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Wavelet Improved Option-Implied Moments: An Empirical Study

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This study investigates the performance of option-implied moments, realised from the model-free Bakshi-Kapadia-Madan (MFBKM) with an improvement using wavelet transform. So far, little attention has been paid in utilising continuous wavelet transform in denoising the option-implied moments, especially within the model-free hybrid framework. Thus, this study primarily seeks to outline the important steps involved in the continuous wavelet transform data-regenerating by assuming that the best fit among the values considered is the best fit model for all. The sample data extracted from Dow Jones Industrial Average index options data is empirically examined throughout this study. This study finds that the wavelet-denoised higher moments record smaller approximation error in most cases compared to the noisy higher moments. It is shown that wavelet transform improves both consistency and error approximation of the signal.

Keywords: denoised signal; option-implied; options; wavelet transform

I. INTRODUCTION

The use of wavelet transform in data analysis has amplified significantly in recent years. Wavelet transform is predominantly preferable in either data analysis, series modelling to time series forecasting due to its special extraction capability feature of multiresolution decomposition of analysis. According to Rioul and Vetterli (1991), wavelet transform enables a certain series to be decomposed into partition of series with different resolution. In other words, the authors can examine the data in various number of resolutions of structure. A series may consist of both coarse and fine structures. However, in most cases only coarse structure appeared and most study tends to disregard the fine structure, hence produce a biased analysis. The multiresolution element of wavelet transform ensures that a data series is smoothed out without destroying the series fine structure during the data denoising process. As a matter of fact, Shik Lee (2004) and Arino and Vidakovic (1995) in their study pointed out the superiority of wavelet transform in series modelling and forecasting. Similar claim was made by Hsieh, Hsiao and Yeh (2011), in which they highlighted that the wavelet transform is so powerful that it has been applied to many fields hitherto. Wavelet transform has undeniably become an interesting topic to be explored.

To date, there are not many studies attempted to utilise the wavelet transforms, especially in the realm of option-implied information. Somehow, it is not clear why the studies on wavelet transforms in financial derivatives are less available despite its multiple advantages. It comes with several advantages. First, wavelet transform is viewed to be appropriate with non-stationary data, in which mean, and autocorrelation of the signal varied over time. Relying on the fact that most of the financial time series data is non stationary, it was sufficiently highlighted the appropriateness of wavelet transform in this case. Secondly, this wavelet transform involves decomposition of signal. It decomposes the given signal into several other signals with different resolution levels. Using this process, the original time domain is preserved,

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without obliterate any information that can lead to further noises. Yu, Kim and YH Song (2001) demonstrated that wavelet transforms involve reverse process, i.e. the inverse wavelet transform. In the process, the signal is reconstructed without being loosed. In fact, there are many areas of application in which researchers employ wavelet transform.

The subsequent sections are developed as follows. A brief background of study is already provided in the first section. Previous related studies are reviewed in Section 2. Sampled data considered throughout this study is described in Section 3. In Section 4, the authors detail the methodology involved in investigating the effect of wavelet transform on the sampled data. The empirical results are presented in Section 5. Finally, the conclusion is drawn in Section 6.

II. LITERATURE REVIEWS

Many instruments have been proposed in denoising financial time series. In fact, there are many areas of application in which researchers employ wavelet transform. Even though the literature covers a wide variety of such available tools, study that employed the continuous wavelet transform is very uncommon. For that reason, the following literature reviews attempt to primarily focus on the wavelet transform as a whole. A number of comparative studies of wavelet transform with varies methodologies have been recorded; either based on the stand-alone wavelet transform function or its hybrid with other models.

