TOWARDS REALIZING MALAYSIA’S SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS (STEM) INITIATIVES: PROFILING STUDENTS’ CONCEPTUAL UNDERSTANDING OF “OPERATIONS ON INTEGERS

BY
MADIHAH KHALID
SUPIAH SAAD
JOHARRY OTHMAN
MOHD BURHAN IBRAHIM
YUDARIAH BT MOHAMMAD YUSOF
ZULMARYAN EMBONG

INTERNATIONAL ISLAMIC UNIVERSITY
MALAYSIA

2019

ABSTRACT

Previous studies have shown that students have difficulties and misconceptions in dealing with many areas of mathematics including in various topics of the number system such as integers. Students' difficulty with the concept of integers causes them to struggle in solving mathematical problems, especially those involving the four basic operations. This study aims to diagnose students' errors in the operations of integers, subsequent to validating the Errors Identification Integers Test (EIIT) which can identify the types of misconceptions that students possess in dealing with the operations of integers. The EIIT which consists of multiple-choice questions involving different combinations of positive and negative numbers was adapted to suit the Malaysian context. This study also determines whether there is any difference in terms of errors which may have been committed among students from rural and urban schools as well as gender. The population of this study is all Form One students from selected public schools in Peninsular Malaysia. A total of eight schools were involved in the data collection as samples. Cluster sampling was employed in order to ensure that the selected schools represented the population. The Rasch Model was used to improve and validate the instrument used in this study. In addition, teachers' and students' interviews were conducted to find and confirm the misconceptions of the operations of integers. Carelessness, poor knowledge, inability to assimilate concepts and surface understanding were identified as the types of misconceptions in this study. Meanwhile, this study found that parenthesis misapprehension, poor mathematical language, calculator hooking, superficial understanding and external limitation were the causes that led to the misconceptions. From the diagnostics test, it was determined that there was a significant difference between students' performance in different areas (rural and urban) and gender.

Keywords: *misconceptions, errors in integers, operations in integers, diagnosing errors in integers, Rasch model*

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

Mathematics is known as an abstract subject which constantly develops and changes from time to time (McEwan, 2000). Despite being one of the most important subjects, many students enter high school with severe gaps in their understanding of basic concepts and skills in mathematics. These weaknesses make it difficult for them to understand higher levels of mathematics. One of the basic concepts which functions as a precondition to the higher levels of mathematical concepts and skills involves a specific part of the number system which is the integer. Integers are positive and negative numbers and the numbers must not be in the form of fractions or decimals. They can be even or odd. For instance, -10, 500, and 0 are all integers, while one-half ($\frac{1}{2}$), 4.3 and pi are not integers. The important skill required for integers is performing basic operations on integers which involve signs of the numbers and the signs of the required operation.

Basic operations of integers seem simple, yet, according to Alsina and Nelson (2006), the students tend to get confused and struggle when they are asked to solve simple mathematical problems. It is difficult for the students because they have been taught to follow rules and procedures in a very abstract manner without going through models for better conceptual understanding. Hence, it is desirable that the students should grasp the fundamentals of mathematics so that they are able to learn the advanced mathematical processes far more easily. In addition, having good mathematical skills will ultimately save the students' time in examination and reduce the need for tutoring or remediation. Moreover, since each process builds upon prior knowledge and successful application of these skills, it is extremely important that the fundamentals are solid for every school student.

Another important element in building a strong fundamental in mathematics is the teaching methods used in the classroom. Since every public school in Malaysia is using the same syllabus, the only difference among them is the teacher's methods of teaching. Each teacher has his or her own ways of teaching in order to encourage students' learning and their participation in acquiring knowledge. Teachers play a vital role in ensuring that students understand the mathematical concepts systematically and comprehensively. Teachers are strongly encouraged to be flexible and creative throughout the teaching and learning process to make teaching mathematics effective. Beside that, teachers should know the nature of students' learning styles, strengths and weaknesses so that an effective teaching and learning environment can be designed. Recognizing the students' misconceptions in solving the mathematical problems, for instance, will assist the teachers to improve students' achievement in mathematics.

This study, subsequently, aims to identify the students' errors in operations of integers, following the development and validation of the instruments which can identify the types of misconception in integers.

1.2 STATEMENT OF THE PROBLEM

Malaysian students' performance in the "Trends in Mathematics and Science Study" (TIMSS) and "Program in International Student Assessment" (PISA) has resulted in a great worry that it would undermine the nation's aspiration for the Vision 2020 (Ideasorgmy, 2014). Much has been talked and reported about the Malaysian students' achievements in these two international tests and the major concern is pertaining to the teaching and learning of mathematics in our school system. Many Malaysian students seem to depend on rote memorization in learning mathematics and the teachers seem to teach the students using rules and procedures in order to get the correct answers, hence, neglect their conceptual understanding (Lim, 2011). She expressed that many teachers teach the students for the sake of passing the examinations instead of emphasizing on the understanding of concepts. Undoubtedly, she recommended that this situation occurred due to the challenging nature of teaching for conceptual understanding which requires extensive preparations and good content knowledge from the teachers. Lim also viewed that Mathematics teaching in many schools in Malaysia can still be characterized as teacher-centred.

On the other hand, the Ministry of Education recommended the focus of five elements in teaching and learning of mathematics which include problem solving, communication, reasoning, mathematical connections and application of technology (MOE, 2003). However, in the case of operations of integers, teachers prefer to provide the students with rules and ask them to memorize, and then drill them with enough practice to make them stick to the rules. This practice might lead to poor understanding and misapplication of the rules since the students will get confused with so many rules that they have to remember. For example, those who answer $6 + (-2) = -8$ argue that 2 added to 6 is 8, yet there is a minus sign which makes the answer to be negative.The fact that the rules are only applied to multiplication of a positive and a negative integer and not for addition of integers is lost without proper understanding.

However, it is important to explore the possible reasons of why students answer the question in such a way and it is more interesting if the errors form a certain pattern that can explain the students' thinking or their conceptual understanding in this case. In many situations, the students tend to use their previous knowledge and strategies that they used to apply for whole numbers in addition and subtraction when dealing with integers. This makes the teachers' approaches in teaching integers as an important investigation in understanding how teachers think when they teach this subject and what their level of knowledge in this topic is. By conducting such investigation, a proper solution could be identified to overcome more problems with regards to students' misconceptions of integers.

This study is a part of the diagnostic exercise to identify gaps in teachers' content knowledge and pedagogical skills as promoted in the Malaysian Education Blueprint (2012). Teachers are expected to understand their students' thinking processes and should be able to correct them at the earlier stage so that the problems shall not persist as they grow up into adults. It is evidenced by Sadler (2012) who found a significant proportion (38%) of adult students between 18 to 25 years of age who gave wrong answers to routine problems on operations of integer due to many different reasons which could be resolved if certain measures were taken to improve the situation.

In addition, teachers lack in terms of instruments which can be used to diagnose the types of errors that students perform in solving problems involving operations of integers. Some studies related to this topic (such as Sadler, 2012; Schindler & Hubmann, 2013; Rubin, Marcelino, Mortel & Lapinid, 2014; and Egadowatte, 2011) seem to rely on self-constructed instruments that have not yet been verified or validated. Therefore, this research produces a validated diagnostic instrument that can be used to identify students' errors and misconceptions in solving problems involving addition, subtraction, multiplication and division of integers, together with a full guideline or manual of how to use the instrument. The instrument was developed based on some existing researches and also this specific research. Teachers may use the instrument and the suggestions of how to teach and counter students' misconception by emphasizing on their conceptual understanding.

Briefly, this study aims to address the above-mentioned problems focusing on the operations of integers. In achieving this aim, the types of errors that the students performed were identified, the causes of errors were examined and a teaching model is proposed. The instrument was also validated to ensure that the students and teachers are able to distinguish the misconceptions in solving the mathematical problems.

1.3 OBJECTIVES OF THE STUDY

This research aims to investigate the students' errors in operations of integers and to help the teachers identify and design their instructional strategies in order to help students reconceptualise the topic. Specifically, the objectives of this study are to:

1. Validate a diagnostic tool suitable for use in investigating errors in the operation of integers among Malaysian Form 1 students.
2. Examine whether the data fit the Rasch model usefully well for the purposes of measurement.
3. Identify the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception?
4. Identify the causes of errors and misconceptions in the operation of integers.
5. Identify the differences between students from the various school settings, in their understanding of operation on integers?

1.4 RESEARCH QUESTIONS

The research questions of this study are as follows:

1. Is the EIIT on operation of integers for the Malaysian Form 1 students valid?
2. Do the data fit the Rasch model usefully well for the purposes of measurement?
3. What are the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception?
4. What are the causes of error in solving problems in operation of integers?
5. What are the differences between students from the various school settings, in their understanding of operation on integers?

1.5 THEORETICAL FRAMEWORK

This study is guided by the constructivist view of learning as the theoretical framework. According to Bruner (2001), constructivism asserts that students construct their own understanding of knowledge according to their experience or prior knowledge. The principles of constructivism are grounded in the established works of Piaget (1965) and Vygotsky (1978). Piaget's theory of cognitive development focuses on the role of individuals as active constructors of knowledge in the learning process (Piaget, 1964). Piaget believed students learn by doing through manipulating objects and connecting experiences to their prior knowledge in order to construct new meaning (Tunca, 2015). Both Piaget and Vygotsky valued the role of social processes and interactions as essential components to shape learning (Yuliani & Saragih, 2015). Learning from a Vygotskian perspective involves the process of internalisation, within the zone of proximal development (ZPD). In an educational context, Vygotsky believed in the important role of the learning environment, whereby a student interacts with the teacher and their peers, thus, the experiences and processes become internalised as their own belief and understanding (Tandiseru, 2015).

However, knowledge does not simply arise from experience. Rather, it arises from the collaboration between students' previous experience and present knowledge structures. Students need to construct their own understanding of each mathematical concept, thus, the primary role of teacher is not only focused on lecturing, explaining, or transferring mathematical knowledge, but to create situations for students to foster their making of the necessary mental constructions. As students are unable to interpret knowledge by themselves, then the role of teachers is to make knowledge into large units of interrelated concepts called schema. Schemas are valuable intellectual tools, stored in memory, and which students can retrieve back and utilised when it is needed. Therefore, according to constructivism, learning then basically involves the interaction between a student's schemas and new ideas.

This interaction involves two interrelated processes which are assimilation and accommodation. Assimilation is when the student learns new knowledge but if it is identifiably familiar. Hence this new idea can directly integrate into an existing schema. These schemas development helps in expanding existing concepts and creating new distinctions through differentiation. Meanwhile, accommodation occurs when the students were engaged with new ideas that are somewhat different from the existing schemas. They cannot assimilate the new idea due to irrelevant information. Therefore, it is necessary to re-construct and re-organise the schema. The re-construction leaves earlier knowledge intact, as part or subset or special case of the new modified schema. However, the previous knowledge is never erased. Thus, to understand an idea means to incorporate it into an appropriate existing schema. However, sometimes some new ideas may be so different from any available schema, that it is impossible to link it to any existing schema. In such a case the learner creates a new "box" and tries to memorise the idea, which can be called rote learning. Students engage in rote learning because the current ideas are not linked to any previous knowledge. As a result of the new idea not being well understood and isolated, therefore it is difficult to remember. This rote learning is cause of many mistakes in mathematics as students try to recall partially remembered and distorted rules. Since mathematics is a cumulative subject where the new knowledge gained is linked to the previous knowledge, hence, if a student is unable to assimilate and accommodate the new ideas, this creates a gap in the learning of the concept, and in turn, leads to mathematical errors or misconceptions.

Errors are wrong answers due to poor planning or understanding. A planning must be systematic so that students are able to apply the right ideas in certain situations. According to Roselizawati and Masitah (2014) and Radatz (1980), errors are the symptoms of the fundamental conceptual structures that become the cause of errors. The underlying beliefs and principles in the cognitive structure that become the cause of the systematic conceptual errors are known as misconceptions. Therefore, when teachers explain about the students' misconceptions, they have to look at the current students' schema and how they interact with each other, with instructions and also experience.

Making errors or misconceptions in mathematics is one of the significant learning barriers to the students. It is however also one of the best way to learn, in essence by making mistakes. It leads to deepening of the students' knowledge and a challenge to the students' thinking. However, the misconceptions must be dealt in decent ways. Most student errors are not of an accidental character, but are attributable to individual problem solving strategies and rules from previous experience in the mathematics classroom, that is incompatible with the teachers' instructions or techniques, or students observed patterns and inferences during instruction. The students fail to make connection with what they have already known. This happens when students receive wrong information at a certain stage. Hence, their schemas are already tainted with the wrong understanding. Students may connect pattern with a misconception and thereby learn an erroneous procedure.

Ashlock (2002) further stated that misconceptions and erroneous procedures are results of overgeneralisation and overspecialisation of rules in an effort to make sense of new information. Unless pedagogical actions are taken or interventions are done by teachers, some of these errors will persist for a very long time. There are beliefs held by the students that inhibit learning from errors and one of the beliefs is they should not learn from their mistakes. Students establish a robust structure that there is no connection between right and wrong ways of doing mathematics, and those beliefs drive them back to the beginning of a question and ignore errors in the solution. Another belief is that mathematics consists of disconnected rules and procedures. Students who hold such beliefs perceive mathematics as something which is not meant to make sense.

In addition, student errors are unique and reflect their understanding of a concept, problem or procedure. Analysing student errors may reveal the erroneous problem-solving process and thus, provide information on the understanding of and the attitudes towards mathematical problems. Upon analysing performance tests in solving text problems diagnostically, erroneous patterns demonstrated by students which are due to other language difficulties, inadequate understanding of texts, or incorrect number manipulation can be determined. Student errors are usually persistent unless the teacher intervenes pedagogically. By examining each of their written work diagnostically, teachers would be able to look for patterns and hence find possible causes for errors and misconceptions. Subsequently, teachers will develop strategies which can be used to encourage students to reflect on their understanding. According to Skemp (1976), concepts and schemata are stable once they are formed and are held to be resistant to change. Thus, good examples of concepts are required in order for proper concepts to be established. However, students are not always successful in acquiring or developing correct conceptual structures which resulted in misconceptions. Misconceptions and errors must not be seen as obstacles or 'dead ends', but must be regarded as an opportunity to reflect and learn. Teachers should recognise these misconceptions then prescribe them in an appropriate instructional strategy to be more diagnostically oriented in order to avoid any subsequent major conceptual problems. Diagnosis should be continuous throughout instruction.

Having identified students' misconceptions, the question then becomes how to deal with them. According to Cakir (2008), Longfield (2009) and Savion (2009), adopting a student-centered pedagogy is the best way to address misconceptions. In contrast to traditional, teacher-centered methods, which position the teacher at the literal and figurative center of the room, student-centered methods aim to place students at the center of their learning process, and to empower them as agents of their own learning. In addition, Bloom's Taxonomy is also a very worthwhile tool to promote the creative and critical thinking in mathematics. Using a range of problem solving activities is a good place to start since teachers can use some shorter activities and some extended activities depending on students' necessities. Open-ended tasks are easy to implement because they provide all students the opportunity to achieve success, together with the critical thinking and creativity. This is the essence of this research.

Another tool and skill involve in students' understanding is using multiple representations. Dealing with multiple representations and their connections play a key role for learners to build up conceptual knowledge in the mathematics classroom. According to Duval (2006) and Goldin and Shteingold (2001), representations play a special role in mathematics. As mathematical concepts can only be accessed through representations, they are crucial for the construction processes of the learners' conceptual understanding. Multiple representations are ways to symbolize, describe and refer to the same mathematical object. They are used to understand, develop, and communicate different mathematical features of the same object or operation, as well as connections between different properties. Multiple representations include graphs and diagrams, tables and grids, formulas, symbols, words, gestures, software code, videos, concrete models, physical and virtual manipulatives, pictures, and sounds. Therefore, representations are thinking tools for doing mathematics. What these methods have in common is that, in placing students at the center of the learning process, they engage them in an authentic process of discovery. It shows that when students are presented with compelling and authentic learning problems, they become more motivated and engaged. Activity-based methods also heighten the likelihood that students will challenge each other, or their own misconceptions, which is thought to have a more transformative effect compared to having one's ideas challenged by the teacher (Goldsmith, 2006). Representations such as concrete, verbal, real world and pictures, will help the students to comprehend the symbolic stage, at the same time enhance their conceptual understanding.

1.6 CONCEPTUAL FRAMEWORK

Many students, despite of having a good understanding of mathematical concepts, are inconsistent at computing. Errors occur due to misreading of signs, carrying incorrect numbers, not writing figures clearly, or putting figures in the wrong place. These students often struggle, especially at the primary school level where basic computation and the “right answers” are emphasized. Often these students end up in remedial classes, even though they are having potential for a higher-level of mathematical thinking. In addition, students have difficulties in making meaningful connections within and across mathematical experiences. For instance, a student may not comprehend the relationship between numbers and the quantities they represent. If this connection is not understood, mathematical skills may not be anchored in any meaningful or relevant manner. This could only make it harder for them to recall and apply mathematics in new situations.

Another constraint faced by the students with mathematical problems is their inability to easily connect the abstract or conceptual aspects of mathematics with the reality. Understanding what mathematical symbols represent in the physical world is important in determining how well and how easily a student will remember a mathematical concept. Holding and inspecting an equilateral triangle, for example, will be much more meaningful to a student than simply being told that the triangle is equilateral because it has three equal sides. And yet student with this problem find connections such as these painstaking at best. In addition, students find difficulties to effectively visualize mathematical concepts. Students who have this problem might not be able to judge the relative size among three dissimilar objects. This confusion has obvious disadvantages, as it requires a student to rely almost entirely on rote memorization of verbal and written descriptions of mathematical concepts that most people take for granted. Moreover, some mathematical problems also require students to integrate the higher-order cognition with perceptual skills.

Constructivism is the base for the current research. Constructivism dictates that students learn new mathematical concepts through previous knowledge structures. This theory has helped in guiding curriculum, instruction, and assessment across all disciplines covered by our formal educational system. It emphasizes on student-centered learning by encouraging teachers to provide guide for the students in discovering knowledge on their own. Hence direct instruction is not encouraged. This type of learning process gives opportunities for the teachers to understand their students' nature and needs.

In the world of mathematics, constructivism plays a vital role in the development of the students' thinking. This philosophy of learning is effective for the students who will learn better in a hands-on environment as it helps them to be able to relate the information learned in the classroom to the real life experience. The curriculum, according to constructivism, should cater the students' prior knowledge, encourage teachers to spend more time on their students' favorite topics, and allow teachers to focus on important and relevant information if necessary. Constructivism also encourages the students to work in groups as this approach will assist them to learn social skills, support each other's learning process and value each other's opinions and inputs. Constructivism can be promoted in cooperative learning, creative and critical learning, and multiple representations.

Cooperative learning activities can be used to supplement textbook instruction by providing students with opportunities to practice newly introduced topic or to review skills and concepts. Teachers can use cooperative learning activities to help students make connections between the concrete and abstract level of instruction through peer interactions and carefully designed activities. Meanwhile, students will work together to help each other understand content, solve problems or create projects and products with the teacher working as a moderator or facilitator. Collaborative learning are designed based on the understanding that interactivity and collaboration in small groups produces stronger solutions that would have not been reached individually and encourages sharing of research for enhanced learning. Further, it encourages trust building, communication, practical learning or application, and acceptance and enhances problem-solving skills. It also develops higher level thinking skills because then the students are allowed to think and diagnose the solutions of problems in their own ways with the teacher's guidance. When the students are able to think by themselves, they can make their own judgment and decisions. This process is important to build students' self-esteem and promote a positive attitude toward mathematics.

