Year 7 Students' Interpretation of Letters and Symbols in Solving Routine Algebraic Problems

Madihah Khalid Dr.
International Islamic University Malaysia, madihahkhalid@iium.edu.my

Faeizah Yakop Dr.
Sengkurong Sixth Form Centre, Brunei Darussalam, madihahk61@gmail.com

Hasniza Ibrahim
International Islamic University Malaysia, hasniezaibrahim@gmail.com

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Abstract

In this study we focused on one of the recurring issues in the learning of mathematics, which is students' errors and misconceptions in learning algebra. We investigated Year 7 students on how they manipulate and interpret letters in solving routine algebraic problems to understand their thinking process. This is a case study of qualitative nature, focusing on one pencil and paper test, observation, and in-depth interviews of students in one particular school in Brunei Darussalam. The themes that emerged from interviews based on the test showed students' interpretation of letters categorized as "combining" - which involved the combining of numbers during addition, "equating a letter as 1" – where a letter in any algebraic expression is considered to be equal to 1, "misconception of equal sign" – where students see equal sign as taking the value or letter which is closest to the equal sign, "inconsistency with own rule" – where students made up their own misguided rules but utilize other rules for similar situation and finally, "seeing letters as an abbreviations for objects" – where students consider the letters as representing objects.

Keywords

Letters and Symbols, Algebra Misconception, Students' Errors in Algebra, Case Study, Qualitative Research

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Acknowledgements

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Madihah Khalid
International Islamic University Malaysia

Faeizah Hj. Yakop
Sengkurong Sixth Form Centre, Brunei Darussalam

Hasniza Ibrahim
International Islamic University Malaysia

In this study we focused on one of the recurring issues in the learning of mathematics, which is students' errors and misconceptions in learning algebra. We investigated Year 7 students on how they manipulate and interpret letters in solving routine algebraic problems to understand their thinking process. This is a case study of qualitative nature, focusing on one pencil and paper test, observation, and in-depth interviews of students in one particular school in Brunei Darussalam. The themes that emerged from interviews based on the test showed students' interpretation of letters categorized as "combining" - which involved the combining of numbers during addition, "equating a letter as 1" – where a letter in any algebraic expression is considered to be equal to 1, "misconception of equal sign" – where students see equal sign as taking the value or letter which is closest to the equal sign, "inconsistency with own rule" – where students made up their own misguided rules but utilize other rules for similar situation and finally, "seeing letters as an abbreviations for objects" – where students consider the letters as representing objects. Keywords: Letters and Symbols, Algebra Misconception, Students' Errors in Algebra, Case Study, Qualitative Research

Introduction

Algebra is part of mathematics which deals with letters and symbols, where rules are usually used to manipulate those letters and symbols. In learning algebra, students are expected to comprehend the concept and the structures that affect the manipulation of symbols and how symbols are used (Suwito et al., 2016). Competency in interpreting and manipulating letters is very important and vital for students in order to pursue higher level mathematics (Fey & Smith, 2017). Understanding how to add, subtract, multiply or divide letters is usually acquired in the topic of early Algebra. However, making sense of the letters is found to be one of the underlying problems in learning Algebra. According to Usiskin (1988) letters can stand for functions, points, matrices, and vectors. Although letters have many different definitions and interpretations, students tend to manipulate letters with minimal sense and logic. There are different ways of using letters in mathematics. They are fundamental in generalized arithmetic and an understanding of letters forms the foundation for the transition from arithmetic to algebra (Schoenfeld & Arcavi, 1988). Since letters play a very significant role in learning algebra, it is necessary for this study to investigate how students cope and struggle with the interpretation of letters across tasks that require different levels of algebraic understanding.

Some students in many countries have an extremely shallow and imperfect knowledge of algebra that is usually taught around the middle school years (Lim, 2000; Katz, 2007; Knuth, et al., 2006; Witzel, 2016). The use of letters and signs make it an abstract subject and because of this, algebra is considered to be a difficult area of mathematics (Greens & Rubenstein, 2008). From the researchers' experience, misconceptions when dealing with algebraic tasks led to the adoption of incorrect strategies and committing errors becomes more common. This study investigated the nature of misconceptions involving letters and symbols that students commit when they were asked to solve various routine algebraic problems. Students tend to carry these misconceptions as they progress, which could possibly contribute to poor understanding in learning algebra in particular, as well as other topics in general. Interpreting letters is essential to the "core activities of algebra" (Kieran, 2004) and making sense of "symbols" (Arcavi, 2005). A deep understanding of letters will contribute to students' proficiency in algebraic activities. Since interpretation of letters is a complex process, teachers who teach algebra need to carefully and strategically introduce and nurture the interpretations of letters to the students. Hence, the research question that was formulated to guide this study is "How do students interpret symbols, letters and equal signs in algebra?"