For instance, a study done by Sun and Meinl (2012) illustrated the denoising analysis imposed on high-frequency data of Germany equity market. They proposed Local Linear Scaling Approximation Algorithm (LLSA) which based on the Linear Maximal Overlap DWT or MODWT. Sun and Meinl (2012) concluded that with flexible settings, the proposed LLSA appears to be having superior performance in decomposing the pattern and noises. Sun, Chen and Yu (2015) modelled the volatility of the US equities returns by proposing a new wavelet-based methodology, i.e. Generalised Optimal Wavelet Decomposing Algorithm (GOWDA). They found that the GOWDA methodology tends to generate more accurate forecasts of volatility compared to the traditional methods. A study by Renaud, Starck and Murtagh (2005) employed multiresolution prediction in capturing the time series. The approach which is analogous to Kalman filter is revealed to be superior in noise filtering and time series forecasting. Similar studies were conducted by Cristi and Tummala (2000) and Hong, Cheng and Chui (1998). Pan *et al.* (1999), Bashir and El-Hawary (2000) and Lotrič (2004) proposed the wavelet transform based on a neural network. Wavelet transform with Multiscale AutoRegressive is employed by Daoudi, Frakt and Willsky (1999).

A paper by Al Wadia and Tahir Ismail (2011) which studied the wavelet transforms in forecasting Amman stock market based on ARIMA model discovered an improved accuracy in the time series forecasting. Similar findings were sought by Alrumaih and Al-Fawzan (2002) in denoising and forecasting the Saudi stock index and Ababneh, Al Wadi and Ismail (2013) that based their study on Amman stock market. Few other studies have also considered the combination between the wavelet transform with ARIMA. Al Wadi, Hamarsheh and Alwadi (2013) utilised the Maximum Overlapping Discrete Wavelet Transform (MODWT) with the combination of ARIMA in modelling and forecasting the Amman stock market. Alwadi (2015) has further extended the work proposed by Ababneh *et al.* (2013) and demonstrated the noteworthy performance of using MODWT compared to another discrete wavelet transform, i.e. Haar and Daubechies. MODWT was also employed by Ismail, Audu and Tummala (2016) The study compared the forecasting performance produced by GARCH alone, in relative with the MODWT-GARCH methodology, which was claimed to produce a better forecast.

In a later study, Alrumaih and Al-Fawzan (2002) considered three denoising technique, i.e. Haar, Daubechies and Biorthogonal wavelet on the Saudi stock index time series. They revealed that a superior performance can be achieved by using soft thresholding with white noise assumption. Agarwal *et al.* (2016) proposed a wavelet transform based mixed model and found the model to be better than the other three models, which are Haar, Daubechies and Biorthogonal. In fact, they highlighted that the most outperform wavelet is Daubechies wavelet of second order, which is similarly sought by Alrumaih and Al-Fawzan (2002). Few others have focused their studies on using the Haar and

Daubechies wavelets, such as Manchanda, Kumar, and Siddiqi (2007). Pan *et al.* (1999) and Soltani (2002) studied the wavelet transform using the traditional undecimated Haar transform. In another study, Jammazi and Aloui (2012) carried out a study on the UK, France and Japan stock markets using A'Trous Haar wavelet transform and Markov Switching Vector AutoRegressive (MSVAR) analysis. Somewhat resembling study was performed by Ismail, Karim and Alwadi (2011) which studied the Malaysian KLCI index. As a matter of fact, there are few other studies cited that revealed the superiority of the wavelet transform and its mixed models. However, none of the study has so far considered a continuous wavelet transform in denoising the option-implied moments using a model-free hybrid framework. Thereby, this study seeks to outline the important steps involved in a continuous wavelet transform data-regenerating by assuming that the best fit among the values considered is the best fit model for all.

Realising that, this research differentiates itself from other existing literature by investigating the performance of higher order moments, realised from the model-free Bakshi-Kapadia-Madan (MFBKM). This study intends to empirically investigate the index options data, specifically those that able to directly proxy the global index options market. For that reason, the Dow Jones Industrial Average (DJIA) index options data is utilised in this study. DJIA is the most cited and the most extensively accepted stock market index. This study generally focused on examining whether the use of wavelet transform in estimating higher moments allows for improvement in the pricing performance.