Another method of teaching that constructivists emphasise in mathematics is creative and critical thinking. Creative thinking is the process of coming up with new ideas or new approaches. It can be seen as a critical life skill and something worth to be developed in the students' thinking styles. Besides, it helps to enhance the students' imagination and concentration, as well as giving them the ability to view the world differently. Meanwhile, critical thinking include a complex combination of skills which requires students to reason, analyse, evaluate, solve problems and make decisions. According to Paul and Elder (2001) in the Foundation for Critical Thinking, students who are critical thinkers will display several characteristics such as being naturally skeptical. They approach texts with the same skepticism and suspicion as they approach spoken remarks. They will not simply accept all the information given to them, rather, they evaluate it in a good manner. In addition, students who are critical thinkers are active in the classroom. During teaching and learning process, they will actively ask the teacher about unfamiliar information and analyse it thoroughly. They consciously apply tactics and strategies to uncover meaning or assure their understanding. Moreover, they do not take an egotistical view of the world. They are open in receiving new ideas and perspectives. They are willing to challenge their beliefs and investigate competing evidence. Therefore, critical thinkers go beyond memorization of facts. In learning mathematics, both creative and critical thinking are essential for the students because they build higher-order thinking skills by prompting the students to relate new knowledge to their prior understandings, think in both abstract and conceptual terms, apply specific strategies in novel tasks, and understand their own thinking and learning strategies.

Constructivists also encourage teachers to use multiple representations, which help a lot in shaping students' thinking in mathematics. The strength of the representations assists teachers in explaining, demonstrating and evaluating mathematical problems to the students. However, the limitation of each representation does not make it work all the time. The misconceptions in the operations of integers are determined by giving students the EIIT. The EIIT will give the right and wrong answers and at the same time detect the misconceptions in this area.

The conceptual framework as illustrated in Figure 1 articulates a context for this study. The framework has been designed and assembled as an indication for this research. This research will confirm the causes of misconceptions through various methods of data collections. The hypothetical causes in Figure 1 acts as a guide and model in the investigation undertaken by this research.

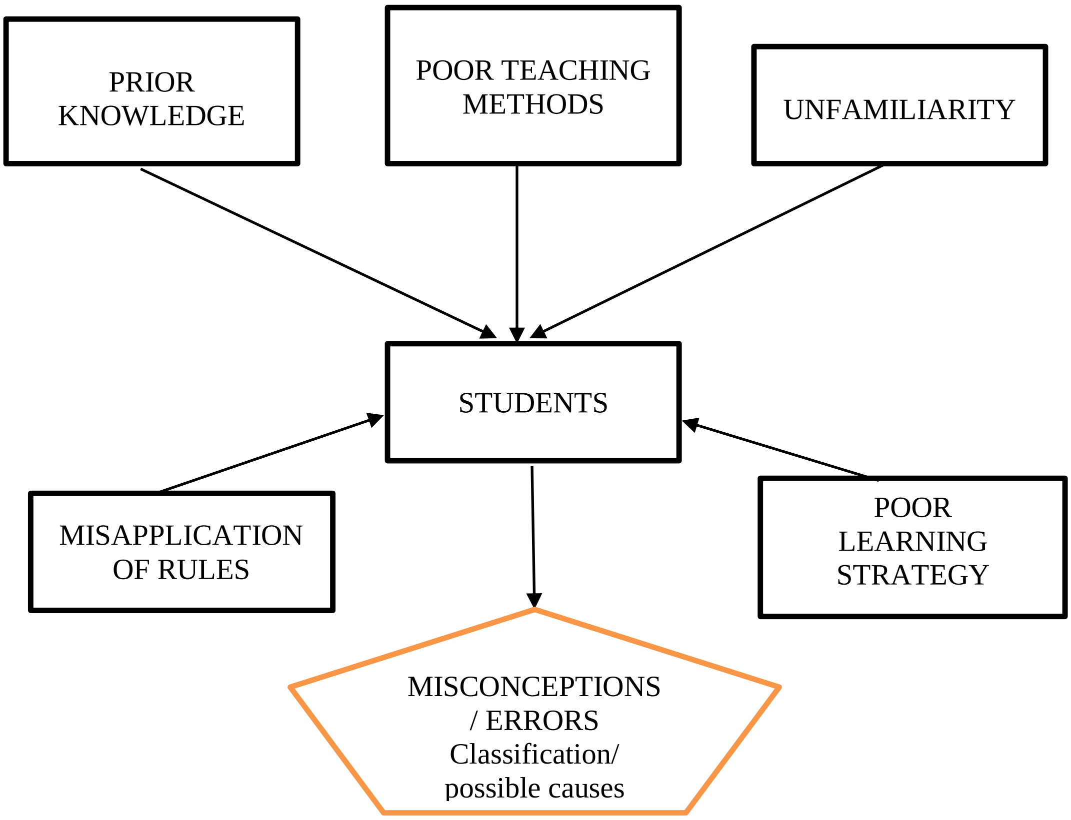


Figure 1: Conceptual framework of the research

1.7 SIGNIFICANCE OF THE STUDY

This study attempts to investigate the students' errors in solving problems involving integers. This study can be an eye opener for the teachers to improve the mathematics education by examining the errors and misconceptions performed by the students in solving routine problems of integers. In addition, this research inspires teachers to have systematic objectives, methodologies, and the diagnostic instrument to determine the students' weaknesses. As a result, the diagnostic instrument is expected to provide more effective information to the teachers in identifying the students' misconceptions. In addition, this EIIT also allows teachers to understand their students' knowledge, skills and behaviours with regard to the operations of integers, a known 'hot spot' of difficulty for many students. It will allow comprehensive collated data to be available, which can be compared within schools to better understand the students' achievement and monitor their progress.

Furthermore, this study also helps the teachers to develop students' understanding of mathematical concepts in dealing with operations of integer. According to Dean (2007), since the students are normally being taught using rules and procedures when dealing with the operations of integer, they are not critical and creative in solving other problems. In the worst case, the students might have difficulties when they are in the middle of the examination where they do not have clear ideas about mathematical concepts of integer. Bny Rosmah (2006) also agreed that students are taught using a set of rules. These rules are reinforced through drilling and practices. Most of the students tend to follow the rules without understanding and knowing why the methods work.

Another problem faced by the students in solving operations of integers is negative numbers. According to Bny Rosmah (2006), the types of error made by the students were mainly in addition of negative with positive numbers and addition of negative with negative numbers. The common error made in addition of negative integers was the students tend to add two given integers by first, ignoring their signs and second, placing a sign in front of the answers following some mixed-up rules of addition, subtraction or/and multiplication. Therefore, this study is important to diagnose the nature of the misconceptions in operational of integers, so that the students' potential errors are estimated.

In addition, this study also looks at the differences between the performances of students from different types of school. This is important in order to identify the differences of the methods used in teaching the operations of integer in different schools. There might be a chance that the different results and different types of students' error are linked by different teaching methods used. Therefore, this research is significant for both the teachers and the students whereby it helps the teachers to diagnose the students' difficulties at the earliest stage and give remedies immediately. Meanwhile, the students can take benefits by learning self-reflection and sharing responsibilities of their own learning.

1.8 DELIMITATION OF THE STUDY

In conducting this research, there were some boundaries that the researchers had to build. First, the first phase of validating the EIIT, this research was delimited to Form One students in four states in Malaysia, and from eight different schools only. This happened due to time and budget constraints. The researcher tried to include all states in Malaysia in the study, but, it seemed impossible to access. The schools were selected using stratified random sampling in order to get a good representation of respondents from all over Malaysia. Second, for the second phase which was the lesson observation and interview process, both students and teachers were involved. Thus, the researchers had to be careful to avoid any disturbance of schools' schedules. Only one class from each school was observed. However, the researchers observed the teaching and learning of integers from the beginning until the end. As for the interview, again, to avoid disturbance of the teaching and learning process, only a selective number of teachers and students were interviewed.

1.9 DEFINITION OF TERMS

Error or misconception in operations of integers refers to the unexpected behavior or misunderstanding that may be exploited to cause wrong calculation which lead to incorrect answers.

EIIT refers to the diagnostic instrument that will assist the researcher to identify students' misconceptions in operations of integers. In this study it is a set of test with forty (40) multiple choice questions.

Traditional learning means that students will be taught by their teachers using number lines and generalized rules. According to Sarah (2012), teachers prefer to use these methods when they attempt to make their students understand the concept of operations of integers. In this study, teacher was use number line method as suggested in a Form 1 textbook.

Cognitive constructivism refers to the type of constructivism established by Jean Piaget through his study and analysis of the epistemological stages of learning and cognitive development (Huitt & Hummel, 2003). It focuses on changes within individuals during the construction of knowledge and beliefs. For the purposes of this study, it is one of the constructivist theories used to support the importance of constructive learning in mathematics. Mathematics activities must be reflective of previous interactive learning rather than following directions regarding earlier skills because developmental cognitive learning must be constructed (Van de Walle, 2004). The foundation for cognitive constructivism is also supported by the National Council of Teachers of Mathematics (NCTM, 2000) which supports the growth of the cognitive stages in children that accounts for developmentally appropriate strategies used when providing mathematics instruction for elementary students (NCTM, 2000).

Constructivism refers to the theory base of this study. According to Marlow and Page (2005) and Midgley (2000), constructivism theory is the learning process to the mastery based instruction where students construct their own knowledge from provided information, learn new knowledge by reflecting on previously learned information, and learn through engagement and discussion of personal thinking with their classmates and teacher.

Social constructivism refers the type of constructivism associated to Lev S. Vygotsky, and additional supporting theorists that affirms the importance of social interaction during the cognitive learning process. It creates the foundation for the construction of future knowledge and beliefs (Vygotsky, 1978). For the purposes of this study, social constructivism is used as a theoretical foundation because of its alignment with the standards of the National Council of Teachers of Mathematics (NCTM, 2000). Through social interaction, discourse, and collaboration students expand learning as they exchange pertinent information with their peers and make connections with their own cognitive learning during their developmental progression (Dangel & Guyton, 2004; NCTM, 2000).

Rasch Model refers to the procedure to identify the probability of a correct response of the difference between the person and item parameters. In this research, Rasch model will be used to validate the EIIT and to identify students' misconception in solving operations on integers.

Academic achievement is the outcome of curriculum that shows the extent to which students, teachers, or institutions have achieved their educational goals. According to Ahmed Gubbad (2010), it is commonly measured by examinations or continuous assessments.

1.10 SUMMARY

This research focuses on diagnosing and confirming the students' misconceptions while solving problems related to the operations of integers. In the background, this research attempts to show that there are many problems faced by both the educators and the students especially when they are sitting for the international level examinations such as TIMSS and PISA. Hence, this research aims to validate a EIIT so that the teachers can use it as a tool to detect their students' weaknesses and misconceptions in the operations of integers. In addition, this study wants to identify the types of misconceptions and determine the causes of the errors.

CHAPTER TWO

LITERATURE REVIEW

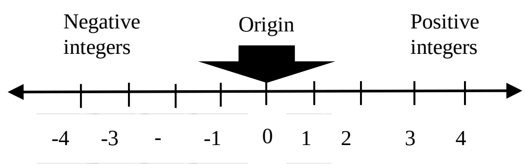
2.1 INTRODUCTION

Despite the shortage of research on teaching and learning of integers in the secondary schools in Malaysia, this chapter reviews some local and international research literatures which are related to the main research questions of the current study. The focus will be on the issues pertaining to the main problems in learning integers especially the students' understanding and their skills in the operations of integers. Some literatures regarding the methods of teaching integers will also be critically discussed.

2.2 WHAT ARE INTEGERS?

Integers are a special group or category of numbers that consists of the set of numbers {… -4, -3, -2, -1, 0, 1, 2, 3, 4…}. They are all positive and negative whole numbers, which do not include any fractional or decimal part. An integer is a whole number that can be either greater than 0, called as positive, or less than 0, called as negative. Zero is neither positive nor negative but it is also considered as an integer. Two integers that are at the same distance from the origin in the opposite directions are called as opposites.

Figure 2.1: Number Line of Integers



Integers can be illustrated in a number line as shown in Figure 2.1. The arrows on each end of the number line show that the line stretches to infinity in both the negative and positive directions. We do not have to include a positive sign (+) when we write positive integers. However, we do have to include the negative sign (-) when we write negative integers. Meanwhile, zero is called the origin, the centre of the number line.

Many students are more familiar with the positive numbers which make them uncomfortable with the negative numbers. Negative numbers are the numbers with negative signs, which have a value less than zero. To introduce positive and negative numbers to the students, we need to relate these numbers to the daily life situations. Students need to be able to apply integers in the real-world applications. Teachers, on the other hand, need to develop understanding of the concepts of integers to their students. However, Hayes (1996) argued that teaching for understanding of negative number concepts and operations was generally considered as one of the difficult topics. Although this topic is introduced at the primary level, the students at the secondary level still find it difficult and have no confidence in solving problems pertaining integers. It seems like effective learning is not achieved. This is evidenced by the results of the "Trends in International Mathematics and Science Study" (TIMSS) in 2012 which indicated that the levels of mathematics achievement in Malaysia for the secondary level had not increased since 2001. In addition, Dean (2007) reported that students had difficulties in understanding new topics and new concepts since they failed to imagine and visualize those concepts.

2.3 DIFFICULTIES IN LEARNING INTEGERS

Many studies describe different strategies used by teachers and students in the teaching and learning integers (Dean, 2007). Yet, it is found that the students are still having difficulties in solving problems of integers. Thus, the effort to develop effective teaching strategies for integers is ongoing. In order to make students understand integers, we have to extend their knowledge, help them to make a logical connections with what they have already known, and use appropriate learning strategies.

The difficulties in learning integers especially the four rules of integers arise from the confusion between binary operations of plus and minus and the unary operators which are positive and negative (Sarah, 2012). This confusion is due to many texts using the same symbols for both plus and positive, and minus and negative. Moreover, students always ask ‘Why do they have to learn negative numbers? And what is the use of negative numbers in our everyday life?’ This shows that students are not given anything to relate to, other than a set of rules governing the combination of negative and positive number for the operations. Hence, they cannot make sense of the multiplication of a negative number with a negative number and why the product of two negative numbers becomes positive. Silver and Marshall (1990) emphasized that when students are taught new concepts, they must know how to adapt and extend their existing understanding by both connecting new information to their current knowledge and constructing new relationships within their knowledge structure.

Teachers also find it easier to teach the rules than to teach for meaning and they hope the students’ understanding will develop as they operate successfully with the relatively ‘simple rule’. However, some students find it difficult to establish the rules themselves. Therefore they just rely on remembering them instead of understanding. This leads to rote learning where students only know how to solve the problems of integers but do not understand why it happens in such a way. Baroody and Ginsburg (1990) described that understanding in mathematics learning involves knowing the concepts and principles related to the procedures being used and making meaningful connections between prior knowledge and the knowledge units being learnt. Meanwhile, Hart, Brown, Kuchemann, Kerslake, Ruddock and McCartney (1981) argued that the difficulty in learning mathematics stems from the need to work consistently with such rules without recourse to an external, concrete reference, which most secondary school students seem unable to do. Realizing the students' difficulties, the researcher feels that teachers should teach their students how to apply mathematical thinking in order for them to understand the topic, rather than applying the method blindly and mechanically without any awareness of the significance of the answers.

2.4 INTEGERS IN THE MALAYSIAN'S MATHEMATICS CURRICULUM

In 2013, a new curriculum was introduced to all public schools in Malaysia. The Minister of Education, Datuk Seri Mahdzir Khalid stated that the curriculum for primary and secondary schools were revised to embed a balanced set of knowledge and skills such as creative thinking, innovation, problem-solving and leadership among the students. For the primary level, the Standard Based Curriculum for Primary Schools (KSSR) was introduced while the Standard Based Curriculum for Secondary Schools (KSSM) was introduced for the secondary level. According to Malaysian Education Blueprint (2012), the curriculum needs to be revised and upgraded to ensure that the new curriculum is always parallel with the global demands. Datuk Seri Mahdzir Khalid (2016) also said that the new curriculum emphasizes on student-centred learning and focuses more on problem-solving, project-based assignments, updating subject or theme and implementing formative assessments. In addition, the students who need additional guidance would continue to have access to the right assistance to ensure their success. Teachers with high leadership qualities will be placed at schools nationwide to ensure that the students would be able to develope in a holistic manner, in line with the National Education Philosophy.

Integers are made as the first topic for the first chapter of the KSSM mathematics curriculum (KSSM, 2013). This shows that integers are vital for the students to master, before they learn more complicated mathematical skills. Teachers need to ensure that the students are familiar with addition, subtraction, multiplication and division of integers since they are the foundation of the learning of algebra which happen to be the subsequent topic after that. Besides that, teachers must engage students in activities that will help to enhance their mental arithmetic using integers. Moreover, teachers are required to engage the students with the daily use of integers. The lessons could help the students to explore some laws governing the operations of integers and mathematical models are used to reinforce the algorithms that they commonly use.

2.5 WHAT ARE MISCONCEPTIONS?

Confrey (1990) reviewed the literature on misconceptions in the fields of science, mathematics and programming. She noted that some terms were used in these fields including alternative conceptions, student conceptions, pre-conceptions, conceptual primitive, private concepts, alternative frameworks, systematic errors, critical barriers to learning and naive theories. She commented that the dominant perspective was that 'in learning certain key concepts in the curriculum, students were transforming in an active way what was told to them and those transformations often led to serious misconceptions, language, and informal knowledge' (p.19).

Graeber and Johnson (1991) presented the characteristics of misconceptions (p.3-15) as:

1. Self-evident where the person does not feel the need to prove them.

2. Coercive where the person is compelled to use them in an initial response.
3. Widespread where it happens among both naive learners and more academically able students.

According to Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989), in making inferences and interpretations, students were very likely to make at least temporary errors. Errors are intrinsic to all-learning where at least as a temporary phenomenon because they are a natural result of students' efforts to interpret what they are told and to go beyond the actual cases that are presented. In addition, errors are intelligent construction based on what is more often incomplete than incorrect knowedge. Therefore, errors in instruction cannot be avoided. Hence, teachers need to be aware of their existence and ensure that their students do not persist with these misconceptions for a long period of time.

2.6 SOME COMMON MISCONCEPTIONS OF INTEGERS

Students find integers and operations of integers as difficult and challenging. The fact that the value of -27 is less than -12 is contrary to the students' experience with the positive whole numbers. To understand this concept, students need to build mental images and models that allow them to visualize the comparisons and relationships.

The operation of subtraction, especially subtracting a negative, is difficult for the students to make sense of. The idea of subtracting a negative number which gives the same result as adding the opposite of negative numbers, is difficult for many students to comprehend. When students have a little understanding of subtraction of negative numbers, they may end up just blindly follow the rules. A study by Hart et.al (1981) found that when students were faced with the expression like $+8-(-6)$ many of them used the rule to work out the appropriate sign, operated with it (in this case by adding 8 and 6), and ignored the starting point. This may be applicable for some cases but not for some others such as in solving $(-2 - -5)$ where students would give 7 as the answer. Hayes (1999) found that slight misapplications of the rules, such as applying 'two negatives make a positive' to $-4 + -2$ to get $+6$, are common.

Teachers have to be very careful of the language that they use when teaching integers, especially to properly use the word 'negative' when they need to address the negative numbers such as 'negative one or negative two' and the word 'minus' should be avoided. The use of the term 'minus' instead of 'negative' could make the students confused between the sign and the operations.