Literature Review

Since decades ago, algebra is used in our daily lives to represent numbers and quantities in mathematical formulae and equations (Kilpatrick & Izsak, 2008; Saleh & Rahman, 2016). Algebra is also a gate-keeper course determining for entry into most colleges and whether the students can go on to the higher-level STEM courses (Remillard et al., 2017). Understanding algebra material is the key to success in learning the next mathematical elements (Star et al., 2015). A lack of sufficient mathematical skill and understanding might affect one's ability to make critically important educational, life, and career decisions (Sherman et al., 2015).

However, despite its importance, algebra can be very 'challenging to students because it introduces more abstract representations, more complex relationships between quantities and also it can increase the misconceptions that have their roots in earlier instruction' (Booth et al., 2015, p. 8). Students struggle with representing unknown quantities, understanding that variables can be a range of quantities, and that when one value changes another does as well (Lucariello et al., 2014; Stacey & Macgregor, 1997;). In regard to numbers and symbols, many students mistakenly believe that algebraic symbols are static, the numbers are literal, each number can only have one value, and they have a sign bias which means that unless there is a negative sign, the student assumes the variable is positive (Christou & Vosniadou, 2012).

Furthermore, solving problems using symbolic algebraic language is also difficult for some of the students (Bohlmann et al., 2014). In order to solve problems that contain symbolic expressions, some of them lack the awareness of both the structural and operational aspects related to algebraic symbolization (Meyer, 2013). Hence, misconceptions and incorrect procedures in solving algebraic problems have been treated as "bugs" in conceptual and procedural knowledge, respectively. According to Samo (2010), misconceptions and common errors generally arise from students' struggle in making sense of symbols. His study highlighted students' perceptions, which were embedded in the multiple meanings or roles that some symbol assumed in different contexts. The core competencies of algebra such as symbolic interpretations and manipulations are crucial when engaging with generalized arithmetic. Activities encompassing the idea of algebra involving equations, expressions and using algebra as an instrument/tool will be meaningless with poor understanding of symbols in generalized arithmetic.

This study also investigated whether students were having difficulties in using, analyzing, or understanding symbols in different situations. It is therefore useful here to closely

examine and discuss students' perceptions about the use of symbols in algebra. Symbols are considered as an impetus for thinking algebraically. Study on equal signs by Molina, Ambrose, and Castro (2004) suggested that students interpreted the equal sign as a directive to produce an answer which is also called "command from left to right and command from right to left" (parra. 20) In addition, two different misinterpretations were observed in the study by MacGregor and Stacey (1997):

1) Writing the same number that is in the closer position on both sides of the equal sign. For example, in the sentence $14 + \square = 13 + 4$, 13 would be written as the answer in the space provided
2) Answers to the other side's operation (when the unknown quantity is in the first or fourth place).

In some cases when the unknown quantity is in the first place, the answer to the operation on the right side was written as the unknown number in the other side. For example, for $\square + 4 = 5 + 7$, one of the answers was written as 12. Similarly, when the unknown quantity was last, the answer to the left side operation was placed in the box. For example, 19 would be the answer to $12 + 7 = 7 + \square$. This can be interpreted as an adaptation of the misconception of the equal sign as a command to produce an answer (Peter & Oloaye, 2013).

In addition, previous researchers also found that students often have difficulty in working with algebraic equations. They found the ways in which the symbols should be manipulated to reach solutions hard to learn, even in simple equations (Meyer, 2013). According to Wang (2015), students' interpretation of equations can be influenced by prior experiences in arithmetic. Their background of arithmetic has been built on a foundation in which the equal sign means "gives" or "makes" as in "3 plus 5 gives 8." Teachers see evidence of this interpretation when students used multistep calculations. They frequently use the equals sign for partial answers, moving from left to right. This restricted but familiar use for the equals sign is an obstacle in understanding equations.