III. DATA

This paper utilises options on the Dow Jones Industrial Index (DJIA) traded daily on the Chicago Board Options Exchange (CBOE). The investigation includes all call and put options traded from January 2009 until December 2015. The DJIA index options comprise track 30-blue chipped-companies index and equity options within the US economy.

IV. RESEARCH METHODOLOGY

This study relies on two core strands of literature, i.e. Bakshi, Kapadia and Madan (2003) and Buss and Vilkov (2012). The approaches used in the two studies are mainly adopted in this research with several adjustments and modifications for a better MFBKM performance. In order to obtain the option-implied moments values, the authors adopt the same methodology as in Buss and Vilkov (2012), which is from the estimated moments of the market index return. The authors control the noise embedded in the MFBKM by considering a wavelet signal de-noiser. Eleven denoised signals are generated for both cases of call and put options for comparison purposes. The denoised signals are first compared against the original noisy data. The comparisons are to include 8 other smoothing filters. The new signal realised from the best performed wavelet is assessed based on two criterias- the Signal-to-Noise Ratio (SNR) and Root-Mean-Square-Error (RMSE).

In addition to that, further analysis on the pricing errors between those generated from wavelet-improved data series against that of non-wavelet-improved data series are performed. The results are delivered in terms of RMSE, mean value of the relative pricing error (MRPE) and mean value of the absolute relative pricing error (MARPE). Finally, the wavelet-enhanced signal is assessed in approximating the model-free higher-order moments of MFBKM. The true values obtained from a normal distribution are set as a benchmark. The approximation errors are measured using both absolute and relative errors. Primary steps involved in this study are highlighted in Figure 1.

A. Wavelet Denoising

The signals denoised using wavelet signal denoiser only consists of significant signals that have been cleaned or flattened. In view of comparisons, the performances of 11 denoised wavelet signals are recorded with respect to both call and put options. The parameters are manipulated to include various combinations of wavelet family, denoising method, threshold rules as well as noise estimate level. The 11 denoised signals and their respective parameters are detailed in Table 1 and Table 2, for call and put options, particularly.

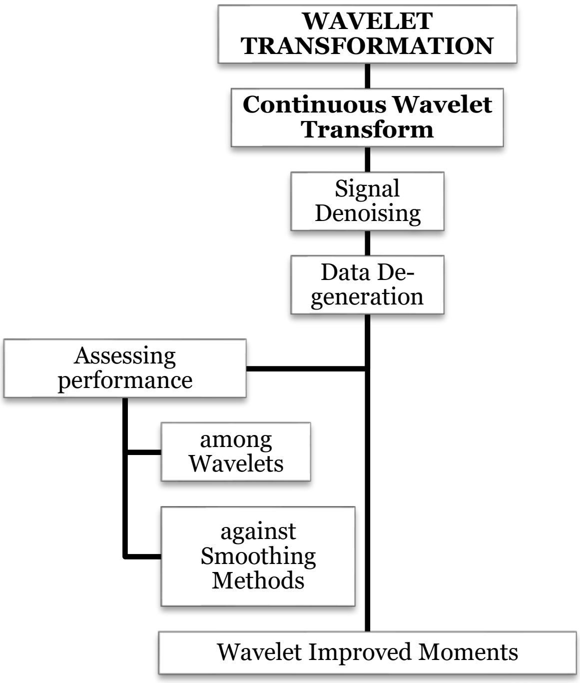

Figure 1. Research Methodology Flow

The gNoise(n) depicts the denoised signals of call options, for n=1,...,11. In the same manner, fNoise(n) are the denoised signals of put options. Five wavelet families are considered in this study with a variant of finite family of wavelets. For instance, the Symlets wavelet ranges from Symlet 2, Symlet 3, Symlet 4, up to Symlet 8. The Daubechies wavelet ranges from Daubechies 1 until Daubechies 10. The range of values that can be taken by the different wavelet family are described in Table 4.1. The level represents the number of times the wavelet is decomposed and the number of vanishing moments.