According to Kuchemann (1981), understanding does not necessarily flow from the use of number line because it is an abstracted representation of abstract ideas. In order to develop mathematical concepts, it is important to develop the students' understanding by linking the new concepts with the known concepts and everyday experiences. The researcher's experience of teaching integers indicates that students are taught that adding two negative numbers will produce a negative answer but multiplying two negative numbers will result in a positive one. It is difficult for the students to understand this concept, the reason why the numbers behave that way, which causes their confusion. The difficulty in understanding the multiplication of negative numbers is due to the reason that it is not something that the students do in their everyday lives. The only practice that they use everyday is multiplication of positive numbers.

2.7 OTHER RESEARCH ON ERRORS IN INTEGERS

Developing strong mathematical skills early in life is necessary for all students. Mathematics also helps students develop general problem-solving skills. Furthermore, according to the National Council of Teachers of Mathematics or NCTM (2000), without a strong foundation in early mathematics, students are not prepared to enrol in more advanced mathematics courses at the high school and college level. Mathematics courses such as algebra, geometry, statistics, and calculus provide an essential foundation not only for careers in fields of science, technology, engineering, and mathematics (STEM), but also in social science research, business, and accounting, for example. Schools in rural areas, however, do not always have access to the same level of federal funding as urban and suburban schools, which can limit the opportunity students in rural schools have for learning mathematics (Patterson, 2010). Nine percent of rural school district budgets are covered by federal funds, compared to 11 percent of budgets in urban school districts (Provasnik, KewalRamani, Coleman, Gilberston, Herring, & Xie, 2007). According to Waters (2005), low salaries, threats of consolidation, and the geographic isolation of many rural areas make it a challenge for rural districts to attract and retain highly qualified teachers, particularly in high-need subjects such as mathematics. Young (2006) found from his study that the location of the school had a significant effect upon student achievement, as students attending rural schools were not performing as well as students from urban schools. Owoeye and Yara (2011) and Ijenkeli, Paul and Vershima (2012) also found that students in urban areas exhibit better performance than their rural counterparts in mathematics, reading, and science.

Some studies found that gender differences do not affect achievement in mathematics. For example, Fennema, Carpenter, Jacobs, Franke, and Levi (1998) revealed that from their research on Grade 3 (8 to 0-year olds) students, gender differences did not affect the students' ability to solve mathematical problems. However, they found that female students tend to understand mathematics better by using modelling and concrete methods. Meanwhile, male students tend to solve mathematical problems by using abstract ideas. Leahey and Guo (2001) also discovered that the male students performed better in numbers compared to female students. The authors believe that the male students have more skills and capabilities to solve mathematical problem. In England, Cooper and Dunne (2000) in their study found that the means in mathematics examination for boys were higher than those for girls.

However, in Germany, Brunner, Krauss and Kunter (2007) found that girls slightly outperformed boys on reasoning ability, but on specific mathematics ability, boys had a significant advantage over girls. It was supported by Neuville and Croizet (2007) in a study on 7 to 8-year olds conducted in France, in which they found that when gender identity was noticeable, girls performed better than boys on easy problems.

2.8 ERRORS IN THE OPERATION OF INTEGERS

There are several errors and misconceptions found in the operation of integers. According to Bny Rosmah and Khalid (2006), the misconceptions in the operation of integers were divided into four categories; addition of integers, substraction of integers, multiplication of integers and division of integers.

2.8.1 Addition of Integers

1. Most students have no problem when two positive numbers are added. Students who made mistake in the addition of two positive numbers seemed to give a negative sign to the first number to get the answer (probably because of being unsure of the position of the unsigned number).
2. When positive and negative numbers are added together, the most common mistake the students make is, the numbers are added together by ignoring the negative sign for instance $3 + (-2) = 5$. In this case, the students give the

answer without considering the negative sign. Sometimes, a negative sign is added later to the sum. For instance, $3 + (-2) = -1$, where the students might think that negative and positive produce negative. The second mistake is usually executed by the students who use the number line. The addition operation makes the students move right and ignore the negative sign. The third mistake is, the students add correctly but change the sign of the answer (maybe they think that the positive and negative produce negative).

3. When two negative numbers are added together, the most common mistake is to make the answer as positive, possibly due to the thought that two negatives produce positive. The second mistake is made when the students use number line and move right because of the addition operation or they apply the distributive law wrongly. Another mistake, although is not frequently done, is to multiply these two numbers together.

2.8.2 Subtraction of Integers

1. Most students have no problem when two positive numbers are subtracted. Those students who made mistakes in the subtraction of two positive numbers seemed to subtract correctly, but placed the negative sign to the result (thinking that negative and positive produce a negative answer, and because this is a test on integers). When a bigger number is subtracted from a smaller number, their mistakes include adding the two numbers then give it a negative sign and a few students put a positive sign to the total or some even multiply these two numbers together.

2. When a positive and negative number are subtracted, the most common mistake that students do is to move left in the number line because of the subtraction for $2 - (-6)$ to give -4 for those who uses number line. For $-6 - 2$, many

students give the answer -4 (by taking away 2 from 6 and adding the negative sign because it is the sign of a bigger number 6 or wrongly applying distribution law). The next common mistake is to add the two numbers and then giving the answer a positive sign (negative and negative make positive) or negative sign (it is the sign of a bigger number 6). Other mistakes include multiplying the two numbers and give a positive answer (because of the negative sign and the subtraction operation), and also subtract the two numbers by ignoring the signs and then make the answer positive (two negatives make positive).

3. When two negative numbers are subtracted, the most common mistake is to add the two numbers (ignoring the signs) and give the answer a negative sign (because there are three negatives or because they move left on the number line due to subtraction operation) or a positive sign (less common but multiplying the operation with the sign of the first number and then with the second). The next common mistake is for them to multiply the two signs of the number to become positive and then subtract the numbers. The least common is multiplying the numbers together to give a negative answer (because of three negative signs).

2.8.3 Multiplication of Integers

1. When two positive numbers are multiplied, students do not have much problems. A small number of students (6.7%) made mistakes by adding the two numbers.
2. When a positive and a negative number are multiplied, most mistakes are in the sign of the correct answer. Some students believe that the answer should carry the sign of the bigger number. The other mistake is students tend to add the numbers together.

3. When two negative numbers are multiplied, the most common mistake is the wrong sign, with similar reasons as in (2). Other mistakes include students' tendency to add or subtract the two numbers.

2.8.4 Division of Integers

1. Students have no problem when two positive numbers are divided. Those who make mistakes tend to subtract the two numbers.
2. When a positive and negative numbers are divided, again the most common mistakes are in putting the signs for the answers. Some students might believe that the quotient should carry the sign of the bigger number. Other mistakes include adding or subtracting the numbers.
3. When two negative numbers are divided, similarly, the most common mistake is in putting the signs for the answers, with similar reasons of doing that as in (2). Other mistakes include adding, subtracting or even multiplying the numbers.

2.8.5 The Mistakes in the Mixed Operations

1. Many students make mistakes in the subtraction part, for example $4 - (-2) = 2$ and add this 6 to make 8.
2. In answering the question $(-4 + 6) \div -2$, more than 30% of the students gave 5 as the answer due to the mistake that they made by adding $(-4 + 6)$ to get-10.
3. Confusion of the signs.
4. Most mistakes are made in putting the signs when the students do subtraction of integers.

2.9 SUMMARY

There are a number of studies that have been reviewed in this chapter that have provided various factors that might lead to errors and misconceptions in the operations of integers. In addition, some of the previous studies reviewed clearly state that types and causes of misconceptions. Lastly, in order to minimise errors, algebraic tiles has been suggested as a viable way to alleviate the errors.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

This present study aims to identify the students' errors and misconceptions in operations of integers among the Form One students by using mixed-method that employ both quantitative and qualitative methods of collecting data. This chapter outlines the methodology employed in the study, including the research design, sample, instruments, procedure for data collection, and procedure for data analysis.

3.2 RESEARCH DESIGN

This study aims to identify the students' error in operations of integers and validate the diagnostic instruments that lead to the identification of the types of misconception made by the students in the operations of integers. The process of identification involved the development of a EIIT that is suitable to be used in Malaysian context and the validation of the test as a diagnostic instrument. In addition, students who give outstanding answers in EIIT were interviewed in order to recognize the students' way of understanding. Quantitative and qualitative methods were engaged to gather the informative data. Therefore, there are three phases which were employed throughout this study.

Phase 1 - Developing diagnostic tool (Review and Exploratory in Nature)

During this phase, the researcher was employed in documenting the literature, organizing round table discussions to gather as much information as possible that lead to the errors in operation of integers, collecting data and then validate it. The data collection from the EIIT was pilot-tested using one class from a school in Klang Valley area. After preliminary analysis and refinement of the EIIT (*see Appendix A),* the tests were given to the selected eight schools in four states in Malaysia which are Kedah, Johor, Terengganu and Selangor. These schools represent the four regions in Malaysia and were selected through stratified random sampling where the strata identified is the location of the school (urban and urban). According to Creswell (2013), stratified random sampling is helpful for various subgroups in the population. Stratified random sampling divides a population into strata, and random samples are taken, in proportion to the population, from each of the strata created. The members in each of the stratum formed have similar attributes and characteristics. This method of testing is broadly utilized and exceptionally useful when the target population is heterogeneous. In this study, Kedah represented the north region of Malaysia. Meanwhile, Johor represented the south region, Terengganu represented the east region, and lastly Selangor represented the west region of Malaysia. Each state has two schools for reprentating them. These two schools then were divided into two areas which are rural and urban schools. This is needed for this research to find if there is any differences in students' performance according to the location of the schools.

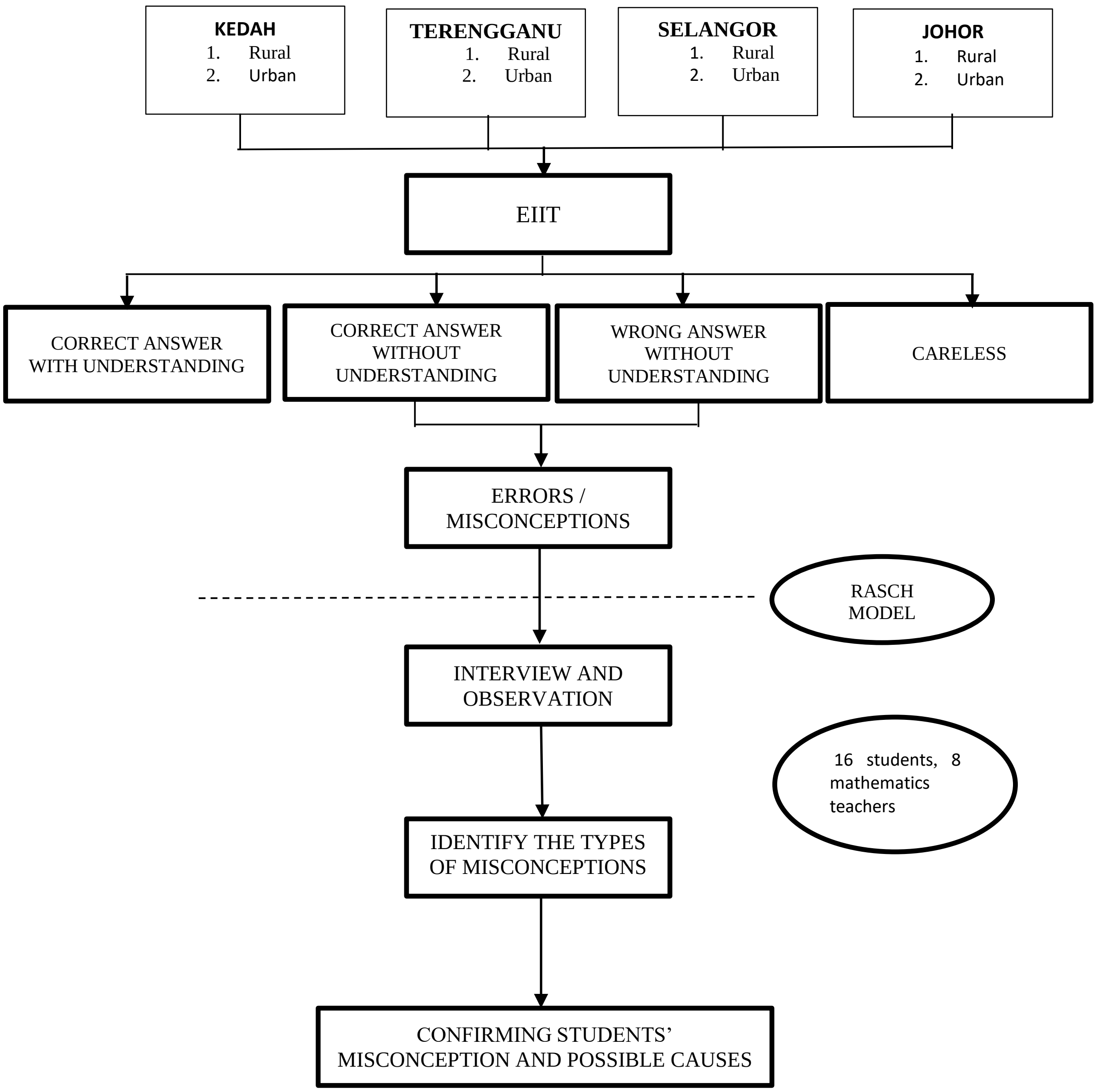
Phase 2 - Validating diagnostic tool (Descriptive Confirmatory)

The data collected were analyzed to validate the diagnostic tool, and to find the errors that were exhibited by the students in solving operations of integers' problems. The validation of the EIIT was done using Rasch model in order to find the reliability and validity of the items in the test, and hence whether the data fit the Rasch model. The validity includes the difficulties index and discrimination index of the tests.

Phase 3 – Classroom Observations and Interviews

Observation in the classroom were employed in order to understand students' behaviour during integers' class and to examine teachers' way of teaching integers. After the lesson, the mathematics teachers were interviewed to gather the information regarding the students (*see Appendix B*). Later, the researcher interviewed selected students who gave outstanding data (*see Appendix C*). Two interview sessions were held for every school by focusing on students who performed particular errors, who were asked to explain their thinking from the errors they made. The interview data was supplemented with observation data where the researcher observed and video-taped the teaching and learning process. The result of interviews were put together with the EIIT as a way to make comparison between different schools in terms of students' errors in operation of integers.

Figure 3
Framework of the study



3.3 POPULATION AND SAMPLE

3.3.1 Population

The population of this research is defined as Form One students from public secondary schools in Peninsular Malaysia. In an attempt to ensure a representative sample, states were identified for participation based on geographic location and congressional district. Next, school districts were selected to ensure diversity of socio-economic status of students. Based on these factors, eight schools from four states in Malaysia were invited to participate. Meanwhile for the participating teachers, they must be wel-trained teachers who taught mathematics. The researcher also obtained the consent letter from the ministry in order to complete the data collection.

3.3.2 The Sample

There were eight schools of Form 1 in four states in Malaysia which participated in this study for the first phase. The sample of this study was selected by using stratified random sampling. This covered 622 students which are the required sample for the form one students' population of 429273 in Malaysia according to Malaysia Educational Statistics (2014). Krejcie and Morgan (1970) propose this figure for sampling error of 5% with a confidence level of 95%. The sample classes were chosen using stratified random sampling method so that students from various background will be represented to give a better picture of the whole population. According to Creswell (2008), stratified random sampling assists in producing a smaller error of estimation compared to a simple random sample of the same size. The small error is due to the population which was divided into strata by important categories relevant to this study. Firstly, Form One students are chosen because they already learned the basic operation skills such as addition and subtraction during primary level. In addition, they also had completed the

Malaysian National Primary Certificate (UPSR) which made them having the same level of proficiency. Furthermore, the schools will be stratified by four regions in Malaysia so that this study was obtain significant data because all parts of West Malaysia will be covered.

. However, since this study was using Rasch model analysis to find the validity of the test, where the sample is independent, therefore, the most reliable interpretation is with at least 50-100 subjects (Granger, 2008). Wright and Stone (1979) also agreed that the size can be as small as 100. However, they further stated that by using Rasch model analysis, the study must consider two aspects: the examinees and the number of items. They recommend a minimum of 200 examinees for a minimum of 20 items in a particular test.

3.4 INSTRUMENTATIONS

3.4.1 The instruments

There were a few instrument that were used in this study to get all the information. Firstly, the researcher used pencil-and-paper instruments to determine errors and misconceptions. The Integer Achievement Test or EIIT comprised of 40 questions. This test consisted entirely of 40 questions and is shown in Appendix A. The EIIT that was administered was adapted from Bny Rosmah (2006). The researcher then makes some changes according to Malaysian context and transform the test into multiple choice questions. The 40 questions on this test covered the integers' components as outlined in Table 3.1. The maximum score on the EIIT is 40 and the minimum possible score is 0. The tests item are constructed in accordance to the syllabus content of the topic.

Table 3.1: Topic area covered in integers test

Table 3.1 (a): Addition of integers. There are eight different patterns for addition of integers considered.

Questions	Types	Answer
1	(positive) + (positive)	Positive
2	(positive) + (negative)	Positive
3	(positive) + (negative)	Negative
4	(positive) + (negative)	Zero
5	(negative) + (positive)	Positive
6	(negative) + (positive)	Negative
7	(negative) + (positive)	Zero
8	(negative) + (negative)	Negative

Table 3.1 (b): Subtraction of integers. There are eight different patterns for subtraction of integers considered.

Questions	Types	Answer
9	(positive) - (positive)	Positive
10	(positive) - (positive)	Negative
11	(positive) - (positive)	Zero
12	(negative) - (positive)	Negative
13	(positive) - (negative)	Positive
14	(negative) - (negative)	Positive
15	(positive) - (negative)	Positive
16	(negative) - (negative)	Negative

Table 3.1 (c): Multiplication of integers. There are five different patterns for multipication of integers considered.

Questions	Types	Answer
17 & 22	(positive) × (positive)	Positive
18 & 23	(positive) × (negative)	Negative
19 & 24	(negative) × (positive)	Negative
20, 21 & 25	(negative) × (negative)	Positive

Table 3.1 (d): Division of integers. There are four different patterns for division of integers considered.

Questions	Types	Answer
26 & 29	(positive) ÷ (positive)	Positive
27 & 30	(positive) ÷ (negative)	Negative
28 & 31	(negative) ÷ (positive)	Negative
32 & 33	(negative) ÷ (negative)	Positive

Table 3.1 (e): Problem solving on integers (word problem). There are four different patterns for problem solving of integers considered.

Questions	Types	Answer
34	(positive) + (positive)	Positive
35	(negative) − (negative)	Negative
36	(positive) − (positive)	Positive
37	(negative) + (negative)	Negative

38	(positive) + (negative)	Negative
39	(negative) + (positive)	Negative
40	(positive) – (negative)	Positive

Furthermore, another instrument used in this study was teachers and students interviews protocol. Interviews were done to elicit more information from teachers and students about the misconception in operations of integers. A semi-structured interview questions (*see Appendix B and C*) were used based on their outstanding answers. The last instrument used in this study was the observation checklist for semi-structured observation.

3.4.2 Interview Protocol

Apart from the quantitative data, eight teachers and sixteen students were interviewed to investigate teachers' teaching methods and to gather some insights on students' outstanding answers in the EIIT in order to support the findings respectively. Every school was represented by two students with different levels of achievements with respect to their answers in EIIT. Meanwhile, the teachers were asked about the content and methodology that they use in the classroom. The complete interview transcripts is included in Appendix B and Appendix C.