Another research study had reported the same perceptions about the use of equal signs (Cooper & William, 2004). Since much of primary school arithmetic is answer oriented, students would interpret the equal sign as a signal to compute the left side and then to write the result of this computation immediately after the equal sign. These researchers suggested that the proper interpretation of the equal sign helps students in algebraic manipulation. Another possible origin of this misconception is the "=" button on many calculators, which always returns an answer. This assumption was made from the discussion among the researchers.

The aim of this research is to investigate misconceptions related to the central aspect of letters which forms the basis of middle school algebra in Brunei Darussalam. In Years 7 and 8, all students were required to work with letters in different contexts such as in Geometry, Perimeter and Area, Trigonometry etc. As they proceed to upper levels, more contexts which involve Algebra will be introduced in the Mathematics curriculum. Therefore, the presence of letters was strongly embedded in the middle year Mathematics Curriculum and research involving letters is crucially important. There are many international studies done on students' misconceptions in algebra but not locally. The researchers could not find any study for students in Brunei Darussalam that focus particularly on letters and symbols on the middle school students particularly Year 7. The researchers felt that it is meaningful to look in depth on students' understanding of algebra particularly on the letters, variables, and equal sign.

According to many studies in Brunei Darussalam, students had an extremely shallow, and usually imperfect, knowledge of Algebra. They also show that students lack relational understanding and relied mainly on rote learning (Law & Shahrill, 2013; Pungut & Shahrill, 2014; Sarwadi & Shahrill, 2014). From two of the researchers' experience, students often showed misconceptions when dealing with algebraic tasks. Adoption of incorrect strategies

and errors by the students were the result of misconceptions. Hence, the researchers tried to investigate the nature of misconceptions in algebraic settings that could possibly have contributed to students' understanding in learning algebra.

Methodology

Research Design

This study was an exploratory case study of qualitative nature as it requires analysis involving looking for patterns and themes in the data. According to Cohen, Manion, and Morrison (2013, p. 289), "case studies can penetrate situations in ways that are not always susceptible to numerical analysis." The purpose of the study was to gain insight into year 7 students' interpretations of letters and misconceptions in algebra, which requires an in-depth understanding of the situation. The case study approach allows this and include multi-faceted explorations of issues in their real-life settings (Crowe et al., 2011).

Sample

The researchers started the sampling process by listing all fifty high schools in the country. It was then narrowed down to fifteen public school from one particular district in Brunei Darussalam. The selected school, which was randomly chosen from the fifteen schools, has eight Year 7 classes. Year 7 students (12–13 year-old) were the selected sample of this study because algebra is formally introduced at this time, and thus fits the aims of the study in examining the very basic problem in learning algebra. After discussions with the Principal of the school, two medium ability classes with a total of sixty students were selected and were given the paper and pencil test. Following the test, each class was observed three times, where three topics – representation of unknowns using symbols and letters, evaluation and simplification of algebraic expressions and solving linear equations were taught. Later, the researchers conducted in-depth interview sessions with nine students, who were purposely sampled, and identified according to the errors that they exhibit from the test. These interviews were conducted in "seeking to maximize understanding of events and facilitating the interpretation of data" (Hitchcock & Hughes, 2002, p. 296).

Research Instruments

The main instruments for data collection used in this research include paper and pencil tests, interview protocol and observation checklist. All instruments used were tried and tested during the pilot study and were found to be reliable and valid for use. Table 1 shows the nature of 20 routine algebraic problems from the paper and pencil test, categorized according to the specification established and used in an earlier research (Kano, 2009; Kuchemann, 1981).

Table 1
The 20 routine algebra problem from the paper and pencil test categorized into 5 parts

Category	Questions No.
Evaluation of letters	6, 7, 8, 15,
Letter as specific unknown	1, 2, 3, 5, 10, 11, 12, 17
Letter as generalized number	4, 13, 14, 18, 19
Letter as variable	20
The Use of Equal Sign	8, 9, 15, 16, 17, 18

Beside the answers from the test, video, and audio recordings of the participants' activities during observation and interviews were the main data. However, data were also obtained from field notes, which also include ongoing notes, notes with predetermined themes, sketches or diagrams, observation checklist, and data from debriefing sessions with other observers as aide-memoire, as suggested by Lincoln and Guba (1985).

The nature of this qualitative study involved "looking for patterns, themes, consistencies and exceptions to the rule" (Hitchcock & Hughes, 2002, p. 296). Themes were established as misconceptions frequently appear from the interviews. The three tentative themes from the paper and pencil test as well as two other themes were strengthened and hence five themes were confirmed.