However, a different definition of level is applied to Fejer-Korovkin wavelet. The level in this type of family wavelet denotes the number of filter coefficients. All levels are designated as positive integers. The minimum level is 1, whereas the maximum number of decomposition of each wavelet is floor $(\log_2(N))$. N is the number of samples in a data.

There are six different denoising methods considered in this study – the Empirical Bayesian, Block James-Stein, Universal Threshold, Stein's Unbiased Risk Estimate (SURE), False Discovery Rate (FDR), and Minimax Estimation. Each method specifies how the thresholds are denoised in the data. The Empirical Bayesian method based its assumption on the independency of prior distribution that can have on the measurements. This is provided by a mixture model.

A rough estimate of the model weight is assessed by the measurements. Thus, this method is efficient for high-sampled data. The threshold rule undermining the Block James-Stein method is rooted on finding the best block thresholding estimator size and threshold. Optimal global and local adaptivity are achieved concurrently from the block product (Cai, 2002).

Table 1. Denoised Signals and Respective Parameters for Call Options

Denoised Signal	Wavelet Family		Level	Denoising Method	Q-value	Threshold Rule	Noise Estimate Level
	Name	No.					
gNoise1	Symlets	8	5	Empirical Bayesian	-	Soft	Dependent
gNoise2	Daubechies wavelets	10	5	Block James-Stein	-	James-Stein	Independent
gNoise3	Daubechies wavelets	10	6	Universal Threshold	-	Hard	Dependent
gNoise4	Fejer-Korovkin filters	22	4	Universal Threshold	-	Hard	Dependent
gNoise5	Biorthogonal wavelets	6.8	7	Stein's Unbiased Risk Estimate	-	Soft	Dependent
gNoise6	Coiflets	5	4	Universal Threshold	-	Soft	Dependent
gNoise7	Symlets	4	4	Empirical Bayesian	-	Soft	Dependent
gNoise8	Symlets	8	6	Block James-Stein	-	James-Stein	Independent
gNoise9	Fejer-Korovkin filters	22	4	Empirical Bayesian	-	Soft	Dependent
gNoise10	Biorthogonal wavelets	6.8	4	False Discovery Rate	0.05	Hard	Dependent
gNoise11	Coiflets	5	6	Minimax Estimation	-	Soft	Dependent

Table 2. Denoised Signals and Respective Parameters for Put Options

Denoised Signal	Wavelet Family		Level	Denoising Method	Q-value	Threshold Rule	Noise Estimate Level
	Name	No.					
fNoise1	Daubechies wavelets	10	4	Empirical Bayesian	-	Soft	Dependent
fNoise2	Daubechies wavelets	10	4	Universal Threshold	-	Soft	Dependent
fNoise3	Fejer-Korovkin filters	22	4	Universal Threshold	-	Soft	Dependent
fNoise4	Biorthogonal wavelets	6.8	4	Universal Threshold	-	Soft	Dependent
fNoise5	Coiflets	5	5	Universal Threshold	-	Soft	Dependent
fNoise6	Symlets	8	6	Universal Threshold	-	Soft	Dependent
fNoise7	Symlets	4	4	Empirical Bayesian	-	Soft	Dependent
fNoise8	Symlets	6	4	Block James-Stein	-	James-Stein	Independent
fNoise9	Fejer-Korovkin filters	22	5	Empirical Bayesian	-	Soft	Dependent
fNoise10	Biorthogonal wavelets	6.8	4	False Discovery Rate	0.05	Hard	Dependent
fNoise11	Coiflets	5	5	Minimax	-	Soft	Dependent