3.4.3 Observation Checklists

After the EIIT, the observation in the classroom took place. In this research, the observation part is important to identify students' behaviour and development in the classroom in learning operations of integers. Observation assists the researcher to determine each students' interests, skills and needs. The researchers will know the students as individuals so that they can monitor each of student's strength and weakness. Observation also helps in measuring students' growth and development over time. It allows the researcher to see how students are progressing cognitively, physically, socially and emotionally during the lesson. Besides, observation can make the changes to the environment. By observing the way students use play spaces and materials, the researchers can determine whether materials are meeting the students' needs. In addition, observation helps in identifying the concern. The researchers can see if students have special requirements that need to be addressed. These can range from a hearing problem to a need for extra attention. Therefore, observation acted as part of communication between the researchers and students in the classroom. *See Appendix E* for Observation Checklist.

3.5 PROCEDURE

The EIIT for this study were distributed to students in January until May of 2017 for pilot study. Permission was obtained from the Ministry of Education (*see Appendix F)* and also the permission from each State Education Office (who holds school's legal authorities for each state in Malaysia) (*see Appendix G*). Then, the researcher obtained the permission from all principals of the schools that participate in this study. This permission is needed in order to let the principals know the flow of the research and also gave the advocate from the ground. The researcher hand-delivered the EIIT to the district offices and schools the week after the obtained State Education Office's approval, which was the third week of January 2018 of school year. The district office representatives and school principals distributed the instruments to all Form One students in each school. The EIIT was returned by each participating students to the return box strategically placed in a central location by the selected teachers. The test took approximately thirty minutes to be completed by the students. The researcher arranged a date within two weeks after the tests with the district office representatives and school principals for collecting the completed EIIT. After getting the complete EIIT, the researcher marked and evaluated the result of each school. The EIIT was analysed using Rasch model in order to find the reliability and validity of the test. Then, outstanding results from the students were chosen so that they can be interviewed by the researcher. The researcher then selected sixteen students who give different answers from others. This procedure is to identify the misconceptions that students committed during the test.

3.6 PILOT STUDY

In this study, the pilot test was conducted in July 2016 on Form 1 students at a school in Klang Valley. The purpose of the pilot test is to examine the reliability and validity of the questionnaire. The cronbach alpha was used to determine the strength of questions and any questions which are considered confusing or misleading were identified and thus were corrected or omitted. The researcher also was familiar with the existing knowledge and level of understanding of Form 1 students who have studied the topic of integers. In addition, pilot test is important to examine the credibility and transferability of the interview question.

The main data collection instrument of the EIIT was pilot-tested with two classes in Klang Valley area consisting of forty students. These students were not included in the actual research as samples. Although the sample of the pilot test was a different batch of students, the researcher still maintain the characteristics of the samples. The researcher found that the result of EIIT did not clearly show the error made on the four operations of integers since most of the errors are made on calculation and problem solving questions. The researcher decided to modify the EIIT by using Malay translation and change the questions more to the Malaysia context. The reason for this is that the researcher noticed that the students were confused with the lengthy question. Students also found the Malay translations to be helpful for them in answering the question.

Another analysis that the researcher did is face and content validity of the EIIT. The mathematics teachers of the school in this study and the school in the Pilot study were asked whether the questions in the test were suitable for their students. They commented that all the questions on the tests were valid in the sense that the students might reasonably have been expected to answer the question correctly as a result of studying the topic integers. The teachers also thought that this EIIT was appropriate for assessing students' knowledge and understanding on integers. The duration of the pilot test was 30 minutes. The reliability of the instrument was determined in term of internal consistency (cronbach alpha reliability). It was found that the internal item consistency of the EIIT using Rasch Model for the pilot test was 0.91 which shows a good reliability score. No question was omitted from the study. Meanwhile for the person reliability, it shows the result of .79. It means, the questions are meaningful to diagnose students' understanding in integers. Analysis of the reliability achievement test of the pilot study can be referred in Appendix H. Appendix I shows that the questions in the EIIT was scattered and assessed most of the students. However, there were students who are able to answer the difficult questions. Therefore, the questions in EIIT need to be improvised in order to let every student can be tested. After preliminary analysis and refinement of the diagnostic instrument, the instrument was sent to the selected school. A pilot test was also conducted for the semi-structured interview protocol. This pilot test is important to ensure if there is any flaw within the interview design. This pilot test is also vital to the researcher to make necessary revisions towards the interview questions.

3.7 DATA COLLECTION AND ANALYSIS

After the data was collected, the data was organised and analysed to answer the research questions of the study. All the analyses of the performance data and EIIT were computed using Rasch model and SPSS package (Statistical Package for Social Science), version 21.0. At the preliminary stage, content analysis were employed to collect all the details concerning the knowledge and understanding of integers among Form 1 students. The contents consist of the following information:

1. Lower secondary Form 1 syllabus – The EIIT used the syllabus from the current curriculum as a guideline to ensure that the questions will not deviate from the lesson planning.
2. Past examination including PMR and PT3 examination – The questions from the EIIT were created based on the past year examination inclucing PMR and PT3.

After the EIIT, interview were held with sixteen students (those who seemed to be having problems with integers). The interview was one-to-one and tape-recorded. An interview schedule, based on the students interview guide, was prepared for the interview with the students (*see Appendix C*). After the students has been asked to answer the selected questions, they were asked to verbalise the thinking which generated the answers. In this manner, their level of understanding of some of the intergers tested in the integer test was probed. Although the interview questions for students were prepared in advanced, additional questions were asked during the interviews, when this is deem necessary by the researcher. In particular, any comments made by the informants which appear to reveal important aspects of their attitude towards integers, were noted and probed by the researcher. These interviews were conducted to determine the difficulties and the types of errors made by the students. According to Cohen, Manion and Marrison (2000), the most practical way of achieving greater validity with interview is to minimize the amont of bias as much as posibble. The sources of bias are the characteristics of interviewer, the chracteristics of the informants and the substantive content of the questions. More particularly they include the attitudes, opinions and expectations of the interviewers; a tendency for the interviewer to see the informant in her own image; a tendency for the interviewer to seek the answer that support her preconceived notions; msconceptions on the part of interviewer of what the informants is saying; and misunderstandings on the part of the informants of what is being asked. The researcher tried to minimize all of bias aspects and maximise the validity of the interview. Table 3.2 shows the data collection and method of analysis based on the objectives of this study.

Table 3.2 Data Collection and Analysis

Research Objective	**Data Type**	**Data Collection Strategy**	**Validity**	**Data Analysis**
Validate a diagnostic tool suitable for use in investigating errors in operation of integers among Malaysian Form 1 students.	Quantitative	• Diagnostic Instrument • Interview • Observation	• Statistical test • Model fit • Membercheck • Persistent Obs	• Winsteps

Examine the data fit the Rasch model usefully well for the purposes of measurement.	Quantitative	• Diagnostic Instrument	• Statistical test • Model Fit	• Winsteps
Identify the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception.	Qualitative	• Interview (student + teachers) • Observation	• Triangulation • Member check • Persistent Observation	• Theme Searching
Identify the types of errors and misconceptions in the operation of integers.	Quantitative Qualitative	• Diagnostic Instrument • Observation • Interview	• Triangulation • Member check	• Statistical • Theme searching
Identify the differences between students from the various school settings, in their understanding of operation on integers.	Quantitative	• EIIT	• Statistical test • Checklist	• ANOVA

3.8 SUMMARY

The eight schools from the four states geographically dispersed throughout Malaysia were used as representative samples in the research study. Data from these participants were analysed to explore the causes of the misconceptions in operations on integers of participating secondary school students. After the return and collection of EIIT, and following the guidelines and procedures established by the researcher and school districts and school principals, data were entered into SPSS and then exported to the Winsteps for further analysis using a Rasch model. Through the Rasch model, the students who misfit, if any, among were identified. Then, one-to-one interviews with teachers and students were employed to identify the causes of misconceptions in operations on integers.

CHAPTER FOUR

4.1 INTRODUCTION

This chapter provides the results of the data analysis of the study. The researcher engaged a set of EIIT, an interview with the outstanding answers, and classroom observation. Then, for the phase of the EIIT, Rasch model was utilized in order to investigate the validity of the test. In addition, the data were analysed using Statistical Package for the Social Science 18.0 (SPSS) to identify the significance difference in students' performance in rural and urban area. For the second phase of interview and observation, the data were analysed by using thematic method.

4.2 RESEARCH FINDINGS

In this chapter, the findings of the study are discussed in five parts, which are:

1. Part I: Demographic background of the respondents.
2. Part II: The result from Rasch model.
3. Part III: The Rasch model from the EIIT.
4. Part IV: The Interviews with the teachers.
5. Part V: The interviews. The information from the interviews are used to identify the misconceptions in operations of integers.
6. Part VI: The result from ANOVA.

For Part I, descriptive statistical procedure was used to examine the demographic background of the respondents. Next, for Part II, the result from the EIIT are used to identify the reliability and validity by using Rasch model. From this analysis, the difficulties index and discrimination index were examined.

In addition, the findings of this study are also based on the five formulated research questions on misconceptions in operations of integers among Form One students and the effectiveness of algebraic tiles in teaching integers.

The questions are as follows:

1. Is the EIIT on operation of integers for the Malaysian Form 1 students valid?
2. Do the data fit the Rasch model usefully well for the purposes of measurement?
3. What are the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception?
4. What are the causes of error in solving problems in operation of integers?
5. What are the differences between students from the various school settings, in their understanding of operation on integers?

4.2.1 Part 1: Demographic

This study employed cluster sampling in the selection of four schools that represent four regions in Malaysia (i.e. Kedah, Johor, Terengganu and Selangor). Two schools from each state (one urban and ne rural) were chosen as representatives. The distribution of respondents for each of the participating school is summarised in Table 4.1.

Table 4.1 Number of Respondents for Each School

School	Rural	Urban	Percentage (%)
Kedah	79	81	17.4
Johor	55	53	25.7
Selangor	83	94	28.4
Terengganu	108	69	28.5
Total	**325**	**297**	**100**

Table 4.1 shows that a total of 622 respondents of Form One (13 years old) students from eight schools were involved at this stage of the study. All of the students were Malaysian and they included Malays, Chinese, Indians and other ethnic groups. A total of 294 (47.3%) students involved in this study were male, while 319 (51.3%) students were female. Meanwhile, nine students (1.4%) did not specify their gender. The distribution of respondents is summarised in Table 4.2.

Table 4.2 Profile of the Respondents

Respondents	Age	Nationality	Total Students	Gender	Frequency	Percentage (%)
Form One	13 years old	Malaysian	622	Male	294	47.3
				Female	319	51.3
				Did not specify	9	1.4

Meanwhile, eight teachers and sixteen students were interviewed for the interviews. All the teachers were teachers in mathematics. From the eight teachers, only one was male. Meanwhile, seven (43.75%) of the interviewed were male and nine (56.25%) were female. The distribution of the interview respondents is summarised in Table 4.3.

Table 4.3 Profile of Respondents Interviewed

Teacher	Nationality	Total Teachers	Gender	Frequency	Percentage
Teacher	Malay	8	Male	1	12.5
			Female	7	87.5
Student		16	Male	2	12.5
			Female	14	87.5

4.2.2 Part II: Validating the EIIT

Research Question 1: Is the EIIT on operation of integers for the Malaysian Form 1 students valid?

This study was designed to identify students' errors and misconceptions in the operations of integers. Therefore, the reliability and validity of the EIIT need to be determined to ensure that the test is valid and reliable for use. To determine the reliability and validity of the EIIT, three indicators were examined. They include item polarity, item fit and construct validity. Rasch model was used to ascertain these factors.

4.2.2.1 Item polarity

Item polarity, which is denoted by the point-measure correlation coefficient, will indicate the extent to which test items are working in the same direction to define the measured construct. Negative and zero values indicate that items or students are working in the wrong direction. Thus, in the investigation of item polarity, relatively high positive values are desired (Linacre, 2010a).

Table 4.4 shows the point measure correlation (PTMEA CORR.) for the 40 items in the EIIT. The results show no negative and zero values. That means all items defined the measured construct in the same direction. Although all items had positive point measure correlation coefficients, 10 of them were below 0.3 (between .02 - .29). The low correlation coefficients indicate that these items did effectively discriminate between persons with high ability and those with low ability. However, the EIIT items show that they were working in the same direction to measure the construct.

Table 4.4 Item Polarity Statistics: Correlation Order (Operations in Integers)

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PT-MEASURE CORR.	PT-MEASURE EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	ITEM
35	184	597	1.99	.10	1.43	8.4	1.89	9.5	.02	.42	63.3	74.3	Q35_SU
26	498	606	-.87	.11	1.16	2.3	2.11	6.6	.09	.34	83.3	82.9	Q26_DI
1	582	620	-2.23	.17	1.08	.6	1.76	2.5	.12	.23	93.9	93.9	Q1_AD
23	463	600	-.50	.11	1.15	2.7	1.63	5.1	.17	.37	80.5	78.7	Q23_MU
27	428	598	-.14	.10	1.19	3.9	1.44	4.6	.20	[illegible]	[illegible]	[illegible]	[illegible]
38	153	593	2.30	.10	1.20	3.7	1.51	4.9	.20	[illegible]	[illegible]	[illegible]	[illegible]
11	580	620	-2.16	.17	.99	.0	.98	.0	.23	[illegible]	[illegible]	[illegible]	[illegible]
37	384	597	.25	.09	1.19	4.6	1.24	3.4	.25	[illegible]	[illegible]	[illegible]	[illegible]
36	500	602	-.93	.12	1.07	1.0	1.21	1.5	.26	[illegible]	[illegible]	[illegible]	[illegible]
40	139	593	2.45	.11	1.10	1.7	1.40	3.6	.29	.40	76.7	78.8	Q40_SU
34	521	601	-1.25	.13	1.00	.0	1.06	.4	.30	.31	87.5	87.0	Q34_AD
8	303	604	.96	.09	1.17	4.7	1.21	3.8	.30	.44	62.4	69.0	Q8_AD
17	576	615	-2.17	.17	.94	-.4	.64	-1.5	.30	.23	93.7	93.6	Q17_MU
22	565	609	-2.02	.16	.92	-.6	.69	-1.4	.33	.24	92.8	92.8	Q22_MU
39	306	594	.91	.09	1.11	3.1	1.18	3.3	.34	.44	66.2	68.8	Q39_AD
25	388	603	.26	.09	1.06	1.6	1.16	2.2	.35	.42	72.5	71.2	Q25_MU
9	543	618	-1.40	.13	.96	-.5	.79	-1.2	.35	.30	88.7	88.1	Q9_SU
16	285	610	1.14	.09	1.06	1.8	1.09	1.8	.38	.44	68.0	69.3	Q16_SU
31	450	602	-.34	.10	.97	-.6	1.03	.3	.40	.38	78.4	76.9	Q31_DI
29	520	609	-1.13	.12	.90	-1.3	.77	-1.6	.41	.32	86.9	85.8	Q29_DI
18	499	606	-.88	.11	.93	-1.1	.76	-2.0	.42	.34	84.5	83.1	Q18_MU
19	455	601	-.41	.10	.96	-.7	.86	-1.5	.42	.38	78.2	77.6	Q19_MU
30	443	601	-.28	.10	.95	-.9	1.00	.1	.42	.39	78.2	76.2	Q30_DI
10	461	616	-.36	.10	.97	-.7	.83	-1.8	.43	[illegible]	[illegible]	[illegible]	[illegible]
24	495	604	-.85	.11	.90	-1.6	.74	-2.2	.44	[illegible]	[illegible]	[illegible]	[illegible]
14	249	608	1.43	.09	.97	-.8	1.05	.9	.45	[illegible]	[illegible]	[illegible]	[illegible]
28	445	597	-.33	.10	.91	-1.8	.82	-1.9	.46	[illegible]	[illegible]	[illegible]	[illegible]
15	364	613	.50	.09	.94	-1.7	.90	-1.7	.48	[illegible]	[illegible]	[illegible]	[illegible]
13	347	609	.63	.09	.94	-1.8	.91	-1.7	.49	.43	72.1	69.4	Q13_SU
20	400	601	.13	.10	.91	-2.3	.82	-2.6	.50	.41	75.4	72.1	Q20_MU
32	404	601	.11	.10	.91	-2.4	.82	-2.6	.50	.41	76.2	72.4	Q32_DI
21	406	605	.10	.10	.90	-2.4	.79	-3.1	.51	.41	75.5	72.4	Q21_MU
33	402	599	.11	.10	.87	-3.3	.80	-2.8	.52	.41	76.8	72.3	Q33_DI
6	318	611	.86	.09	.90	-2.9	.87	-2.7	.52	.44	73.5	69.0	Q6_AD
3	416	617	.07	.09	.86	-3.5	.79	-3.1	.53	.41	76.8	72.6	Q3_AD
4	425	617	-.02	.10	.85	-3.8	.75	-3.5	.54	.41	77.1	73.3	Q4_AD
12	227	611	1.62	.09	.86	-3.6	.86	-2.4	.54	.44	80.7	71.8	Q12_SU
5	276	613	1.21	.09	.85	-4.5	.84	-3.2	.57	.44	76.2	69.5	Q5_AD
2	344	619	.68	.09	.83	-5.3	.79	-4.2	.58	.44	78.2	69.2	Q2_AD
7	358	607	.52	.09	.83	-5.1	.75	-4.6	.58	.43	77.4	69.8	Q7_AD
MEAN	402.5	606.2	.00	.11	.99	-.3	1.04	.0			78.1	77.0	
S.D.	114.2	7.7	1.16	.02	.13	2.9	.34	3.2			7.9	7.7	

Low correlation

High correlation

4.2.2.2 Item fit

Next, fit statistics were calculated to help detect discrepancies from the Rasch model expectation to ensure that the items were contributing meaningfully to the measurement of the variable or construct. The two major fit statistics used were the infit and outfit

Mean-square statistics. These statistics indicate the amount of “distortion of measurement system” (Linacre, 2010a, p. 514). The recommended range for multiple choice items is 0.7 – 1.3. The items within the recommended range are considered productive or meaningful to the measurement, and values below this range indicate that the items are considered as over-fitting, while those above this range are considered as mis-fitting (Bond & Fox, 2007; Wright, Linacre, Gustafsson & Martin-Lof, 1994).

Table 4.5 shows the infit and outfit mean-square of individual items. For the infit mean-square, all items were within the specified range (0.7 – 1.3). However, the outfit mean-square index shows that seven (7) items have outfit mean-square values above 1.3. There is no outfit item below 0.7. The means of infit mean-square is (.99 logit) and outfit mean-square is (1.04 logit), closer to the expected value (1.0), indicating little variation from the expectation of the Rasch model. The standard deviation of the outfit mean-square (.34 logit), however, was larger than the standard deviation of the infit mean-square (.13), showing more variation in the outfit items. However, in general, the items seem to fit well for this study.

