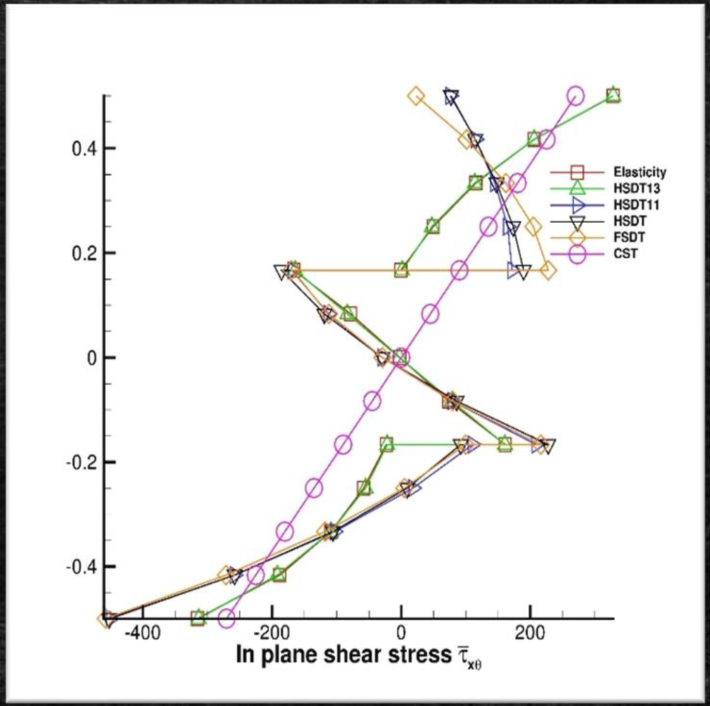


In-plane displacement $\overline{u}$ for (45/-45/45) cylindrical shell under mechanical loading.

Results and Discussion – cont.



In-plane stress $\bar{\sigma}_\theta$ for (45/-45/45) cylindrical shell under thermal loading.



In-plane shear stress $\bar{\tau}_{x\theta}$ for (45/-45/45) cylindrical shell under thermal loading.

Conclusion

- Limits of applicability of commonly used 2D shell theories are assessed with respect of 3D elasticity solution.
- It can be concluded that:

1. CST and FSDT provide good results for thin (R/h > 20) laminate for mechanically loaded shell. HSDT9 and HSDT 11 can provide good result even for thick shell (R/h = 4)
2. For thermally loaded shell, all the theories provide enormous error specifically for thick shell
3. HSDT13 predict accurate results for all the case studies
4. Results presented shows the importance of zig zag function and stretch/contraction terms which has been incorporated in HSDT13.
5. α_T/α_L ratio affect the accuracy of 2D theories.