The threshold rule of the FDR method is built to manipulate false positive detections of the expected ratio into all positives. This method is handy for a less-densed sample data (Abramovich, Angelini & De Canditiis, 2007). Threshold of Minimax Estimation method is selected to return those that return a minimax performance. The performance is assessed based on mean square error. In line with its name, the SURE method chooses threshold using Stein's Unbiased Estimate of Risk. The approach is also known as a quadratic loss function. The threshold value is selected to return the threshold value with a minimum risk. Finally, the threshold from the Universal Threshold method is calculated by the product of a threshold and a factor. The threshold is chosen from those that return a minimax performance, whereas the factor represents a small constant proportional to a $\log(X)$. In this case, X is the length of a sample. The Q-value is an optional parameter assigned only in the case of FDR method. It corresponds to the false positives proportion. The Q-value can take the value from 0 until ½, i.e., $0 < Q \leq 1/2$.

In order to filter wavelet coefficients, the authors depend on the threshold rule. However, only certain threshold rule is applied for certain denoising method. The Method of estimating variance of noise in the data is defined as either Dependent or Independent. Variance of the noise is estimated at each resolution level using Level Dependent. On the other hand, the noise variance is estimated on the highest resolution in the case of Level Independent.

The new signal realised from the best performed wavelet is assessed based on two criteria- the Signal-to-Noise Ratio (SNR) and Root-Mean-Square-Error (RMSE). According to Carlson (1968), SNR is a ratio of a signal to a noise level. In other words, the SNR measures the useful signal level against the irrelevant signal.

$$SNR = \frac{P_{signal}}{P_{noise}} \tag{1}$$

where P_{signal} and P_{noise} denote the power of signal and noise, respectively. Another note to be taken is that the definition of the SNR works under the assumption that the measurements of the desired signal and noise are executed at the output of the bandpass filters, within the same bandwidth of system (Kieser, Reynisson & Mulligan, 2005).

The authors then deliberate the assessment to additionally consider on two other criteria- mean value of the relative pricing error (MRPE) and mean value of the absolute relative pricing error (MARPE). The mean value of the relative pricing error (MRPE) is the sample average of the respective retrieved volatility minus the volatility true value, divided by the true value. RMSE is simply the square root of the averaged squared error (SE). The average is obtained by dividing the SE by the total number of samples used in the calculation. The mean value of the absolute relative pricing error (MARPE) is the absolute sample average of the respective retrieved volatility minus the true value, divided by the true value. S.D. is the standard deviation.

V. RESULT AND DISCUSSION

The performance of how each denoised signal is approximated against the noisy original signal is compared in this section.

Table 3. Performance of 11 Wavelet Denoised Signals of Call Options

Denoised Signal	Signal-to-Noise Ratio (dB)	RMSE (pts)	S.D.	MRPE (%)	S.D. (%)	MARPE (%)	S.D. (%)
gNoise1	14.4049	0.1734	0.0734	4.4879	11.0788	4.8276	10.9350
gNoise2	16.4055	0.1380	0.0002	4.4870	11.3439	4.8534	11.1921
gNoise3	13.6433	0.1887	0.8096	4.4905	10.8955	4.9622	10.6890
gNoise4	14.7273	0.1652	0.1595	4.4875	11.2612	4.9531	11.0643
gNoise5	15.9754	0.1461	0.1419	4.4856	10.9982	4.8113	10.8596
gNoise6	14.0853	0.1786	0.1186	4.4878	10.9059	4.8701	10.7406
gNoise7	16.1039	0.1425	0.0038	4.4874	11.2710	4.8275	11.1296
gNoise8	19.0515	0.1023	0.0001	4.4870	11.3442	4.8537	11.1922
gNoise9	14.8620	0.1624	0.0882	4.4877	11.0025	4.8736	10.8371
gNoise10	15.2549	0.1575	0.0065	4.4871	11.3409	4.8566	11.1877
gNoise11	14.5091	0.1695	0.4670	4.4897	10.6185	4.8770	10.4462

Table 4. Performance of 11 Wavelet Denoised Signals of Put Options.