Table 4.5 Item Statistics: Misfit Order

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PT-MEASURE CORR.	PT-MEASURE EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	ITEM
26	498	606	-.87	.11	1.16	2.3	2.11	6.6	A .09	.34	83.3	82.9	Q26_DI
35	184	597	1.99	.10	1.43	8.4	1.89	9.5	B .02	.42	63.3	74.3	Q35_SU
1	582	620	-2.23	.17	1.08	.6	1.76	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	_AD
23	463	600	-.50	.11	1.15	2.7	1.63	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3_MU
38	153	593	2.30	.10	1.20	3.7	1.51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8_SU
27	428	598	-.14	.10	1.19	3.9	1.44	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7_DI
40	139	593	2.45	.11	1.10	1.7	1.40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0_SU
37	384	597	.25	.09	1.19	4.6	1.24	3.4	H .25	.42	64.5	71.1	Q37_AD
8	303	604	.96	.09	1.17	4.7	1.21	3.8	I .30	.44	62.4	69.0	Q8_AD
36	500	602	-.93	.12	1.07	1.0	1.21	1.5	J .26	.34	83.6	83.7	Q36_MUSU
39	306	594	.91	.09	1.11	3.1	1.18	3.3	K .34	.44	66.2	68.8	Q39_AD
25	388	603	.26	.09	1.06	1.6	1.16	2.2	L .35	.42	72.5	71.2	Q25_MU
16	285	610	1.14	.09	1.06	1.8	1.09	1.8	M .38	.44	68.0	69.3	Q16_SU
34	521	601	-1.25	.13	1.00	.0	1.06	.4	N .30	.31	87.5	87.0	Q34_AD
14	249	608	1.43	.09	.97	-.8	1.05	.9	O .45	.44	74.8	70.5	Q14_SU
31	450	602	-.34	.10	.97	-.6	1.03	.3	P .40	.38	78.4	76.9	Q31_DI
30	443	601	-.28	.10	.95	-.9	1.00	.1	Q .42	.39	78.2	76.2	Q30_DI
11	580	620	-2.16	.17	.99	.0	.98	.0	R .23	.23	93.5	93.5	Q11_SU
10	461	616	-.36	.10	.97	-.7	.83	-1.8	S .43	.38	75.3	76.9	Q10_SU
19	455	601	-.41	.10	.96	-.7	.86	-1.5	T .42	.38	78.2	77.6	Q19_MU
9	543	618	-1.40	.13	.96	-.5	.79	-1.2	t .35	.30	88.7	88.1	Q9_SU
13	347	609	.63	.09	.94	-1.8	.91	-1.7	s .49	.43	72.1	69.4	Q13_SU
15	364	613	.50	.09	.94	-1.7	.90	-1.7	r .48	.43	70.8	69.8	Q15_SU
17	576	615	-2.17	.17	.94	-.4	.64	-1.5	q .30	.23	93.7	93.6	Q17_MU
18	499	606	-.88	.11	.93	-1.1	.76	-2.0	p .42	.34	84.5	83.1	Q18_MU
22	565	609	-2.02	.16	.92	-.6	.69	-1.4	o .33	.24	92.8	92.8	Q22_MU
28	445	597	-.33	.10	.91	-1.8	.82	-1.9	n .46	.38	79.7	76.7	Q28_DI
20	400	601	.13	.10	.91	-2.3	.82	-2.6	m .50	.41	75.4	72.1	Q20_MU
32	404	601	.11	.10	.91	-2.4	.82	-2.6	l .50	.41	76.2	72.4	Q32_DI
21	406	605	.10	.10	.90	-2.4	.79	-3.1	k .51	.41	75.5	72.4	Q21_MU
6	318	611	.86	.09	.90	-2.9	.87	-2.7	j .52	.44	73.5	69.0	Q6_AD
29	520	609	-1.13	.12	.90	-1.3	.77	-1.6	i .41	.32	86.9	85.8	Q29_DI
24	495	604	-.85	.11	.90	-1.6	.74	-2.2	h .44	.34	83.9	82.7	Q24_MU
33	402	599	.11	.10	.87	-3.3	.80	-2.8	g .52	.41	76.8	72.3	Q33_DI
3	416	617	.07	.09	.86	-3.5	.79	-3.1	f .53	.41	76.8	72.6	Q3_AD
12	227	611	1.62	.09	.86	-3.6	.86	-2.4	e .54	.44	80.7	71.8	Q12_SU
4	425	617	-.02	.10	.85	-3.8	.75	-3.5	d .54	.41	77.1	73.3	Q4_AD
5	276	613	1.21	.09	.85	-4.5	.84	-3.2	c .57	.44	76.2	69.5	Q5_AD
2	344	619	.68	.09	.83	-5.3	.79	-4.2	b .58	.44	78.2	69.2	Q2_AD
7	358	607	.52	.09	.83	-5.1	.75	-4.6	a .58	.43	77.4	69.8	Q7_AD
MEAN	402.5	606.2	.00	.11	.99	-.3	1.04	.0			78.1	77.0	
S.D.	114.2	7.7	1.16	.02	.13	2.9	.34	3.2			7.9	7.7	

Higher than range (0.7-1.3)

4.2.2.3 Construct validity

Continuum of increasing intensity

Furthermore, evidence of a continuum of increasing intensity is achieved by the fact that there are no significant gaps in item distribution, and items should be distributed evenly (i.e. no redundant items). Figure 4.1 shows that there are no significant visible gaps between item distributions. The map shows that persons are distributed on the left side of the logit ruler and items on the right side of the logit ruler. It can be seen that a person's mean logit falls slightly on the upper item of mean logit and there are gaps on the left and right side in the upper mean logit line that are known as person and item free respectively. These indicate the test is slightly easy for students with strong ability. This test was prepared to be easier than the students' expected ability due to the fact that this study attempts to identify misconceptions that occur in the operations of integers. Although the questions were easy, only 31 out of 622 students are located at above 3.0 logit. It further shows that most of the students are located between -1 to 2 logit. Meanwhile, the least able students are shown at -2. It is observed that items on addition and subtraction are the most difficult operations while items on multiplication are considered easiest by the majority of the students.

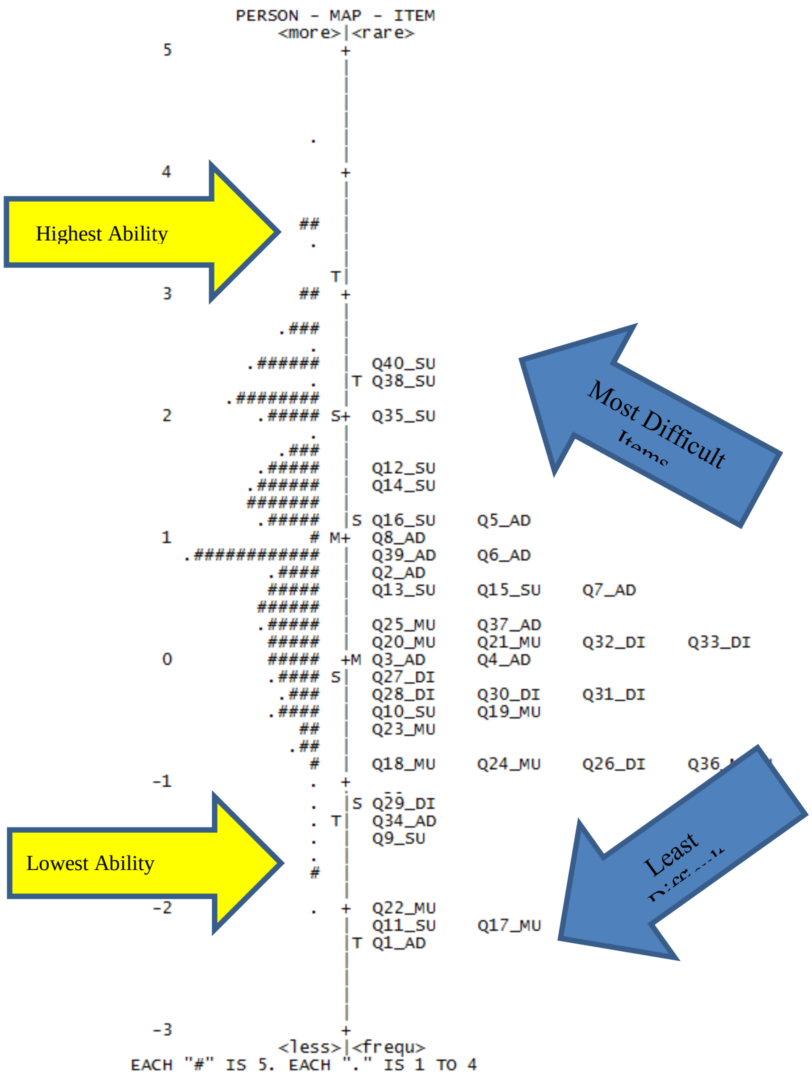


Figure 4.1 Wright Map for Operations of Integers

4.2.3 Part II: Research Question 2: Do the data fit the Rasch model usefully well for the purposes of measurement?

4.2.2.4 Consistency of result with purpose of measurement

Reliability and separation

In this study, the Rasch model test was conducted to check the content reliability of the EIIT. It was determined by using the item reliability statistical method. Table 4.6 shows the summary of 622 measured persons and 40 measured items. The reliability of item difficulty measure was very high (.99) and mean measured item at 0.0 logit. This indicates that the test is slightly difficult for the majority of the students. This also suggests that the ordering of item difficulty is highly replicable with other comparable sample of students and that the items are well-separated in terms of difficulty. The item separation index was 10.28 indicating that the items can be divided into at least ten (10) difficulty levels, which is satisfactory for 40 items.

Table 4.6 Reliability of Item Difficulty Estimates

	TOTAL SCORE	COUNT	MEASURE	MODEL ERROR	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	402.5	606.2	.00	.11	.99	-.3	1.04	.0
S.D.	114.2	7.7	1.16	.02	.13	2.9	.34	3.2
MAX.	582.0	620.0	2.45	.17	1.43	8.4	2.11	9.5
MIN.	139.0	593.0	-2.23	.09	.83	-5.3	.64	-4.6

REAL RMSE .11	TRUE SD 1.16	SEPARATION 10.28	ITEM	RELIABILITY .99
MODEL RMSE .11	TRUE SD 1.16	SEPARATION 10.50	ITEM	RELIABILITY .99
S.E. OF ITEM MEAN = .19				

Table 4.7 reveals that the reliability of student ability measure was also high at .84, suggesting that it is likely high that the ordering of students can be replicated with other items of the same difficulty. The student separation index was 2.22, indicating that EIIT can divide students into two level of abilities. This indicates a moderately acceptable fit with a person's ability ranging from high to low and vice versa. This

suggests a slightly broader continuum for items than persons, meaning that the items are easier than the students' ability.

Table 4.7 Reliability of Person Ability Estimates

	TOTAL SCORE	COUNT	MEASURE	MODEL ERROR	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	25.9	39.0	.97	.42	.98	-.1	1.03	.1
S.D.	7.3	3.6	1.10	.10	.21	1.1	.55	1.1
MAX.	39.0	40.0	4.27	1.03	1.78	4.4	7.93	4.2
MIN.	3.0	7.0	-1.93	.36	.37	-2.7	.22	-2.3

REAL RMSE .45	TRUE SD 1.00	SEPARATION 2.22	PERSON RELIABILITY .83
MODEL RMSE .44	TRUE SD 1.01	SEPARATION 2.31	PERSON RELIABILITY .84
S.E. OF PERSON MEAN = .04			

Form all the three indicators examined (item polarity, item fit and construct validity) and also the person and item reliability, the EIIT is considered reliable and valid for identifying students' errors and misconceptions in the operations of integers. Following this result, the next part involves documentation analysis using the EIIT, students' interviews, and classroom observations which were engaged to determine the types of errors that students commonly made during the test.

A PCA of residuals was obtained for diagnostics test. Figure 4.2 and Figure 4.3 present the result of this analysis. It shows that the variance explained by the measures to be at 31.3% indicating that the diagnostics test had measured 5 constructs. Meanwhile, the variance unexplained for five constructs is 6.0%, 5.1%, 3.7%, 3.6% and 2.6%. Furthermore, both measures for data and modelled expectation were almost equal (31.3% and 31.4%).

Table of STANDARDIZED RESIDUAL variance (in Eigenvalue units)

		-- Empirical --			Modeled
Total raw variance in observations	=	58.3	100.0%		100.0%
Raw variance explained by measures	=	18.3	31.3%		31.4%
Raw variance explained by persons	=	7.9	13.6%		13.6%
Raw Variance explained by items	=	10.3	17.8%		17.8%
Raw unexplained variance (total)	=	40.0	68.7%	100.0%	68.6%
Unexplned variance in 1st contrast	=	3.5	6.0%	8.7%	
Unexplned variance in 2nd contrast	=	3.0	5.1%	7.5%	
Unexplned variance in 3rd contrast	=	2.2	3.7%	5.4%	
Unexplned variance in 4th contrast	=	2.1	3.6%	5.3%	
Unexplned variance in 5th contrast	=	1.7	2.9%	4.3%	

STANDARDIZED RESIDUAL VARIANCE SCREE PLOT

Figure 4.2 Standardised residual variance

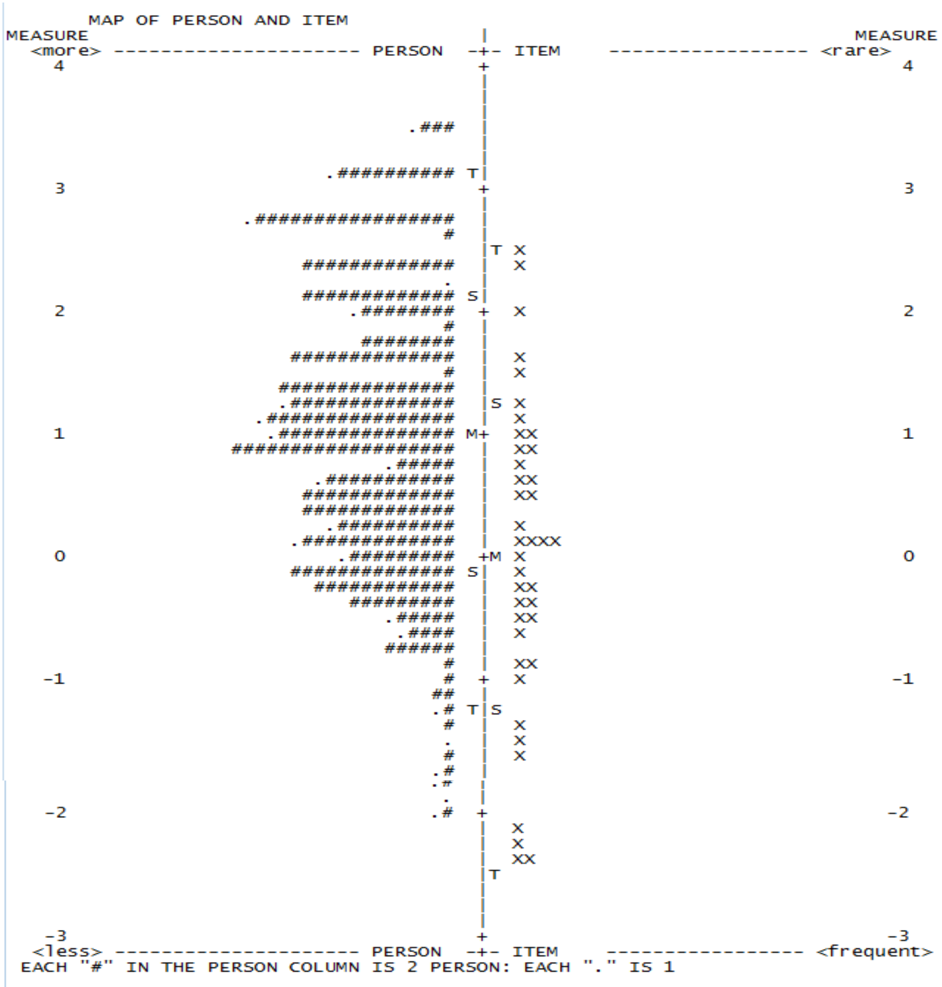


Figure 4.3 Wright map for diagnostics tests

4.2.4 Part III: Research Question 3: What is the types of misconceptions in operation of integers?

This section summarises and analyses the data generated from the EIIT, students' interviews and classroom observations. Table 4.8 illustrates the percentage of correct/wrong answers performed by the students.

Table 4.8 Percentage of Correct/Wrong Answers

	NUMBER OF QUESTION													
	2	5	6	7	8	12	13	14	15	16	35	36	38	40
Correct (%)	55.7	44.7	53.0	59.5	49.5	37.2	57.0	40.3	59.6	46.6	30.5	50.6	25.3	22.4
Wrong (%)	44.3	55.3	47.0	40.5	50.5	62.8	43.0	59.7	40.4	53.4	69.5	49.4	74.7	77.6

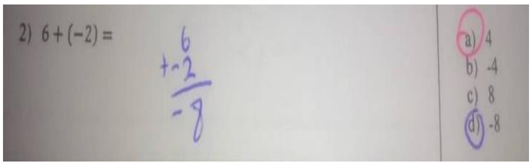
Among the 40 questions of the EIIT given, 14 of them presented the most difficult questions for the students to obtain the correct answer (<60% correct). Almost half of the students failed to answer each of the question shown in Table 4.8 correctly. The Rasch model analysis in Part II already indicates which items that students had difficulty answering. Therefore, upon closer look into the EIIT, several identical misconceptions occurring in some particular questions were identified. From the test, students' erroneous computational procedure was identified by looking at their answers.

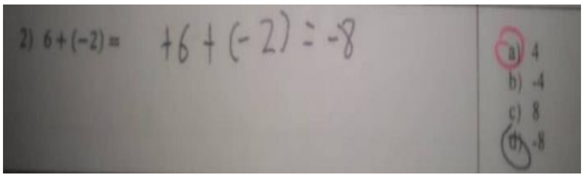
<u>Question 2: Addition</u>

For Question 2, the question asks the students to compute:

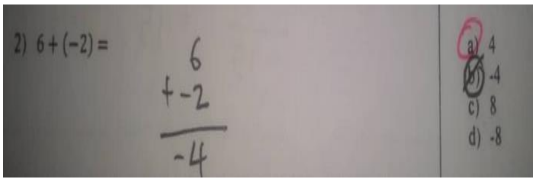
$$6 + (-2) =$$

From the analysis, students commit two types of mistakes in Question 2. Firstly, many students think they should sum up the number of six with two, without considering the negative value of two. After adding them up, the last result must be negative because there is a negative sign in the question. They were confused with the rules of multiplication in that “positive × negative = negative.”





Furthermore, the students were unable to distinguish between subtraction (operation) and negative values. They tend to give a negative result because they think that they should put a negative value at the last answer since there is a negative value even after correctly executing the steps.



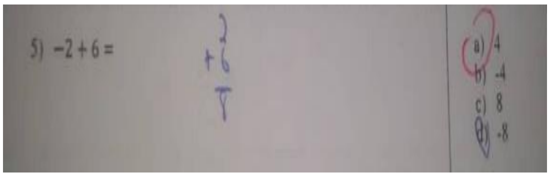
<u>Question 5: Addition</u>

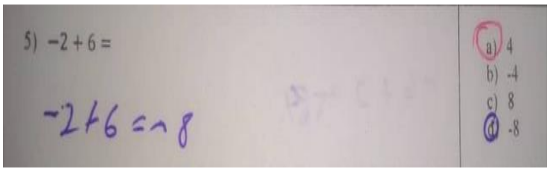
For Question 5, the question asks the students to compute:

$$-2 + 6 =$$

This is another example where students ignored the function of a negative value in integers. Students only focused on adding the number without regarding the negative value in 2. Therefore, some provided the answer 8 as they only considered the numbers.

In other cases, a negative sign was added later to the sum (for instance $-2 + 6 = -8$), because the student think that they should add the negative sign as the final answer. This might have been done due to their belief that since the signs are negative and positive the final answer must be negative.