Validity and Reliability

For this study, the triangulation method was utilized in order to ensure the validity and reliability in data collection. Triangulation can explain the complexity and richness of human behaviour by studying from more than one angle (Cohen et al., 2013) and can also be defined as using two or more methods of data collection in a study or research. To improve data reliability, the data collection techniques and protocols have been standardized in this study. For instance, all of the participants were observed by using the same checklist, prompts and responses as well as the way of listening (active listening). The researchers documented details such as time, date, and settings during the observations for future reference or to be used in the data analysis.

Trustworthiness and Quality Assurance

Lincoln and Guba (1985) claimed that trustworthiness of a qualitative research is significant in evaluating its quality. Lincoln and Guba (1985) proposed 4 criteria to determine the trustworthiness of a qualitative research, which are: (i) credibility, (ii) transferability, (iii) dependability, and (iv) confirmability. Therefore, as proposed by Guba and Lincoln, some criteria for the evaluated quality study were included as follows:

a) Member Checks – In this study, in order to check the data accuracy, member checking among the researchers who were involved was done. The students involved were asked to discuss and read the interview transcripts they participated.
b) Triangulation - Triangulation in this study was applied by using different methods, which are the observation and semi-structured interview as the major data collection methods for qualitative research.
c) Prolonged Engagement - The researcher established a good relationship with the participants and trust in order to gain an adequate understanding. Hence, the cycle of the observations was repeated, and it took approximately 3 months to complete the study.

Ethical Consideration

In this study, ethical considerations were also not neglected as it involves interviewing sessions with children. The main ethical component taken into consideration is the informed consent. Consent letters were signed by the teachers whose classes were observed. Consent forms were also distributed to each parent of the students in the two classes, to inform of the research and consent for their child's involvement. Parents for selected students for interviews

were also asked to sign a special form. Participants are allowed to voluntarily participate in this study and are allowed to withdraw at any time. In addition, the rights of the participants are always observed including the confidentiality and anonymity, and the findings of this research will not affect the participants' life.

Results and Analysis

Students' experienced great difficulty in engaging with the questions. In total, the students managed to get 25% of the solutions correct. Two students scored seventeen out of twenty responses correctly which was the highest mark (85%) while seven students scored zero. Forty-two students scored below 40% (excluding those who scored 0), while twelve students scored between 40% to 60%.

From observations of three lessons, the most common errors that kept turning up were "combining" the unequal terms. Below are some of the errors exhibited during the lesson observations and also classwork:

1. Simplify $3x + 2y$.
   Some of the students responded or wrote the answer as $5xy$ or 5.
2. Simplify $2a + 5b - a$.
   Some managed to get the correct answer. But some of the students responded or wrote the answer as $7ab$, 7 or 6.
3. Solve $3 + y = 7$.
   Some managed to get 4. But some of the students responded or wrote the answer as 7.

Other results that can be interpreted from the observation of classes, written in the filed notes and observation checklist are as follows:

1. Students looked at the letters or variables as something whose values must be given. For example, for question "Give an expression of area of a rectangle of width x cm and length y cm" presented during one of the lessons, some students who didn't understand algebra from the beginning would find the question difficult as they always thought that the values of x and y should be given. So, early misunderstanding of letters and variables had an effect on what the students experience in the learning of algebra.

2. The "combining" error is most dominant among students where unlike terms were added together such as: a plus b is ab, $2x + 3y$ is $5xy$. So from here, the students would solve any algebraic equations wrongly.

3. Students see equal sign as giving one answer or number only. To them, solving equation or writing down an expression as the answer is impossible for example "Simplify $3x + y = $ ____." Students who interpreted equal sign as a sign to give one single answer might give the answer as 4xy and students who interpreted equal sign as a sign to give numerical answer only might give 4 as the answer.

Table 2 shows the five themes found from the interview of 9 students. "×" shows that the theme was not present while "□" indicates that the theme was present in the student's

interview data. S1 – S9 indicates the different students interviewed, who were sample selected based on the common errors made.