Denoised Signal	Signal-to-Noise Ratio (dB)	RMSE (pts)	S.D.	MRPE (%)	S.D. (%)	MARPE (%)	S.D. (%)
fNoised1	16.8853	0.0863	0.0399	0.0821	0.3899	0.2168	0.3339
fNoised2	15.2166	0.1028	0.0313	0.0594	0.3052	0.1621	0.2650
fNoised3	14.5021	0.1127	0.0515	0.0893	0.4601	0.2356	0.4048
fNoised4	16.1669	0.0937	0.0428	0.0631	0.3663	0.1979	0.3143
fNoised5	13.9493	0.1206	0.0246	0.0806	0.3408	0.1958	0.2899
fNoised6	13.6125	0.1208	0.0461	0.0802	0.4121	0.2164	0.3593
fNoised7	16.5549	0.0900	0.0273	0.0646	0.2965	0.1738	0.2484
fNoised8	19.2202	0.0666	0.0184	0.0481	0.2094	0.1227	0.1761
fNoised9	13.0860	0.1326	0.0367	0.0633	0.3568	0.1955	0.3047
fNoised10	18.4424	0.0730	0.0317	0.0648	0.3641	0.2067	0.3062
fNoised11	16.3218	0.0923	0.0366	0.0761	0.3822	0.2128	0.3261

Two primary measures are considered in assessing the denoised signal performance, i.e. Signal-to-Noise Ratio (SNR) and Root-Mean-Square Error (RMSE). Eleven denoised signals are generated for both cases of call and put options. The denoised signals are made to be built on different combination of parameters - wavelet family, denoising method, threshold rules and noise estimate level.

In examining the denoising performance of each signal generated, the denoised signal is plotted together with the original signal, as well as the coarse scale approximation of the signal. The coefficient plot indicates the level of decomposition that has underwent by the denoised signal. The plots of denoised signals against the original signal indicate the approximation plots of most signals are relatively noisy. A relatively clean approximation, however, is evident for the case of the 6-level decomposition wavelet, i.e. the gNoise8 signal. With regards to put options, the coarse scale approximation plots do not apply the same as in the case of the call options. To make an early deduction based on the plots per se can lead to biasness.

Further assessment on the performance of each denoised signal generated from the different combination of parameters as produced in this study is conducted using the SNR and RMSE against the original noisy signal. The respective results of both call and put options are represented in Table 3 and Table 4. From the tables, superior results are achieved by the gNoise8 signal with regards to call options. Both SNR (19.0515) and RMSE (0.1023) are recorded in the case of this 6-level decomposition of Symlet8 wavelet. Concerning put options, the best-performed denoised wavelet signal is realised by the fNoised8 signal.

Indifferent from the call options, the signal which is reconstructed from the 4-level of Symlet6 wavelet decomposition is found to produce the largest SNR (19.2202), and the least RMSE (0.0666).

Based on the comparison among the eleven wavelet signals respective to both call and put options, obvious finding is reached, i.e. the Symlet8 wavelet produces the best denoised signal. Additional pricing error measures based on MRPE and MARPE reported to be comparably small for both cases of studies; hence supporting the robust findings highlighted.

Table 5. Signal-to-Noise Ratio of Wavelet Signal against Smoothing-Method-Produced Signals of Call Options

Denoised Signal	Signal-to-Noise Ratio (dB)
gNoise8	19.0515
Moving average	13.1225
Savitsky-Golay	14.4083
Gaussian	14.4083
Moving Median	12.75360
Linear regression	14.2383
Quadratic regression	15.5252
Robust linear regression	14.0578
Robust quadratic regression	14.6336

Table 6. Signal-to-Noise Ratio of Wavelet Signal against Smoothing-Method-Produced Signals of Put Options

Denoised Signal	Signal-to-Noise Ratio (dB)
gNoise8	19.2202
Moving average	11.9691
Savitsky-Golay	12.9341
Gaussian	12.9341
Moving Median	11.9324
Linear regression	12.5929
Quadratic regression	12.9940
Robust linear regression	12.5929
Robust quadratic regression	12.9940