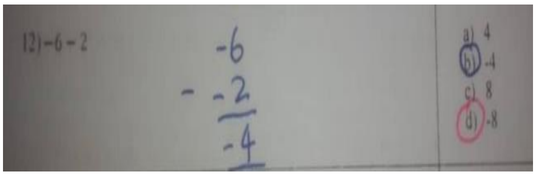


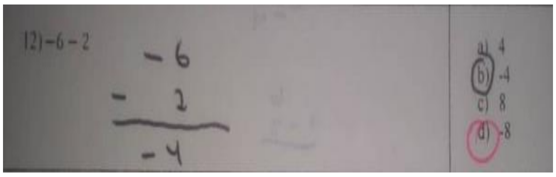
<u>Question 12: Subtraction</u>

Most students have no problem when two positive numbers are subtracted. However, students were confused when a negative number is subtracted from a positive number. For Question 12, the question asks the students to compute:

$$-6 - 2 =$$

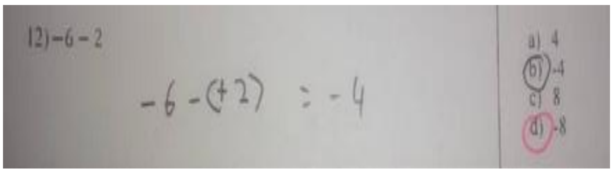
At this point, again the students ignored the negative value and operation.





When a positive and negative number are subtracted, the students would be unsure on whether to add or subtract the value after the operation. For the case of $-6 - 2 =$, -6 is a negative value, while 2 is a positive value. However, many students provide the answer as -4 (by taking away 2 from 6 and adding the negative sign because it is the sign of a bigger number 6, or wrongly applying the distribution law). This is probably due to them not considering the concept of subtraction in which they are supposed to change the direction (if they apply the number line procedure) to the left instead of to the right side. For example, student A had the right concept of the number value for 2, which is a positive value. However, the student made an error because he/she did not consider the subtraction operation that might affect the answer.

Student A

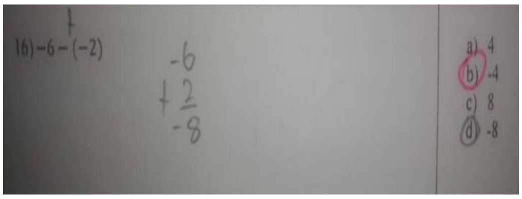


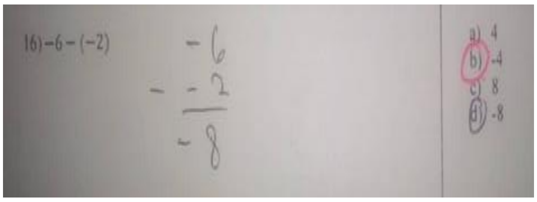
Question 16: Subtraction

For Question 16, the question asks the students to compute:

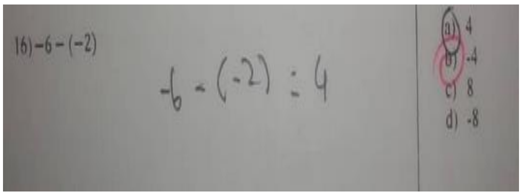
$$-6-(-2) =$$

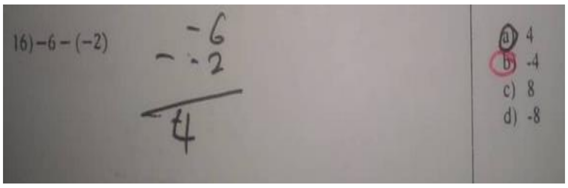
When two negative numbers are subtracted, the most common mistake was for students to add the two numbers (ignoring the signs) and make it a negative (because there are three negative signs, or because they moved left on the number line because of the subtraction operation). Using that technique, students automatically added both numbers first, and then placed the negative value at the end of the answer.

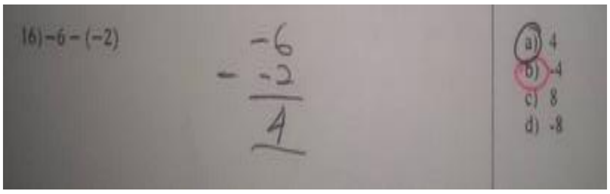




The next common mistake relates to the presence of parenthesis. The negative numbers after the subtraction operation are usually put into the parenthesis. Most teachers will ask students to memorise that the parenthesis means multiplication. Therefore, students must remove the parenthesis first in order to get the sign of the operation by using multiplication. In this case, after removing the parenthesis, students would subtract both numbers without considering the negative value of 6. They might think that the negative value at -6 would also disappear together with the parenthesis. Most of them answered -4 because they think that the number was changed when the parenthesis is removed.







The documentation analysis of EIIT necessitates further understanding on why students commit these errors. Therefore, students' interviews were handled to deeply understand why these errors happened. Table 4.9 shows the possible reasons for errors from the examination of students' answers in the scripts and interviews. The possible types of misconceptions are illustrated in Table 4.9.

Table 4.9 Test Items and Students' Justification for Answer Given

Student	Errors	Student's Reasons	Types of Errors
1	• Q8: -2-6 = 8 • Q16: -6-(-2)=4 • 8/-4=2	• The number is not negative (1:57) • Negative meets with negative becomes positive (71)	• Surface understanding • Poor knowledge • Ignored negative sign
2	• Q8: -2-6 = 8	• Negative minus negative becomes positive	• Rule mix-up
3	• Q8: -2-6 = 8 • Q16: -6-(-2)=4	• -6-(-2)=8 is negative minus negative becomes positive (69)	• Rule mix-up • Poor knowledge
4	• Q8: -2-6 = 8 • Q16: -6-(-2)=4	• Negative meets with negative becomes positive (86)	• Rule mix-up • Poor knowledge
5	• Q8: -2-6 = 8 • Q16: -6-(-2)=-8	• Negative + negative=positive (75)	• Rule mix-up • Poor knowledge
Table 4.9 continued			
6	• Q8: -2-6 = 8 • Q16: -6-(-2)= 4 • Q19: -2x6 = -12	• Negative meets with negative becomes positive (48)	• Rule mix-up • Inability to assimilate

	• Q32: -8/-4 = -2	• Although correct, the student didn't know how to get it (101) • Because there is negative, so the result must be negative (121)	• Surface understanding
7	• Q16: -6-(-2)= 8 • Q19: -2x6 = 12 • -8/-4 = -2	• Negative meets with negative becomes positive (80). There is a formula for addition & subtraction (82) • Ignored the sign • 8/4 =2, so it changes to -2 because there is negative (125)	• Rule mix-up • Surface understanding • Poor knowledge
8	• Q8: -2-6 = 8 • Q19: -2x6 = -12 • Q32: -8/-4 = 2	• Negative meets with negative becomes positive (51) • Although correct, student didn't know how to get it (99), (123)	• Rule mix-up • Inability to assimilate
9	• Q8: -2-6 = 8 • Q16: -6-(-2)= -4 • Q19: -2 x 6 =8 • Q33: -8/-2	• Negative minus negative equals positive (50) • Although correct, student didn't know how to explain it (80) • Wrong operation (addition) (95); (subtraction)	• Rule mix-up • Inability to assimilate • Carelessness
10	• Q8: -2-6 = 8 • Q16: -6-(-2)= -8	• Negative minus negative equals positive (44) • -6 minus -2 become -8 (72)	• Rule mix-up • Ignored parenthesis
11	• Q8: -2-6 = 8 • Q16: -6-(-2)= 4	• Negative minus negative equals positive (46) • Negative – negative = positive (70)	• Rule mix-up • Surface understanding
12	• Q8: -2-6 = -8 • Q16: -6-(-2)= -4	• Although correct, student can't explain how –ve happen correctly. Just put –ve because there is –ve (74) • Although correct, student explained (6-2=4). Then put –ve because there is a negative sign (84)	• Inability to assimilate
13	• Q8: -2-6 = 8	• Negative + negative = positive (68)	• Rule mix-up

14	• Q16: -6-(-2)= 8	• Negative + negative = positive (90)	• Rule mix-up
15	• -2-6 = -4 • Q16: -6-(-2)= -8	• Negative + negative = positive (41) • Negative - negative = positive	• Rule mix-up • Poor knowledge
Table 4.9 continued			
16	• -2-6 = -4 • Q16: -6-(-2)= 4	• Negative + negative = positive (50) • Confused over the rules of parenthesis	• Rule mix-up • Poor knowledge

<u>Rule Mix-Up</u>

For question 8:$-2 - 6 = 8$, fourteen (14) of the students got the incorrect answer of 8 as the answer. According to Student 2, Student 3, Student 4, Student 5 and Student 10, the answer to question 8 is 8 since a negative number minus a negative number should result in a positive number.

> *"Negatif jumpa positif, negatif. negatif + negatif = positif. Jadi positif jawapan dia." (5:74)*
> "Negative meets positive is negative. Negative plus negative is positive. So positive is the answer" (5:74)

> *"Negatif tolak negatif jadi positif." (10:44)*
> "Negative minus negative equals positive" (10:44)

Students 1, Student 6 and Student 7 also mixed-up the rules of integers. They stated that the answer to Question 16 is negative because since a negative number meets with a negative number, it thus becomes a positive.

> *"Jadi jawapannya +4 lah. So tulah bila negatif dia jumpa dengan positif jadi positif." (1:69)*
> "The answer is +4. This happens because when a negative meets with a positive, the result is positive" (1:69)

> *"Negatif dengan negatif jadi positif" (6:48)*

“Negative with negative becomes positive” (6:48)

“Negatif sama negatif pastu ada tolak jadi 8” (7:80)
“When negative numbers meet with negative numbers, then they become negative. Therefore, the answer is 8” (7:80)

Therefore, in this case, they seem to have poor knowledge about the operation of symbols, negative numbers and also how both symbols and negative numbers are combined.

Surface Understanding

Another type of error made by the students is when they have surface understanding about integers. According to Student 1, for question 8, the result is 8 because the answer is not negative since both numbers are negative

“Sebab nombor dia bukan negatif.” (1:56)
“Because the number is not negative” (1:56)

Meanwhile, Student 7 believes that there are rules for subtraction as well. However, the rules of integers are only applicable for multiplication and division. The student thinks that the rules can also be used for addition and subtraction. Thus, the student made a mistake due to this misunderstanding.

“Sebab ni ada formula tolak ni” (7:82)
“Because there is a formula for subtraction” (7:82)

For Question 32: $-8/-4 = -2$, Student 6 stated that the result becomes -2 because of the negative sign present. Hence, the result must be negative. The student’s failure in memorising the rules of integers caused him/her to make this error. Negative integers apparently make them confused.

"Negatif 8 bahagi negatif 4, 8 bahagi 4 dapat 2. negatif tu tak berubah" (6:121)
"Negative 8 divided by 4, 8 divided by 4, we get 2. Negative does not change" (6:121)

<u>Inability to Assimilate</u>

Students also find it difficult to assimilate their previous knowledge with the current knowledge. Although Student 6, Student 8 and Student 9 managed to get the correct answer, when interviewed for details, they did not know how to explain it. The same goes for Student 8 who was unable to explain how he/she obtained the answer.

"pakai buat" (8:123)
"Just do it" (8:123)

"tak tau nak explain" (9:80)
"I don't know how to explain (it)" (9:80)

<u>Carelessness</u>

Lastly, another type of error that student tend to commit is carelessness. They made mistakes due to confusion with the operation symbols. Some of them saw multiplication operation as an addition operation, and vice versa. Meanwhile, some of them saw the division symbols as subtraction. Thus, they commit errors because of silly mistakes which affected their performance.

"tersilap tengok simbol darab" (9:95)
"I thought it was multiplication" (9:95)

"silap lagi" (9:105)
"another mistake" (9:105)

Based on students' interviews, a majority of them apparently make the same mistakes. The mistakes that they commit are usually due to rule mix-up, surface understanding about the concept of negative integers and subtraction, inability to assimilate with the previous knowledge, and lastly, carelessness.

4.2.5 Part IV: Research Question 4: What are the causes of error in solving problems in operation of integers?

This section summarises and analyses of data generated by the classroom observation and researcher's interview with eight mathematics teachers teaching Form One classes of four states in Malaysia. After administering the tests, the researcher went to schools to observe the teaching and learning of integers. The observation was needed in order to understand the classroom setting, teachers' methodologies in teaching and students' engagement towards integers, as well as students' behaviour during the lessons. Table 4.10 illustrates the themes from the observation checklist (see Appendix J) throughout the classroom observations.

Table 4.10 Observation of Themes

Main Themes	Sub Themes
Lacks multiple representation	• No concrete representation • 2 – 3 representation
Not much collaborative learning	• Pair • The whole class
Not much active learning	• Discussion with whole students • Monitoring • Teacher as a discussion leader
No incident of creative and critical thinking	• Teacher-centred • Reinforce rules and procedures • Using whiteboard only • Not giving time to think other methods

From the observation data of the field notes, in general, teachers devote two weeks (5 hours) to complete the topic of integers. Within five hours of teaching (2 weeks), most teachers prefer to use only the number line approach as a teaching strategy for the operations of integers. All of them used direct instruction and classroom lecture style in order to explain the concept of integers.

Multiple Representation

From the observation, it was found that teachers used between two to three representations such as verbal and visual or verbal, and visual and real-life representations. The real-life representation is in the form of analogy and this method was used by only four teachers. Examples of their analogies include the concepts of ships in the sea, the lift with underground basement, the theory of debt, and the idea of thermometer. All of these concepts were explained using the whiteboard. Apparently, the teachers did not transform an abstract idea to the concrete. As a consequence, some students were unable to grasp the concepts of integers as they have difficulties to understand them.

Collaborative Learning

Collaborative learning is emphasised in this study. However, from the observation, no evidence of any collaborative learning was found. With only a total of five hours to complete the topic on integers, teachers are not expected to perform elaborate activities. Therefore, to make it easy and convenient for both teachers and students, the former prefers to have simple activities such as solving problems in pairs or the whole class. For them, this can save more time compared to the use of hands on activities or group discussion. In addition, with the huge number of students in a class, it is difficult for teachers to handle their students. This kind of scenario is depicted in the vignette of classroom teaching attached in Appendix K.

Creative and Critical Thinking

From the observation, all teachers were found to use the number line approach in teaching the operations of integers. They would typically ask students to memorise the rules and procedures. There is no room for creative and critical thinking to take place in the teaching and learning process. For teachers, the students must remember how the number line works. The students must know which direction should the move be, either to the left side or to the right side. The teachers expect the students to understand the concept of number line with a lot of practice. For them, when students focus on specific rules or procedures, they will gradually become more comfortable with the rules, and finally would be able to solve similar problems. Teachers think that after students are comfortable with the specific rules, they can then create their own math problems where the rules apply. However, with this style of teaching, students do not get meaningful ideas about integers. Students simply tend to follow the rules and procedures without understanding most of the concepts of integers.

Active Learning

In addition, from the observation, it can be seen that all teachers monitored their students' understanding by praising their students' works or achievements in solving questions on integers. In addition, students worked according to teachers' guidance and teachers acted as discussion leaders. This was to ensure that the classroom was not interrupted and organised. Meanwhile, teachers do not give students the time to work on their own to solve the problems. Teachers rush to finish the important basic skills in mathematics since they think that the students will know them once the topic is completed. Students only follow what the teachers do without asking any question.

4.2.4.2 Teachers' interview

This section summarises and analyses data generated by the researcher's interview with eight mathematics teachers teaching Form One classes at four states in Malaysia. Following the identification of student errors, teachers' perspectives were needed to understand how these errors were committed. Based on the interviews, some recurring problems were identified. Table 4.11 illustrates the details of the teachers' background.

Table 4.11 Demographic Details of Interviewed Teachers

Teacher	Age	Experience	Highest Qualification	Gender
1	35	11	First Degree	Female
2	43	18	First Degree	Female
3	40	15	First Degree	Female
4	31	3	First Degree	Female
5	38	13	First Degree	Female
6	48	23	First Degree	Male
7	33	8	First Degree	Female
8	27	4	First Degree	Female

Subsequently, Table 4.12 lists the themes and subthemes of the interview.

Table 4.12 Themes and Subthemes of the Interview

Main Themes	Sub Themes
Parenthesis Misapprehension	• Think that parenthesis is not important • Ignore parenthesis • Tend to forget the next step • Did not know the function of the parenthesis • Did not consider the role of parenthesis • Confusion between symbol and value when there is a parenthesis

Poor Mathematical Language	• Confused with the subtraction symbols and negative values • Think the subtraction symbol and the negative sign are the same • Unable to differentiate between them • Unable to understand the concept of negative values
Calculator Hooking	• Too dependent on the calculator • Teachers will teach students how to use the calculator, since calculator use is allowed • Students do not want to memorise the multiplication table • Cannot answer the questions without a calculator
Superficial Understanding	• Only understand when in the classroom • Unable to understand the symbols and value • Cannot solve a question without a calculator • Still obtain wrong answers even through the use of a calculator • Do not get sufficient exercise • Do not understand number line

Table 4.12 continued

External Limitation	• Time constraint • Too much syllabus to complete • Different students' abilities • Large number of students • School activities, holidays, natural disasters

<u>Parenthesis Misapprehension</u>

Some students think that the parenthesis is not important and hence, they tend to ignore it. In addition, they are unable to remember why the parenthesis is supposed to be there. The students often tend to forget about them in the very next step. Teacher 2, Teacher 3, Teacher 4, Teacher 5 and Teacher 6 said that their students always commit mistakes when they solve questions with parenthesis. For example, Teacher 2 and Teacher 6 stated that their students always forget the function of the parenthesis when solving a question. They tend to just solve the question directly without considering the value with or without parenthesis. As a result, they end up making a big mistake when answering the question.

> *"Hmm, memang macam tu. Sebab pelajar akan keliru di sini. Mereka tidak buka kurungan dulu" (2:78)*
> "Hmm, it is like that. It is because the students got confused. They did not open the parenthesis first" (2:78)

> *"Macam hari ni, bila melibatkan operasi, jadi pelajar mungkin akan keliru dari segi penggunaan kurungan, jadi yang tu kena diulang balik" (6:20)*
> "Like today, when it involves operations, the students appear confused with the function of parenthesis, so I need to teach them again" (6:20*)*

Teacher 3 also added,

> *"Mereka tidak faham kurungan itu macam mana, bila guna kan. cara betul nak guna kurungan. Kita nak terangkan macam ni, tapi bila masuk kurungan mereka mula pening" (3:24)*
> "The students do not understand how parenthesis works, when to use it and the right way to use it. When I am explaining, they get confused whenever there is a parenthesis" (3:24)

Meanwhile, Teacher 4 always reminds her students to remember the rules. She also tells her students that parenthesis means multiplication. However, her students will still make mistakes. This is because they just ignored the function of parenthesis that might influence the final answer. However, when it comes to using the calculator, the students can answer the question correctly because they just have to follow and punch in the exact signs and symbols of the questions.

> *"Lagi satu kalau ada yang bracket*-bracket *kan, contoh negatif 2 tambah dalam kurungan negatif 3, ((-2)+(-3)) kan, tekan kalkulator boleh la. Kadang mereka ni, kita bagitahu dah, kurungan darab, masa tu ok. Bila ujian, hmmm." (4:68)*
> "One more problem, when there are parentheses, for example (-2)+(-3), they can get the answer right by using the calculator. Although I would remind them that the parenthesis means multiplication, they only remember it at that time. But in the exam, hmm." (4:68)

Teacher 5 also has difficulties in explaining to her students the concept of parenthesis. The students are always confused with the symbol and operation when there is a parenthesis. For them, parenthesis does not give any value to the question. Therefore, the mistake keeps recurring.