Table 2
Themes that emerged from 9 Interviewed Students

Student	S1	S2	S3	S4	S5	S6	S7	S8	S9
Theme									
Combining	×	✓	✓	✓	✓	✓	✓	✓	✓
"Equating a letter as 1"	✓	✓	✓	✓	✓	✓	✓	✓	✓
Misconception of equal sign	✓	✓	✓	✓	✓	✓	✓	✓	✓
"Inconsistency with own rules	✓	✓	✓	✓	✓	✓	✓	✓	✓
Letter as abbreviation for objects."	✓	✓	×	✓	✓	✓	✓	✓	✓

Combining

The theme of combining was evident in many of the answers to the paper and pencil questions. Many students used "combining" in solving routine problems such as: $4 + m + 5 = 9m$ and $4 + 3n = 7n$. Combining is adding of the seen letters to seen numbers, and combining the letters was evident in all interviewed students' data except S1, S2 and S3 both gave $8xy$ as answers for simplifying $2x + 5y + x$. Another question that led to the misconception of "combining" is when they were asked to "add 4 to $3n$." Three of the students: S2, S5 and S6 answered this as $7n$. They combined 4 to $3n$ without considering that they were not of the same terms. The following interview dialogue illustrates how three of the students combined the letters without considering that they were not of the same terms:

> Researcher: How did u get 7n?
> S2: 4 plus 3 is 7 and plus n… I get the answer, 7n.
> S5: Just plus all… 4 plus 3 plus n equal to 7n.
> S6: Like this… add altogether 3n is like 3 plus n, so plus altogether with 4, I get 7n…

Questions which involve the equal sign where an algebraic concept of the letter was needed were also tested in the paper and pencil test. The reason why the students in the sample tend to "combine" the letters was because they were seeing an equal sign as a sign to combine the letters so that a single answer solution is obtained. The "single answer solution" is elaborated further in the next paragraph.

From the instruments that had been adapted in this study, the data collected shows that students had the sense to find a single answer (one term expression or one single number only). The misconception of "combining" which also emerged in many of the interviews seemed to be a fundamental cause for the whopping majority of students' single term answers in the paper and pencil tests. This tendency to find single answer solutions suggests a link to arithmetic thinking where answers are commonly single terms. The belief that the equal sign always gives a single numeric answer also led the students to give single answer solutions in algebraic task. For example, for question "add 4 to m + 5", students who had the belief of always having a single numerical value would "combine" to get 9m and finally ignored or equated m as "1."

Misconception of Equal Sign

In question "if a + b= 43, then a+ b+ 2 = _," the students picked the number which was closer to the equal sign rather than replacing a + b with 43 to get 43 + 2 = 45. Three students (S1, S4, and S9) who were interviewed explained their reason by choosing the number which was close to the equal sign. S1, S8 and S5 also committed the same error for "If a– 246 = 762, then a– 247 =_."

Meanwhile, S4 applied his incorrect rule of "seeing the equal sign as picking the number which was closer to the equal sign." He answered "0" for "what can you say about x if x + 4 =6" instead of 2. For "if a – 246 = 762, then a – 247 = ?," he provided 247 as the answer which was obviously the number which was closest to the equal sign. Below are the excepts:

> Researcher: How about in question 16? Why 247?
> S4:   Same, a – 247 =, must choose number near to equal, 247.
> Researcher: For question 15, how did you get 0?
> S4:   You see, x + 4 = 6... there is no number here (pointing at the right-hand side of 6), no number meaning 0...
> Researcher: Why did you choose the number here? (pointing at the right-hand side of 6)
> S4:   Oh, because 4 and 6 is near to equal sign but x is far, so 0 is also far from equal sign.

The students perceived the equal sign as a sign that returned only one value on one side. If there were expressions, the final answer should be the one which was closest to the equal sign. This misconception might have been embedded during their learning of arithmetic since the equal sign is thought as a signal to compute the left side and then to write the result of this computation immediately after the equal sign (Cooper & William, 2001). The researcher discussed with some teachers who agreed that the use of a calculator was also another possible origin of this misconception as the "=" button on many calculators, which always returns a single answer.

Equating a letter as 1

The questions from the paper and pencil test requires students to interpret letters as specific unknowns, generalized numbers or variables. However, most students in the sample seemed to have no algebraic concept of the letters and the language rules of algebra, maybe because of unfamiliarity with the concept. Therefore, the findings of this study conflict with what is prescribed for year seven students in the algebra curriculum. Due to lack of a conceptual understanding of letters in algebra, students created their own ways to give meaning to the letter. Therefore, the misconception of "letter is equal to 1" were common as all interviewed students mentioned the phrase "letter is equal to 1" with little algebraic understanding. It seems like the phrase was learned in the arithmetic class but with a limited understanding of algebra.