Additional investigation is done to include the comparison of the wavelet signal performance against other smoothing-method-produced signals. Eight smoothing methods are considered in comparing the Signal-to-Noise Ratio against the wavelet signal. The smoothing methods are Moving Average, Savitsky-Golay, Gaussian, Moving Median, Linear Regression, Quadratic Regression, Robust Linear Regression and Robust Quadratic Regression. Based on the SNR comparison results, the gNoise8 signal consistently records the biggest SNR, hence the best-performed signal.

Similar results are achieved in the case of put options. The most superior signal is found to consistently be the fNoised8 signal. The SNR of 19.2202 is recorded to be the biggest value, even against other eight smoothing-method-produced signals. The pricing performance of the wavelet-improved option prices are further compared against the non-wavelet-improved option prices. The authors investigate the pricing errors generated from both wavelet-improved data series as well as non-wavelet-improved data series

Table 7. Summary of Error Analysis of the Wavelet-Improved Option Prices against the Non-Wavelet-Improved Option Prices

Signal	RMSE (pts)	S.D.	MRPE (%)	S.D. (%)	MARPE (%)	S.D. (%)
Panel A: Call Options						
Non-wavelet-improved	0.3599	1.4547	136.6815	332.9900	136.8018	332.9406
Wavelet-improved	0.3572	1.4520	136.6883	330.0533	136.6997	330.0486
Panel B: Put Options						
Non-wavelet-improved	0.3230	0.3689	263.8663	186.2364	263.892	186.2000
Wavelet-improved	0.3149	0.3544	264.0043	171.6148	264.0577	171.5328

The results are expressed in terms of root-mean-square-error (RMSE), mean value of the relative pricing error (MRPE), mean value of the absolute relative pricing error (MARPE), in addition to the standard deviation in Table 7.

The RMSE results in Table 7 confirmed that the option prices generated out of the wavelet denoising process outperform those that do not undergo the process. The performance of how each wavelet-improved moment is approximated against the true values is compared based on three approaches: basic method, adapted method; and advanced method. The skewness and kurtosis in this study are set to be always $\gamma_1 = 0$ and $\gamma_2 = 3$, respectively based on the normal distribution. The option prices of both calls and puts, in which inclusive for both out-of-the-money (OTM) and at-the-money (ATM) moneyness are estimated using the Black-Scholes-Merton option pricing model. The tick size of the strike prices is chosen to be $1, ranging from K = 54 to 150. The wavelet-improved model-free moments estimated by the three methods – basic, adapted, and advanced – are reported in Table 8.

Table 8. Estimated Values of Improved Model-Free Moments

Model-Free Moments	True Values	Estimated Values		
		Basic	Adapted	Advanced
T-Period Variance (σ_T^2)	0.0225	0.0151	0.0154	0.0188
T-Period Skewness (γ_{1T})	0.0000	-2.6699	-2.5947	-2.6353
T-Period Kurtosis (γ_{2T})	3.0000	39.8798	38.3653	48.3879

In order to analyse further on how each estimated value generated from the three methods deviate from the true values, an approximation error based on the absolute method is presented in Table 9. It can be observed that the basic method performs poorly in estimating all model-free moments in all three methods. It occurs in this study that the adapted approach is more accurate compared to the advanced method. Nevertheless, looking at the number per se is quite unreliable. The relative error is found to be much relevant as the different in the absolute error can be quite negligible by number per se. The relative error for model-free wavelet-improved moments estimates is reported subsequently in Table 10. Special condition is applied in the case of skewness, in which the true value is assumed to be 0.100 to cater the zero-denominator problem in finding the percentage value of the absolute error.