> *"(bagi pelajar)Yang ada kurungan tu kan. Bila tolak atau tambah bertemu dengan negatif, kalau tolak bertemu positif kita takde kurungan kan. Bila tolak atau tambah bertemu dengan negatif, simbol dah bertukar kan" (5:30)*
> "(students think)The questions with parenthesis; when subtraction or addition meets a negative, or if a subtraction meets a positive number, we do not need the parenthesis. But when subtraction or addition meets with a negative value, the symbols need to change too" (5:30)

Therefore, based on the teachers' interviews, it shows that most of the students are struggling with parenthesis. Some of them were confused with the concept and the function of parenthesis. Others were unable to relate it with multiplication.

<u>Poor Mathematical Language</u>

From the interviews, five out of eight teachers agreed that students always commit errors when they are solving addition and subtraction questions. One of the reasons of the errors is due to a confusion in the symbol and value. Many students are confused with the subtraction symbols and negative values. Teacher 3 and Teacher 6 mentioned,

> *"Okay, bila dia tak faham konsep, contoh tukar simbol untuk tambah negatif 6, bila dia tak faham konsep, dia akan lari la jawapan dia. Patutnya tolak dia pergi tambah. Sepatutnya tambah dia pergi tolak. Itu la antara masalah dalam integer tu. Tu yang bila sampai algebra dia jadi macam keliru sebab kat integer dia tak betul-betul faham" (3:32)*
> "Okay, when they do not understand the concept, for example, if they need to change the symbol for additional negative 6, when they do not understand the concept, the result is wrong. They are supposed to subtract but they add instead. When they are supposed to add, they subtract. Those are the problems in integers. When the students learn

> algebra, they will get more confused because they do not understand integers" (3:32)

Teacher 3 believes that the most difficult task in teaching integers is to make the students understand the concept of operational symbols and negative sign. The students always make mistakes because they think the subtraction symbol and the negative sign are the same. Meanwhile, Teacher 6 has difficulties in making the students understand how to handle the situation of negative signs and subtraction symbols when both are in the same problem. The students tend to make an error when both mathematical languages are combined.

> *"Manakala bagi operasi macam tambah tolak tu, dia masalah kalau yang mereka jumpa tu dua simbol" (6:28)*
> "Meanwhile, for operation like addition and subtraction, they have a problem when they come across those two symbols" (6:28)

Teacher 8 also faced the same problem. His/Her students get confused with the symbol and operation itself.

> *"Kebiasaannya mereka keliru dengan simbol ataupun operasi tu sendiri" (8:32)*
> "Usually, they are confused with the symbols and operation itself" (8:32)

The same teacher continues to state that the students are unable to differentiate between them and believe that the subtraction symbol and negative values are the same.

> *"Macam mereka tak boleh nak bezakan antara operasi dan simbol." (8:56)*
> "They are unable to differentiate between operation and sign" (8:56)

> *"Mereka ingatkan sama je tolak dengan negatif tu" (8:58)*
> "They think that subtraction and negative are the same" (8:58)

The concept of subtraction alone confuses many students. Teacher 7 stated that the students are not able to understand the concept of negative values, in which how can a smaller number be subtracted with a bigger number and becomes a negative number.

> *"Bila saya suruh buat kadang-kadang mereka bertukar kalau macam kadang-kadang mereka tak faham lagi operasi tambah tolak tu kan, masalah dah di situ. Macam kalau 1D tu pula, bila kita tanya 3 tolak 8 berapa jawapannya? Tak boleh cikgu (student answered the question). Ha, mereka punya mindset nombor kecil tak boleh tolak nombor besar. Ha, tu mindset mereka dah tu" (7:26)*
> "When I asked them to solve the question, they do not understand and wrongly swap the addition and subtraction operation. Like in 1D, when we ask what is 3 minus 8? They say, "cannot solve teacher!" In their mindset, small numbers cannot be subtracted with a big number" (7:26)

Students are used to deal with whole numbers and were told during their primary school years that it is not possible to take away a bigger number from a smaller number. Hence this is set in their mind in that the concept of a negative number does not exist. Therefore, they are unable to solve integers-related problems involving negative numbers.

Calculator Hooking

Another cause of errors in this research relates to students' dependency on the calculator. According to Teacher 5 and Teacher 6, they prefer to make their students understand the concepts. However, since the schools allow the use of calculators, they will teach them to use the calculator after teaching the concept.

> *"Kita dah dibenarkan guna kalkulator kan. Jadi kita guna kalkulator jelah tapi basic dia, saya suruh tengok jugak kepada sifir. Kalau macam kelas-kelas yang ni katalah tengoklah ok sifir ni cari sifir 2 yang mana jawapnnya 6 kan. Diorang jumpa jugaklah tapi lambatlah. Nak kena tulis sifir dulu kan pastu nak cari. Tapi dah boleh guna kalkulator ni gunalah" (5:126)*

“We are allowed to use calculators, so we just use it. But basically, I will ask them to memorise the multiplication table. For a weak class, I will ask them to check the answer using the multiplication table. They will solve it although it will take some time. In addition, I will ask them to write the multiplication table first and then find the solution. But since we can use the calculator, so, we just let them use it” (5:126)

“Ya, mereka kena faham dulu. Saya bawak contoh sesetengah pelajar 4 bahagi 2, guna kalkulator. Sebabnya sekarang ni dalam kepala mereka apa-apa pengiraan boleh guna kalkulator” (6:52)
“Yes, they need to understand it first. For example, when I ask the students ‘4 divided by 2,’ they will use the calculator to find the answer. This is because, nowadays, in their mind, all calculations can be done with the use of a calculator” (6:52)

However, since the use of calculator is allowed in secondary schools, some of the teachers consider this as an opportunity to let their students use the calculator instead of making them understand the concept. It also makes teaching and learning become smoother. According to Teacher 2, his/her students are allowed to use the calculator as long as they know about integers.

“Sebab sekarang budak-budak dah pakai kalkulator. So, kita tak nampak sangat la kesan, janji dia tahu integer tu” (2:138)
“Because nowadays students use the calculator. So, we cannot see the effect (number line method), as long as they know integers” (2:138)

Teacher 3 also agrees with the Teacher 2, and said that nowadays students can use calculators. Hence, the students just need to key in the numbers and get the result.

“Tu la, sekarang kan dah boleh guna kalkulator, tolak 6 semua tu” (3:26)
“That’s it. Nowadays, we can use the calculator. All those minus 6” (3:26)

Teacher 8 mentioned that his/her students do not want to memorise the multiplication table since they can use calculators.

"Ya, sekarang dah boleh guna. Sebab itu mereka tidak mahu hafal sifir" (8:52)
"Yes, they can use it (calculator) nowadays. That is why they do not want to memorise the multiplication table" (8:52)

From the interview, calculators may be a reason why students commit errors when solving operations of integers. Sadly, although they are allowed to use the calculator, their answers may not be correct all the time.

Superficial Understanding

This is another cause of error identified from the interview with teachers. Five of the teachers believe that the students are unable to answer the questions because of their surface understanding of integers. Teacher 1 stated that his/her students can only solve questions that have been addressed in the classroom. However, when they are asked to answer other types of question, they are unable to do so.

"Ha, tu la kata. Soalan yang kita buat kat depan je dorang boleh faham. Dah tu bila kita bagi latihan, kita suruh buat, "macam mana ni cikgu?" ha, tambah tolak macam mana? Dia macam dah tak ambil port dah kadang-kadang tu" (1:109)
"Yes, they can understand the questions we did in front of the class. But when we give exercises, they will ask, "what should we do teacher?" "how do add and subtract work?" Some of them do not even care anymore" (1:109)

Teacher 3 also faced the same problem. The students are not able to understand the concept of symbol and negative value.

"Ha, dia tak faham konsep la tu. Sepatutnya dia kekalkan simbol ni, jadi la negatif 8. Tu kadang budak pening" (3:22)
"Ha, they did not understand the concept.Supposedly the symbol will remain, so it becomes negative 8. The students are confused" (3:22)

The same teacher furthers stated that the students do not have any problem with multiplication because they just need to memorise two rules. Besides, students can use the calculator. However, without a calculator, they are unable to answer a question. However, there are those who provide wrong answers even though they use the calculator. This is a recurring problem among the students.

> *"Darab takde masalah sebab darab dia kena ingat dua je. Tak sama negatif. Sama positif. Darab dia boleh guna kalkulator. Tapi kalau ini, dia tak guna kalkulator tapi punya salah konsep tu, ha salah jawapan dia. Tu la. Tu antara kesalahan dia la" (3:32)*
> "Students do not have problems with multiplications because they just need to remember two rules. If the number has a different sign, thus it is negative. If the number has the same sign, the answer is positive. In addition, they can use the calculator to solve problems in multiplications. But some of them even give the wrong answer even with the use of a calculator" (3:32)

However, Teacher 6 believes that students are unable to understand the new concept because they do not have enough exercise. This is one of the reasons why they cannot perform when the question is different.

> *"Satu mungkin dari segi kurang dari segi latihan dia sendiri pun, makna dia kalau time belajar je, masa dalam kelas sahaja baru belajar. Jadi bila begitu makna kalau datang soalan yang lain sedikit, dia akan pening" (6:24)*
> "Probably the students do not have enough exercise. They can answer the question in class, but will get confused to solve it elsewhere" (6:24)

The same teacher continued to say that the students are not familiar with the concept of number line method. They are unsure of which way should they move, either the right side or the left.

> *"Maknanya yang prinsip kan kalau untuk operasi tolak, bila nilai positif pergerakan dia mana ke mana tu" (6:36)*
> "For subtraction operations, when the value is positive, they do not know on which side they should move" (6:36)

Therefore, superficial understanding of the concept of integers leads to misconceptions among students. The students are not able to see the whole picture in the operations of integers.

External Limitations

Other causes of errors in the operation of integers can be termed as external limitation such as time constraint. From the interviews, all teachers use the number line method as their teaching preference for the addition and subtraction of integers. Meanwhile, for multiplication and division, they would ask students to memorise the multiplication table and the rules of multiplication and division. Some teachers stated that they have limited time to focus only on integers in their teaching. According to Teacher 1, he/she would just provide his/her students with the rules and procedures to solve the integer problems. This is because they need to finish all the 13 topics in the textbook within a certain time frame. Thus, it is impossible to merely focus on the one topic and neglect the other topics.

> *"Saya suruh hafal yang ni jela, hafal yang ni je. Kalau sama, sebab kita tak boleh nak lama-lama kat situ, sebab dia hanya subtopik. Kita ada 13 bab. Ha, so kita kena kejar bab-bab tu. Kita suruh dia hafal yang ni, dan latihan yang banyak. Tu jela." (1:83)*
> "I asked them to memorise something, and they would just memorise that thing. We cannot be stuck at only one subtopic. We have 13 topics, so we need to rush. So, I would just ask them to memorise and give lots of exercises. That's it" (1:83)

Teacher 8 also agreed with Teacher 1. Since all the teachers need to follow the lesson plans set by the Ministry of Education, with 13 topics altogether, therefore, teachers feel rushed and they need to finish the syllabus within the given timeframe.

“Kena cepat la sebab kita ada lesson plan kena ikut, jadi rushing sikit tapi kena habiskanla.” (8:78)
“We need to be faster because we have lesson plans to follow. So, it is a bit rushed but we must to finish it” (8:78)

For teachers, this is the reason to explain their inability to focus more on only one topic and to not finish the other topics. Therefore, they prefer to use any method of teaching that can reduce the time.

Another limitation is the mixing of students’ abilities in one class. According to Teacher 1, the current ministry favour to place students of differing levels of ability in one class. Hence, a classroom may have students with strong cognitive abilities and students with weak cognitive abilities. Therefore, teachers have to spend more time to cater to the differing abilities of the students.

“Tapi sekarang kelas pun dah sama dah. Dia takde istilah pandai tak pandai. Takde dah. Jadi murid yang dalam kelas tu ada yang level tinggi sikit” (1:58)
“But nowadays the classes are the same. There is no academically-excellent and weaker classes. Each class is comprised of students with different levels of abilities (1:58)

However, for public schools, sometimes the teachers will have a large number of students. Teacher 1 has at least 40 students in a single class. This makes it difficult for Teacher 1 to focus on only one topic.

“Lepas tu dalam satu kelas bukannya dua puluh orang, kan tengok ramai kan. Jadi bila melibatkan ramai tu, kita nk tumpukan sorang-sorang tu, dengan kita lepas habis mengajar lepas tu kita nak fokus pada dorang bila bagi latihan kita nak tengok apa latihan. 5minit je kita boleh bagi tengok mende tu. Sebab kita habis masa mengajar dah” (1:87)
“And then, in one class, we not only have twenty students, you can see many students in one class. When we are involved with many students, we cannot focus on only one person and even after class or during the class activities. We can just spend five minutes with each. All our time used for teaching” (1:87)

In addition, school activities serve as a distraction that teachers have to deal with, apart from school holidays and sometime natural disasters such as flooding. According to Teacher 2,

> *"Kita memang nak pelajar faham. tapi kita kena habiskan bab jugak. mereka dah susun jadual dan bab dengan baik, cuma yang mereka tak ambil kira ialah aktiviti-aktiviti, program sekolah macam-macam ada. Tiba-tiba ada bencana berlaku ke, tiba-tiba banjir macam tu la. Sekolah dah cuti sekolah. Mende-mende tak dijangka tu la" (2:134)*
> "We want the students to understand but we need to finish all the topics. They (ministry) already organised the schedule and topics with a good flow. They just did not consider other school activities. There are a lot of school activities. Then, there are natural disasters like flood, and school public holidays, and sometimes unexpected holidays" (2:134)

Teacher 5 agrees with statement of Teacher 2 and said that school activities slowed the teaching and learning process. With a half an hour class, it is difficult for the teachers to finish all the topics in the given time.

> *"Hmm. Dengan masa setengah jam pastu event sekolah lagi cuti lagi kan" (5:160)*
> "Hmm. With half an hour of lesson, school events and holidays…" (5:160)

Teacher 2 further said that, the 13 topics prepared by the Ministry of Education are only relevant if there is nothing disturbing the teaching and learning time.

> *"Memang 13 bab yang diikaji memang cukup-cukup. Tapi kalau takde program pape. Tapi kita dah melibatkan program, ada bencana, ada masalah, yang tak dapat lari, banyak la tertinggal." (2:134)*
> "The 13 topics are enough for one year of school. But only if there are no other programmes. However, we are involved with many programmes, natural disasters, and other problems that may hinder our aim to finish the syllabus" (2:134)

Furthermore, school activities could lead students' absenteeism. Thus, the affected teachers need to repeat the missed lessons so that the students will not be left out. Catering to the needs of weaker students also disrupts the lesson's progress.

> *"Lepas tu yang dia fikir kata, yang dia buat tu kata budak boleh ikut, tapi kita kena tengok jugak bukan semua pelajar faham dalam masa 3jam tu. Tapi kalau pelajar lemah lagi, kita sebenarnya terpaksa ambil masa lagi, nak bagi dia faham tapi kita tak boleh nak tunggu dorang la" (2:134)*
> "Then we need to follow students' abilities. Not every student can understand something within three hours. Weak students need more time to understand and at the same time we cannot wait for them (due to the syllabus)" (2:134)

From the interviews conducted, the teachers agreed that parenthesis misapprehension, poor mathematical language, calculator hooking, superficial understanding and external limitations are the causes of students' errors and misconceptions in the operations of integers. These causes are difficult to alleviate because teachers think that they are natural occurrences that are out of their control.

4.2.5 Part V: Research Question 5: Is there any significant difference between students' performance in different schools?

This study also aims to determine whether the location of the schools and the students' gender play a reason to affect students' achievement. Therefore, Part VI is allocated to answer this question by using the ANOVA.

<u>Is there any significant difference between students' performance in rural and urban schools?</u>

To investigate the significant difference between students' performance in rural and urban areas, the independent-sample T-test was employed. An independent-sample t-test was conducted to compare the results of the students in rural and urban areas. There was a significant difference in the scores for urban (M=29.2917, SD=5.98379) and rural (M=24.5154, SD=6.80990) conditions; t (449.786) = 8.033, p = .000. These results show that students in urban schools perform significantly better than those in rural schools. The result is shown in Table 4.13 and Table 4.14.

Table 4.1 Group Statistics

	Type	N	Mean	Std. Deviation	Std. Error Mean
result	urban	240	29.2917	5.98379	.38625
	rural	227	24.5154	6.80990	.45199

Table 4.2 Independent Sample Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Result	Equal variances assumed	4.886	.028	8.062	465	.000	4.77625	.59242	3.61210	5.94039
	Equal variances not assumed			8.033	449.786	.000	4.77625	.59455	3.60782	5.94468

Is there any significant difference in mathematics performance based on students' gender?

To investigate the significant difference in performance due to gender, the independent-sample T-test was again employed to compare the results based on students' gender performance. There was a significant difference in the scores for male (M=25.72, SD=56.933) and female (M=27.99, SD=6.63) conditions; t (460) = -3.577, p = .000. These results show that female students perform significantly better than the male students. The result is shown in Table 4.15 and Table 4.16.

Table 4.3 Group Statistics

Group Statistics					
	Gender	N	Mean	Std. Deviation	Std. Error Mean
TOTAL CORRECT	male	201	25.7214	6.93340	.48904
	female	261	27.9923	6.63209	.41052

Table 4.4 Independent Sample Test

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
TOTALCORRECT	Equal variances assumed	.064	.800	-3.577	460	.000	-2.27094	.63482	-3.51846	-1.02343
	Equal variances not assumed			-3.557	420.539	.000	-2.27094	.63850	-3.52600	-1.01589

4.2SUMMARY

This chapter has presented the result of data analysis using Winsteps, thematic and paired-sample t-test. The Winsteps analysis shows that the diagnostics tests were validated. In the thematic analysis, a few findings on the types and causes of misconceptions in the operations of integers were identified.

CHAPTER FIVE

5.1 CONCLUSIONS AND RECOMMENDATIONS

5.1.1 Introduction

This chapter deals with the discussion, conclusion, implication of the findings and the recommendation on diagnotics test towards students' conceptual understanding of operations on Integers. This chapter discusses the findings of this study based on five research questions below;

1. Is the EIIT on operation of integers for the Malaysian Form 1 students valid?
2. Do the data fit the Rasch model usefully well for the purposes of measurement?
3. What are the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception?
4. What are the causes of error in solving problems in operation of integers?
5. What are the differences between students from the various school settings, in their understanding of operation on integers?

5.1.2 Discussion

5.1.2.1 Research Question 1: Is the EIIT on operation of integers for the Malaysian Form 1 students valid?

The findings from the study showed a positive benefit of using the EIIT to improve students' achievement in mathematics. It is clear that the EIIT has been useful and is valid to be used to diagnose misconceptions in the operations of integers. Unlike standardised tests, EIIT is criterion-referenced. This means that the test items and goals are determined according to a fixed set of requirements. They are scored using true test score criteria, meaning that they are not averaged or normed. Each test is scored based on the students' own performance based only on their grade level requirements. Because of this feature, EIIT has the added advantage of being able to pinpoint specific grade level strengths and weaknesses. Beyond this, revealed weaknesses can be properly targeted for remediation. Moreover, EIIT for the operations of integers provides the tools needed to verify adequate understanding of the accomplishments in students' educational experience. Furthermore, students with attention disorders or learning disabilities can be recognised and teachers can help with the time limits. Therefore, teachers are able to grasp the weaknesses and strengths of the students by referring to the result of these tests.