Meanwhile, all students interviewed performed "equating a letter as 1" error. As an example, for question "Add 4 times of x to 10 times of y" three of the interviewed students answered 14 instead of $4x + 10y$.

The same misinterpretation occurred in question "add 4 to 3n" where four students answered 7 instead of 4+3n. Here, three of the students; S2, S5 and S6 said that after combining 4 and 3n to get 7n, they then equated n as 1, and so their final answer was 7. On the contrary, S7 equated n as 1 earlier before adding 4 and 3 together. He said '3n is 3' so he adds 4 to 3 and got 7.

Analysis of S8's responses for the last question showed the error in applying his incorrect own rule where he equated n =1 to 4+3n to get 4+ 3×1 = 7. The examples are shown as follows:

Researcher: For question 11, why did you answer 8?
S8:   Because 3 + 5 is 8, so x and y plus 8 is 8xy and x and y is equal to 1.
Researcher: Ok, how about for question 13, how did you get 10?
S8:   Ok, 4 plus m + 5 is 9 plus m. m is 1, so 9 plus 1 is 10.
Researcher: How about question 14, how did you get 7?
S8:   First, I get 4 + 3n, then n is 1 so I have 4 plus 3 equal 7.
Researcher: Is your method for these three questions the same?
S8:   Yes.
Researcher: In question 11, you put x and y as 1 after you get 8xy. But in questions 13 and 14, you put m and n as 1 earlier. Why?
S8:   It is the same, early or ending, it is still the same, and letter is always 1.

Meanwhile, another example by student S4 is shown as follows:

Researcher: In question 17, how did you get 3?
S4:   Oh, for this one, all of the letters is 1, so 1 plus 1 plus 1 is 3...
Researcher: This time why didn't you choose 'h + i + j=' equals to j? Isn't j the closest to the equal sign?
S4:   Because I remember letter we put as 1.

The researchers also noticed that students demonstrated inconsistency even in applying their own rules which are incorrect. To validate this tentative theme, the researcher includes questions on how the students decide on using their own rules during the interview sessions. Here, S3 "combined" $3x+5y$ to get 8xy. For question 12, she equated $a = b = 1$ to get $2a - b = 2×1 - 1 = 1$ but she combined $4 + 3n$ and got $7n$ before equating $n = 1$ to get the final answer of 7. For question 17, she was expected to combine "h+i+j" to become "hij." She instead equated "h=i=j=1" to get 1+1+1 to get 3 as the answer. The dialogue below shows the example of the student who demonstrated inconsistency:

Researcher: For question 14, why did you answer 7?
S3:   Because 4 + 3n is equal 7n and my teacher said n is same like number 1, so 7   × 1 is 7
Researcher: How about in question 11? How did you get 8xy?
S3:   Like 3 plus 5 is 8, so xy I just put at the back.
Researcher: x is not same as 1?
S3:   Emmm... no... because have two letters... if only one letter, it is 1.
Researcher: How about question 17, how did you get 3?
S3:   This one is simple, all are like 1, h is 1, i is also 1, and j also 1... So, when 1 plus 1 plus 1, becomes 3.
Researcher: But for question 11, you said if there is one letter, then it is 1, but not for two letters. But here, there are 3 letters?
S3:   Oh... mmm. I don't know, I forget.

S8 is another student selected for the interview where inconsistency in applying incorrect own rules occurred in the questions shown above. He answered 8 for $3x+5y$ where he

"combined" 3*x* and 5*y* to become 8xy and then equated *x* and *y* as 1 to get the final answer 8. However, he did not combine *m*+9, but instead equated *x*=1 to get 1+9=10.

Letter as abbreviation for objects

It was also seen that in some instances letters were equated, but not as 1 as mentioned in the previous section. Some students related the order of the letters of the alphabet to the order of numbers. This further suggests that students created their own ways to give meaning to the letters due to their lacking in conceptual understanding of algebra. The students also see letters as the arrangement of alphabetical order and abbreviations to certain objects. "a" stands for apple, "b" stands for banana, "s" stands shirt and "p" stands for pants were mentioned by the students during the interviews. The data analysis suggests that the fundamental issue of algebraic letter is not understood by students in the sample, and this will likely lead to errors and misconceptions in the future learning of algebra.

