# Scopus

### **Documents**

Hosen, M.A.<sup>a</sup> , Chowdhury, M.S.H.<sup>b</sup>

# Accurate approximations of the nonlinear vibration of couple-mass-spring systems with linear and nonlinear stiffnesses

(2019) Journal of Low Frequency Noise Vibration and Active Control, .

DOI: 10.1177/1461348419854625

<sup>a</sup> Department of Mathematics, Rajshahi University of Engineering and Technology, Rajshahi, Bangladesh

<sup>b</sup> Department of Science in Engineering, Faculty of Engineering, International Islamic University Malaysia, Kuala Lumpur, Malaysia

#### Abstract

An analytical technique has been developed based on the harmonic balance method to obtain approximate angular frequencies. This technique also offers the periodic solutions to the nonlinear free vibration of a conservative, couple-mass-spring system having linear and nonlinear stiffnesses with cubic nonlinearity. Two real-world cases of these systems are analysed and introduced. After applying the harmonic balance method, a set of complicated higher-order nonlinear algebraic equations are obtained. Analytical investigation of the complicated higher-order nonlinear algebraic equations is cumbersome, especially in the case when the vibration amplitude of the oscillation is large. The proposed technique overcomes this limitation to utilize the iterative method based on the homotopy perturbation method. This produces desired results for small as well as large values of vibration amplitude of the oscillation. In addition, a suitable truncation principle has been used in which the solution achieves better results than existing solutions. Comparing with published results and the exact ones, the approximated angular frequencies and corresponding periodic solutions show excellent agreement. This proposed technique provides results of high accuracy and a simple solution procedure. It could be widely applicable to other nonlinear oscillatory problems arising in science and engineering. © The Author(s) 2019.

#### Author Keywords

couple-mass-spring systems; Duffing equation; harmonic balance method; homotopy perturbation method; iterative method; Nonlinear stiffnesses; two-degree-of-freedom oscillation systems

#### Index Keywords

Algebra, Control nonlinearities, Degrees of freedom (mechanics), Harmonic analysis, Iterative methods, Perturbation techniques, Stiffness, Vibration analysis; Duffing equations, Harmonic Balance method, Homotopy Perturbation Method (HPM), Mass spring systems, Non-linear stiffness, Oscillation system; Nonlinear equations

Funding details International Islamic University MalaysiaIIUM

#### References

- Moochhala, Y.E., Raynor, S.
   Free vibration of multi-degree-of-freedom non-linear systems (1972) Int J Non-Linear Mech, 7, pp. 651-661.
- Huang, T.C.
  - Harmonic oscillations of nonlinear two-degree-of-freedom systems (1995) *J Appl Mech*, 22, pp. 107-110.
- Gilchrist, A.O.

The free oscillations of conservative quasilinear systems with two degrees of freedom

(1961) Int J Mech Sci, 3, pp. 286-311.

- Efstathiades, G.J.
   Combination tones in single mode motion of a class of non-linear systems with two degrees of freedom

   (1974) J Sound Vib, 34, pp. 379-397.
- Alexander, F.V., Richard, H.R.
   Non-linear dynamics of a system of coupled oscillators with essential stiffness

## nonlinearities

(2004) Int J Non-Linear Mech, 39, pp. 1079-1091.

- Chen, G.
   Applications of a generalized Galerkin's method to non-linear oscillations of twodegree-of-freedom systems (1987) J Sound Vib, 119, pp. 225-242.
- Ladygina, Y.V., Manevich, A.I.
   Free oscillations of a non-linear cubic system with two degrees of freedom and close natural frequencies

   (1993) J Appl Math Mech, 57, pp. 257-266.
- Cveticanin, L.
   Vibrations of a coupled two-degree-of-freedom system (2001) J Sound Vib, 247, pp. 279-292.
- Cveticanin, L. **The motion of a two-mass system with non-linear connection** (2002) *J Sound Vib*, 252, pp. 361-369.
- Dimarogonas, A.D., Haddad, S. (1992) Vibration for engineers, Englewood Cliffs, New Jersey, Prentice-Hall
- Telli, S., Kopmaz, O. **Free vibrations of a mass grounded by linear and non-linear springs in series** (2006) *J Sound Vib*, 289, pp. 689-710.
- Lai, S.K., Lim, C.W. Nonlinear vibration of a two-mass system with nonlinear stiffnesses (2007) *Nonlinear Dyn*, 49, pp. 233-249.
- Hashemi Kachapi, S.H.A., Dukkipatic, R.V., Hashemi, S.G.
   Analysis of the nonlinear vibration of a two-mass-spring system with linear and nonlinear stiffness

   (2010) Nonlinear Anal, 11, pp. 1431-1441.
- Bayata, M., Shahidia, M., Bararib, A.
   Analytical evaluation of the nonlinear vibration of coupled oscillator systems (2011) *Z Naturforsch*, 66a, pp. 67-74.
- Ganji, S.S., Bararib, A., Ganji, D.D. **Approximate analysis of two-mass-spring systems and buckling of a column** (2011) *Comp Math Appl*, 61, pp. 1088-1095.
- Bayat, M., Pakar, I., Shahidi, M. Analysis of nonlinear vibration of coupled systems with cubic nonlinearity (2011) *Mechanics*, 17, pp. 620-629.
- Wang, Y., An, J.Y.
   Amplitude-frequency relationship to a fractional Duffing oscillator arising in microphysics and tsunami motion (2018) J Low Frequency Noise Vib Active Control,
- Ren, Z.F., Hu, G.F.
   He's frequency-amplitude formulation with average residuals for nonlinear oscillators

   (2018) J Low Frequency Noise Vib Active Control,
- Mickens, R.E. (2010) Truly nonlinear oscillations,

Singapore, World Scientific Publishing

- Mickens, R.E. A generalization of the method of harmonic balance (1986) J Sound Vib, 111, pp. 515-518.
- Chowdhury, M.S.H., Hosen, M.A., Ali, M.Y. (2018) An analytical technique to obtain higher-order approximate periods for nonlinear oscillator, 19, pp. 182-191.
- Hosen, M.A., Rahman, M.S., Alam, M.S. An analytical technique for solving a class of strongly nonlinear conservative systems

(2012) Appl Math Comp, 218, pp. 5474-5486.

- Javidi, M. Iterative methods to nonlinear equations (2007) Appl Math Comp, 193, pp. 360-365.
- Wu, Y., He, J.H. Homotopy perturbation method for nonlinear oscillators with coordinate-dependent mass

(2018) Results Phys, 10, pp. 270-271.

**Correspondence Address** 

Chowdhury M.S.H.; Department of Science in Engineering, Faculty of Engineering, International Islamic University MalaysiaMalaysia; email: sazzadbd@iium.edu.my

Publisher: SAGE Publications Inc.

ISSN: 14613484 Language of Original Document: English Abbreviated Source Title: J. Low Freq. Noise Vib. Act. Control 2-s2.0-85074033584 **Document Type:** Article Publication Stage: Article in Press Source: Scopus Access Type: Open Access

ELSEVIER

Copyright © 2019 Elsevier B.V. All rights reserved. Scopus® is a registered trademark of Elsevier B.V.

**RELX** Group<sup>™</sup>