It is obvious that the advanced approach fails to accurately estimate the model-free moments in the case of skewness and kurtosis estimations. The percentage difference between the adapted and advanced method itself is quite small, i.e. 1.35% in estimating skewness and 3.23% in estimating kurtosis. Thus, the adapted method performs the best in estimating the model-free skewness and kurtosis. With the small different in percentage, however, a clear line can be established in drawing conclusion that the advanced method is reliably quite accurate in all cases of estimations. Further comparison analysis is conducted between the wavelet-improved MFBKM moments and the original signals. This study finds that the wavelet-denoised higher moments record smaller approximation error in most cases compared to the noisy higher moments. It is shown that wavelet improves both consistency and error approximation of the signal. The results are observed in Table 9 and Table 10.

Table 9. Absolute Error for Improved and Noisy Model-Free Moments Estimates

Model-Free Moments	True Values	Wavelet-Improved MFBKM			NoisyMFBKM		
		Basic	Adapted	Advanced	Basic	Adapted	Advanced
T-Period Variance (σ_T^2)	0.0225	0.0074	0.0071	0.0037	0.0075	0.0071	0.0040
T-Period Skewness (γ_{1T})	0.0000	2.6699	2.5947	2.6353	2.6522	2.5748	2.7093
T-Period Kurtosis (γ_{2T})	3.0000	36.8798	35.3653	45.3879	36.9404	35.3627	45.0483

Table 10. Relative Error for Improved and Noisy Model-Free Moments Estimates

Model-Free Moments	True Values	Wavelet-Improved MFBKM			NoisyMFBKM		
		Basic	Adapted	Advanced	Basic	Adapted	Advanced
T-Period Variance (σ_T^2)	0.0225	0.3271	0.3141	0.1641	0.3312	0.3177	0.1771
T-Period Skewness (γ_{1T})	0.0000	26.6993	25.9467	26.3527	26.5220	25.7480	27.0930
T-Period Kurtosis (γ_{2T})	3.0000	12.2933	11.7884	15.1293	12.3135	11.7876	15.0161

VI. CONCLUSION

This research focuses on investigating the effect of wavelet on denoising the noisy-embedded signal. Eleven wavelet-denoised signals are considered with regard to both call and put options in controlling the noise of the original data. The wavelet signals are generated based on variant combination of parameters. The parameters are manipulated to include various combinations of wavelet family, denoising method, threshold rules as well as noise estimate level. There are six different denoising methods considered in this study –the Empirical Bayesian, Block James-Stein, Universal Threshold, Stein's Unbiased Risk Estimate (SURE), False Discovery Rate (FDR), and Minimax Estimation. The performances of the wavelet signals are examined against the original strike-option signal. Further comparison is executed to include eight other smoothing-method-generated signals. This study considers the sample data extracted from DJIA index options data, which covers the period from January 2009 until the end of 2015.

The wavelet-denoised signals are plotted with respect to the original signal, along with the coefficients used to reconstruct the denoised signals, for call and put options, respectively. Based on the plot, the approximation plots of most signals are relatively noisy. A relatively clean approximation, however, is evident for the case of the 6-level decomposition wavelet, i.e. the gNoise8 signal. With regards to put options, the coarse scale approximation plots do not apply the same as in the case of the call options. Further assessment on the signals performance based on the SNR, RMSE, MRPE and MARPE depicts superior results are achieved by the gNoise8 and fNoised8 signals, with regards to call and put options. Interestingly, the gNoise8 and fNoised8 signals consistently record the biggest SNR, hence the best-performed signals, even in comparison with eight other smoothing-method-produced signals.

This study finds that the wavelet-denoised higher moments record smaller approximation errors in most cases compared to the noisy higher moments. It is shown that wavelet improves both consistency and error approximations of the signal. The adapted method is found to perform the best in estimating the model-free skewness and kurtosis. It is evident in this study that the superiority of wavelet in producing denoised model-free higher moments effectively. This provides a platform that benefits further research to include wavelet in controlling the noise embedded inside a data.

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