The researcher had earlier discussed the question "Apples cost *a* dollars each and bananas cost *b* dollars each. If Hadwan buys 2 apples and 3 bananas, what does 2*a* + 3*b* represent?" Where 73% of the students responded that 2*a* + 3*b* represented 2 apples and 3 bananas although "*a*" represents the cost of an apple and "*b*" represents the cost of a banana. It was very obvious from the analysis of paper and pencil test that students who have problems with algebra perceive letters as abbreviations or objects. When interviewed, only one out of nine answered the question correctly. One did not respond to the question and seven of them responded that 2*a* + 3*b* represented 2 apples and 3 bananas instead of the *total cost* of the two apples and 3 bananas. When interviewed, all seven stated that letter "*a*" stands for 'a'pple and "*b*" stands for 'b'anana. S5 reasoned it out as follows:

> Researcher: Why do you think *s* stands for shirt and *p* stands for pants?
> S5:   You said... In front of shirt is *s*... in front of pant is *p*...
> Researcher: Why didn't you answer question 5 in the previous test?
> S5:   No time...
> Researcher: Ok, now try to read and answer the question (Question 5 from
> the paper and pencil test) again...
> S5:   Ok, 2 apples and 3 bananas.
> Researcher: Why?
> S5:   *a* is apple and *b* is banana.

It is also interesting that the answer given for question "What is the cost of *h* erasers at *w* cents each?" by S2, S7 and S9 is 184. From the interview data, the researcher now is more convinced that one of the reasons students are weak in the learning of algebra is because of "seeing letter as an abbreviation or object." Below is how S7 explained his calculation:

> Researcher: How did you get 184?
> S2:   Wait... 8 times... 23... wait... yes... 184.
> Researcher: 8 and 23? Where did you get those numbers?
> S2:   8 is *h*, *a* is 1, *b* is 2, *c* is 3,... until *h* is 8, then count until *w*, *w* is 23.
> Researcher: How did you get 184?
> S7:   I count *a, b, c, d, e, f, g, h*, number 8, and then *i, j, k, l, m, n, o, p , q*
>   Number 23. Then times.
> Reseacher: Times what?
> S7:   23 and 8, I get 184.

Discussions of Findings

The following are the summary and discussion that may explain students' errors and difficulties in understanding Algebra, based on the errors identified. An assumption of this study was that misconceptions will be displayed by students of the sample which was confirmed by the data analysis. Misconceptions in prior learning of concepts such as substitution, implicit multiplication and gathering of like terms were seen to be problematic and caused students to make errors in many tasks. Students had poor understanding of the concept of algebraic letters and basic symbolic manipulations and hence did not have 'symbol sense'. This is especially evident when students were seen to perform "combining", which was termed as "conjoining" by MacGregor and Stacey (1997), in their study of Year 7 (11–13 years old) in Australia. The incorrect rules of equating letter to 1 and combining the letters to numbers seemed to be the major problems in the learning of algebra. This resulted in random picking, joining of numbers and letters during addition and the creation of rules that were not consistent. Moreover, this poor algebraic understanding also seemed to have contributed to students creating their own ways to give meaning to the letter such as the "letter is equal to 1" misconception.

The researchers suspect that most of the students have no clear understanding regarding the importance of algebra in their daily life which led them to neglect algebra as a significant subject with real-life problems. Students should have been provided an in-depth understanding of algebra to understand algebraic concepts. This requires conceiving the use of algebra as having links with processes of generalizing the arithmetic. The study highlighted that students were having difficulties in transforming arithmetic thinking to algebraic thinking because they could not set apart between arithmetic and algebra. As a result, their algebraic thinking could not develop, which led to the misconceptions revealed in this research. In the end, the students applied arithmetic procedures for solving algebraic problems. McGregor and Stacey (1997) found that this led to equal sign misconception, which was also confirmed by Molina, Ambrose, and Castro (2004). The researchers found that operation symbols like "+" and "−" with equal sign presented in the questions led them to end up with single answers. The results of this study suggest that many students did not have the ability to interpret letters and equal sign properly and not being able to consider algebraic letters as generalized numbers or as variables. The majority were interpreting letters as specific unknowns. They had that perception that each letter in algebra is a known variable whether it varies or not. It is suggested that students are taught to see the equal sign as a balance rather than as an indication for them to produce an answer (Kieran, 2004) even when they were doing arithmetic (Welder, 2012).

Students also believed that each variable or letter in algebra has a particular and fixed number value. As a result, students had the perception that in solving any algebraic task, the numerical value for the unknown should be given, as was stated by Samo (2010). When the value is not given, students try to give the letters according to certain rules such as equating the letter with the its order in the alphabet such as "a=1, b = 2, c = 3,… etc." The results of the study highlighted that students also use letters as abbreviations or object in some cases especially in the word problems. This can be due to some coding game that they play or come across. The random picking, joining of numbers and letters during addition resulted in the creation of own rules that were not consistent. Moreover, this poor algebraic understanding also seemed to have contributed to students creating their own ways to give meaning to the letter which resulted in other errors. This finding concurs well with that of Samo (2010) in his study where he found misconceptions and common errors generally arise from students' struggle in making sense of symbols. The misinterpretations of letters and equal signs in learning algebra led the students to create their own incorrect rules are inconsistently applied to other algebraic tasks.

Limitations

The following are the limitations of the study:

i.   The study examined Year 7 students in Brunei Darussalam with average academic achievement who were selected using multi stages sampling method. Year 7 students were selected because algebra is formally introduced at Year 7, and thus fits the aims of the study in examining the very basic problem in learning algebra. Hence the result of this study may not be generalized to those students in higher levels than Year 7. Additionally, generalization may not be applied to students with better achievement in mathematics.

ii. As this study has budget and time constraints, only one school from one selected district was involved. Therefore, other students were not included.

iii. Meanwhile, the study focused on students' errors and misconceptions in learning algebra, therefore, other recurring issues in the learning of mathematics were also not included.

Implications

This study investigates how students interpreted symbols, letters, and equal sign in algebra and how their perceptions affect their learning of algebra. This study also explored basic reasons for students' misconceptions and difficulties in learning of algebra. The data of the study revealed that those students of year 7 in a public high school faced many problems in understanding algebra, such that the lack in relevant understanding of the concepts of algebra. They perceived algebra as a blend of numbers and letters. From the results of this study, they do not seem to have ideas on how to use algebra for generalizing arithmetic for solving problems.

The finding of this study will be useful to teachers of mathematics such that it will help them to understand students' difficulties and therefore would help them in planning to teach and help their students. The traditional method of teaching where students are just being told about the symbols and letters seems to be failing in teaching students the concepts of algebra. Hence teachers should think of more innovative methods that could engage the students and make them understand algebra better.

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Author Note

Madihah Khalid is currently employed at the Department of Curriculum and Instruction, International Islamic University Malaysia. She obtained her PhD in Mathematics Education from Curtin University of Technology, Western Australia in 2004. Her areas of interest include Curriculum and Instruction, Education for special needs, Mathematics Education, Teacher Education, Assessment and Evaluation, Lesson Study and Statistics for Social Research. She regularly publishes papers in refereed journals, conference proceedings as well as book chapters. Google Scholar: https://scholar.google.com/citations?user=iq4QqyMAAAAJ&hl=en; Orcid: https://orcid.org/0000-0002-4538-1922; ResearcherGate: https://www.researchgate.net/profile/Madihah_Khalid; Please direct correspondence to madihahkhalid@iium.edu.my.

Faeizah Haji Yakop has been teaching Mathematics since 2000 at Sayyidina Husain Secondary School, Brunei Darussalam after graduating with a B.Sc. in Education (Majoring in Mathematics) from University Brunei Darussalam. She then pursued her Master in Mathematics Education in 2011 at the same university. She was then posted to Sengkurong Sixth Form Center as a Mathematics instructor and has been there until now. Her research interest is in the area of students' misconceptions in mathematics especially when they enter the lower secondary level. She can be reached via her email, pie_74@hotmail.com.

Hasniza Ibrahim is currently a PhD Student (Education) in International Islamic University Malaysia. She acquired her Master's degree in Educational Psychology from the same university. She has plenty of experience in conducting research both in qualitative and quantitative, and published articles in the refereed journals covering educational psychology and mathematics education. She is a registered engineer in the Malaysian Board of Engineers since 2008 and worked in various fields since 2006 after graduating with a B.Sc. in Engineering (Chemical). Google Scholar: https://scholar.google.com/citations?user=YIlOU_0AAAAJ&hl=en; Orcid: https://orcid.org/0000-0003-1044-9445; ResearcherGate: https://www.researchgate.net/profile/Hasniza_Ibrahim; Please direct correspondence to hasniezaibrahim@gmail.com.

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