So far, studies on the misconception of integers had been carried out qualitatively or quantitatively using instruments that are open ended, in which students would provide the answers (Bush, 2011; Faudiah, Suryadi & Turmudi, 2016; Setyawati & Indiati, 2018; Ural, 2016). To the best knowledge of the researcher, this study is the first attempt at diagnosing students' misconception using selected-responses instrument. The options for the responses had been specially selected to enable the researcher to guess the type of misconception that was acquired by the students. For example, student 10 answered $Q8$: $-2-6\ =\ 8$ and $Q16$: $-6-(-2)\ =\ -8$. From the first answer, it can be assumed that, first, the student may have ignored the signs, add the numbers and thinks that since there are two negatives in the question, the answer would be positive. This is confirmed from the answer in Q16. Here, it seems that, again the signs were ignored, the numbers added, and the answer given was a negative sign since there were three negatives (two are signs of the numbers and one is the operation) in the question. The suspicion that the student was applying the integer rule for multiplication was confirmed during the interview.

The study did not discuss the integer word problems because they involve other errors/ misconceptions related to word problems such as misunderstanding the question, transformation problem (inability of students to put the problem in symbolic form), reading ability, and many more. An upgraded version of the instrument can be seen in Appendix L.

5.1.2.2 Research Question 2: Do the data fit the Rasch model usefully well for the purposes of measurement?

From the Winsteps results, it shows that the data is fit the Rasch model. It is useful for the purposes of measurement since it is fit with the conditions. Although the items are too easy for some students, it suitable for the research' aims that want to figure out the causes of the misconceptions rather than to test students' ability.

5.1.2.3 Research Question 3: What are the types of misconceptions in operation of integers and its relationship with the common methods of teaching integer operations that lead to students' errors and misconception?

From the analysis of this study, there are a few types of errors and misconceptions in the operations of integers due to the common methods of teaching integers and operations that lead to students' misconceptions. Table 5.1 illustrates the types of errors between previous research and this study.

Table 5.1 Types of Errors and Misconceptions

Errors according to the Literature Review	Errors Found in This Study
• Ashlock (2002) - overgeneralisation and overspecialisation of rules in an effort to make sense of new information. • Drews, Dudgeon, Lawton and Surtees (2014) - error in changing the abstract to concrete ideas, inadequate	• Rule Mix-up

knowledge and experience related to certain mathematics topic.	

Table 5.1 continued

• Hayes (1999) - misapplications of the rules. • Kuchemann (1981) - understanding does not necessarily flow from the use of number line because it is an abstracted representation of abstract ideas. • Drews, Dudgeon, Lawton and Surtees (2014) - students are unable to understand the question given, misapprehension of symbols or sign. • Graeber and Johnson (1991) - misconceptions are due to the self-evident, where the person does not feel the need to prove them.	• Surface Understanding
• Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) - in making inferences and interpretations, students are very likely to make at least temporary errors. • Drews, Dudgeon, Lawton and Surtees (2014) - unable to see the connection between previous and current knowledge, and leaving the task unfinished. • Graeber and Johnson (1991) - coercive, where the person is compelled to use them in an initial response; and widespread, where it happens among both naive learners and more academically able students.	• Inability to Assimilate
• Drews, Dudgeon, Lawton and Surtees (2014) - error in choosing useful and meaningful information. Errors can be divided into three categories: careless errors, computational errors and conceptual errors.	• Carelessness

Firstly, most of the teachers used the number line in order to teach addition and subtraction operations. This method is a good technique if the teachers explains it well to the students, since this method is clean and easy to understand. However, teachers have to deal with the different level of students' abilities. Not every student can understand the number line method, especially if they are confused with the definition of negative values and subtraction operations since both terms have the same symbol but each of them gives a different meaning. For subtraction operations, the teachers may have to explain to the students that they need to move the numbers to the left. However, the subtraction operation gives a totally different meaning if there is a negative value involved. When students encounter this situation, they may likely be confused and uncertain of which direction they should move the number involved on the number line. Therefore, it might lead to misconceptions in their understanding.

Meanwhile, for multiplication and division operations, all of the teachers used memorisation as method of teaching the operations of integers. They believe that the students only need to remember the multiplication table and integers multiplication rules to answer the related questions. The teachers also used information memorisation based on repetition. Hence, the students need to do lots of drilling so that they can see the patterns of questions. This method might be effective for short term learning but this method is not properly good for long term memory and understanding. The students may not know the exact conception and this may lead to misunderstandings in the future. The result and possible reasons according to students' interviews are categorised as follow. Some errors committed include:

1. Students' carelessness as can be seen in questions number 1, 9, 11, and 17.
2. Poor knowledge of divisions as in questions number 26 and 29 which leads to rule mix up.
3. Students' inability to assimilate the concepts of integers into the schema of whole numbers that they built from primary school which leads to misconception. Hence, they ignored the signs of the integers and performed the operations given, which can be seen when students answered (b) in items 2 and 3, and (c) for item 4, and so on.

4. Surface understanding which leads to misapplication of rules for the multiplication of positive and negative in addition and subtraction questions and also misapplication of distributive law. Students were also confused when the teacher sometimes said that for the ‘addition’ operation, when a positive and a negative number is added, the answer will carry the sign of the bigger number and they apply this mistaken knowledge in problems that involve other operations too. This can be seen when students answered many of the numbers shown in the appendix.

5.1.2.4 Research Question 4: What are the causes of error in solving problems in operation of integers?

The first and most common error found in this study was the use of parenthesis. Students tend to make a mistake when they deal with any questions involving the parenthesis. For example, $2-\ (-5)$ is usually interpreted as $2-5$ or $2\ --5$ or $2+-5$. They will give the result of -3 or -7 or 7 because they take into consideration that the value of 5 is a negative. The students tend to ignore the function of the parenthesis or they do not how it works. Therefore, they give a wrong answer and this leads to misconception. Teachers need to focus more on this problem because the parenthesis is always used in mathematics, especially algebra and other higher-level concepts. This finding supports the result obtained by Balbuena and Buayan (2015) which stated that the parenthesis made students more confused when they were required to add two negative numbers, add a positive integer and a negative integer, subtract a negative integer from a negative integer, and subtract a negative integer from a positive integer.

The second cause of errors was due to poor mathematical language. Mathematics is a language on its own. The understanding of mathematical concepts and application require a specific language skill. Most of the time, the terminology used in our everyday language has other implications in mathematics. Poor mathematical language is one of the causes in the misconceptions of integers. This is due to students' inability to differentiate between the mathematical symbols and signs, especially for subtraction and negative values. Since both of them are written the same way, thus, the students tend to mistake them for one another. This requires teachers' skill in explaining to the students of the difference between them. However, with the time constraint, teachers are unable to explain further to the students. This finding supports the result of Hayes and Stacey (1998) and Sadler (2012) which stated that students would totally disregard the negative signs and the negative numbers were not easy to teach and learn.

Thirdly, based on the teachers' interviews, it was revealed that some of them prefer the use of calculators among students instead of providing the right concept of integers. Since they have 13 topics in the syllabus to cover within the given time, thus, they prefer not to waste time only on teaching integers. However, the use and dependency on calculators in lower level math teaching makes students unable to learn basic facts. In addition, the calculator also prevents students from discovering and understanding underlying mathematical concepts and instead encourages them to randomly try different operations without understanding what they are doing.

Another common error discovered relate to incorrect addition and subtraction. When adding or subtracting integers, especially when one is negative and the other is positive, students fail to add or subtract correctly. For example, when given -4 plus 6, the student may assume the answer to be negative 10, when it really is positive two. Misconceptions involving the subtraction of negative numbers are perpetuated when students rely on previously learned procedures to evaluate these sorts of expressions. Middle school students are sometimes taught to address the subtracting of a negative value using “keep, change, change,” meaning retaining the sign of the first number (keep), changing the minus sign to a plus sign (change), and changing the negative number to a positive number (change). While this shortcut is probably introduced by educators with good intentions, it does not provide the student with the conceptual knowledge to understand why subtracting a negative number results in a positive value. This supports Alsina and Nelsen’s (2006) claim in that students tend to get confused and struggle when they are asked to solve simple and routine mathematical problems.

Lastly, another common cause of errors is external limitations. Teachers may find it difficult to navigate a class with a large number of students, juggling between school activities and events, handling absent students, students with mixed abilities, and natural disasters. The large number of students in a class with different levels of abilities prove to be a challenge for teachers in making the students understand a particular concept. Teachers have to ensure that students do not feel frustrated. The stronger (or advanced students) may feel that they are not being challenged enough and are not learning as much as they can, while the weaker (less advanced) students may feel that the tasks and learning materials are too difficult or the teachers do not assist them enough. Student participation is another worry. In a mixed-ability class, teachers will find that the stronger students generally participate more than the less advanced students. This may be due to the shy nature of the less advanced students or because they are very aware that they are not the top students in the class and are scared of getting the answer wrong. Lack of participation can also cause the less advanced students to perform even less (or worse) in class.

Another challenge is that the teacher might not be able to devote the time and attention needed to less advanced students. In a mixed-ability class, the teacher might find that they spend too much time on the stronger students, and effort should be made to spend more time assisting the less advanced students. Therefore, the limitation of the teaching period might give a big influence to the teachers in order to focus on certain topics and students. In addition, the large volume of syllabuses also affects teachers' performance. Thus, for better education, the syllabus needs to be revised so that important foundations will have more time to be explained.

5.1.2.5 Research Question 5: What are the differences between students from the various school settings, in their understanding of operation on integers?

The result shows that there is a significant difference between students from rural and urban schools. Urban school students exhibited better results than rural students. This is probably due to the better quality in education in terms of the availability of information that they obtain from various sources like electronic and mass media, their educated families and peer groups which also help them to better perform. They have many facilities, better resources and advantages in their education compared to rural students. Students in rural areas are less exposed to the outside world and there is also a lack of knowledge about currents issues.

A probable explanation for the results of this study is that the current government is facilitating the rural areas with needed resources. Another possible reason could be due to the availability of electronic media and especially the internet in rural environments, thus the gap of exposure to the external environment which previously existed is reduced. In addition, the rural communities nowadays know the importance of having good education. This supports the contention of Young (2006), Owoeye and Yara (2011) and Ijenkeli, Paul and Vershima (2012) who found that students in urban areas performed better than their rural counterparts. Similarly, studies conducted by Provasnik, KewalRamani, Coleman, Gilberston, Herring and Xie (2007) and Waters (2005) found that different areas will have different learning access.

In addition, gender also caused a significant difference on the students' mathematics performance in EIIT. Female students performed better compared to male students. This contradicts to the findings by Fennema, Carpenter, Jacobs, Franke, and Levi (1998), Leahey and Guo (2001), and, Cooper and Dunne (2000) who found that male students performed better than female students. However, this research supports Brunner, Krauss and Kunter (2007), and, Neuville and Croizet's (2007) findings in that female students performed better than male students.

As a conclusion, the results of EIIT showed significant differences in the academic performance of rural and urban students and also gender differences, clearly indicating the significant effect of locality and gender on the academic performance of the students.

1.2 LIMITATIONS OF THE STUDY

Firstly, the limitation of this study is due to the Hawthorne effect. As this study used students' and teachers' involvement, this effect naturally took place. This effect causes the students and teachers to change their behaviours and become more productive because of the attention they were getting, and not because of any change in any of the variables such as the classroom environment, teachers' teaching experiences, or the EIIT. Secondly, in the second phase of the research which was the interview process, both students and teachers were involved. Thus, permission from the Ministry of Education was required in order to avoid any overlapping of schools' schedules. Obtaining consent and permission required much time since it involved the approval of the Ministry, State Education Office, and schools' principals. Apart from that, this study only focused on the operations of integers. It does not include word problems which needed extra attention. Therefore, the future study is recommended to focus on this point.

1.3 CONCLUSION

The findings of this study have provided significant evidence to support the importance of EIIT in the operations of integers in increasing students' achievement in mathematics. The analysis in the previous chapter indicates that the EIIT could assist students to score better solving the problems of integers. In addition, the data has also shown that the EIIT encourages teachers to design new activities in identifying students' errors and misconceptions, and subsequently, to diagnose students' weaknesses in integers. Teachers should take into account that learning is an active process in which the teachers should know the weaknesses and strengths of their students so that everyone will get the proper treatment in the future.

Lastly, in order to development a strong new understanding, students need a systematic instruction and they have to make connections between mathematical concepts and the real world using actual objects. Diagrams, graphs, pictures and concrete-based learning could support the teachers' lesson in order to represent the mathematical concepts in the most interesting ways.

1.4 RECOMMENDATION FOR FURTHER RESEARCH

The result of this study supports the findings of previous studies on operations of integers (Bny, 2006; Khalid, 2006). Students should have a sound conceptional understanding of the topic in order to solve the operations of integers and related problems successfully. In the teaching and learning process in the classroom, teachers should adopt a constructivist approach, creative and critical thinking, active learning, and multiple representatives. Students ought to be exposed and guided to understand the concepts well. Teachers need to make the effort to ensure students could grasp the basic skills in mathematics before they delve into difficult problems.

With respect to the findings of the present study and the contributions of previous studies concerning the operations of integers, several recommendations for further study are made. The following recommendations for future research and practice are put forth:

1. This study was conducted on only eight public schools. Thus, the findings from this study cannot be generalised. Further studies are strongly recommended to increase the number of schools involved so that generalisation can be made. Performing the EIIT to different types of schools is also a good step in order to provide more richness in the data on EIIT. The different types of schools can be either religious schools, Chinese-Indian type of schools, private schools, or international schools.
2. More studies should be done with more representative samples to find out possible factors that could obstruct students' performance in solving problems in the operations of integers.
3. In the currently advanced and new curriculum, almost all teachers are still be using the chalk-and-talk method in teaching mathematics. Students are made to sit passively while the teacher delivers a lecture. It leads to the students' inability to grasp key ideas and concepts. In addition, since a teacher has to deliver a fixed number of concepts within a limited time, most classroom activities are sufficed to the presentation stage only. Practice is left for the students to do as homework. As a result, it does not

allow students to experiment with new concepts. Their learning is put to a halt at a certain stage, and they end up cramming concepts, and are unable to produce anything fruitful, except generic answers to exam questions. Furthermore, many students might get stuck while doing problem sets at home. This too thwarts their performance. If they are unable to master one concept, and have been unable to practice it effectively, they will likely be unable to grasp a newer concept based on the previous one. Hence, a point which should be seriously considered.

4. A teacher's lecture is generally one-size-fit-all. However, not every student has the same pace of learning. While some students can follow the teacher's lecture with convenience, others may require time to chow on the information that they are getting. Also, each student has a different learning style. Teachers must not expect a kinaesthetic learner to master a concept by just listening to a lecture. If a visual learner gets worse grades than an auditory learner, it does not mean that the former is slow or dull. It might simply mean that the classroom strategies were designed for the auditory learner only. Poor grades and lagging in classroom performance are a major contributor to a poor self-image and lack of confidence. In fact, the failure of many students to achieve what they are capable of achieving can be attributed to the above factors.
5. The use of the instrument in this study can be limited to Q1 – Q32, if future researchers are only interested in the computation of routine problems in the operations of integers. However, if future researchers are interested in diagnosing students' ability in solving word problems involving the operations on integers, then Q33 – Q40 would be suitable.

BIBLIOGRAPHY

Ahmed Gubbad, A. A. M. (2010). The effect of cooperative learning on the academic and retention of mathematical concepts at the primary school in Holy Makkah. *Journal King Saud University on Educational Science and Islamic Studies*, *22*(2), 13-23.

Akpalu, R., Adaboh, S., & Boateng, S. S. (2018). Towards improving senior high school students' conceptual understanding of system of two linear equations. *International Journal of Educational Research Review*, *3*(1), 28-40.

Alsina, C., & Nelsen, R. B. (2006). *Math Made Visual: Creating Images for Understanding Mathematics.* United States of America: The Mathematical Association of America.

Ana, S. (2011). *Neuroscientist Shares Life-Changing Visual Approach to Math Instruction at TEDX Orange Coast.* California: MIND Research Institute.

Ashlock, R. B. (2002). *Error Patterns in Computation: Using Error Patterns to Improve Instruction*. New Jersey: Pearson Education, Inc.

Badarudin, B. R. H., & Khalid, M. (2008). *Using the Jar Model to Improve Students' Understanding of Operations on Integers*. Proceedings of ICME-11–Topic Study Group 10 Research and Development in the Teaching and Learning of Number Systems and Arithmetic, 85.

Balbuena, S. E., & Buayan, M. C. (2015). Mnemonics and gaming: Scaffolding learning of integers. *Asia Pacific Journal of Education, Arts and Sciences*, *2*(1).

Baroody, A., & Ginsburg, H. (1990). Children's mathematical learning: a cognitive view. In R. B. Davies, C. A. Maher ,& N. Nodding (Eds.), *Constructivist Views in the Teaching and Learning of Mathematics* (pp. 51-64). National Council of Teachers of Mathematics (NCTM).

Battista, M. (1983). *A Complete Model for Operations on Integers*. Retrieved December 15, 2016 from http://www.iit.edu/~smile/ma8718.html

Björklund, C. (2014). Less is more: Mathematical manipulatives in early childhood education. *Early Child Development and Care, 184*(3), 469-485.

Bny Rosmah, B. (2006). *Investigating Teaching and Learning of Integers at Form 1 level in Brunei Darussalam*. Brunei Darussalam.

Bolyard, J., & Moyer-Packenham, P. (2012). Making sense of integer arithmetic: the effect of using virtual manipulatives on students' representational fluency. *Journal of Computers in Mathematics and Science Teaching*, *31*(2), 93-113.

Bolyard, J. J., & Moyer-Packenham, P. S. (2006, November). *The Impact of Virtual Manipulatives on Student Achievement in Integer Addition and Subtraction.* Paper presented at The Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Mérida, Yucatán, Mexico.

Bouck, E. C., Satsangi, R., Doughty, T. T., & Courtney, W. T. (2014). Virtual and concrete manipulatives: a comparison of approaches for solving mathematics problems for students with autism spectrum disorder. *Journal of Autism and Developmental Disorders, 44*(1), 180-193.

Brahier, D. J. (2016). *Teaching Secondary and Middle School Mathematics*. New York: Routledge.

Bruner, J. S. (1966). *Towards a Theory of Instruction*. Cambridge, Mass.: Belkapp Press.

Brunner, M., Krauss, S., & Kunter, M. (2007). Gender differences in mathematics: Does the story need to be rewritten? *Intelligence*, *36*(5), 403-421.

Bush, S. B. (2011). Analysing common algebra-related misconceptions and errors of middle school students. *Electronic Theses and Dissertation Paper 187*. Retrieved from http://doi.org/10.18297/etd/187

Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, *18*(2), 99-111.

Cakir, M. (2008). Constructivist approaches to learning in science and their implications for science pedagogy: a literature review. *International Journal of Environmental & Science Education*, *3*(4), 193-206.

Castro, S. (2017). *Algebra Tiles Effect on Mathematical Achievement of Students with Learning Disabilities* (Unpublished master's thesis). California State University, Los Angeles.

Cavendish, M. (2010). *Singapore Math: A Visual Approach to Word Problems*. Retrieved January 9, 2017 from http://www.hmhco.com/shop/education-curriculum/math/math-in-focus-singapore-mathematics

Cculy, J., Scully, B., & LeSage, J. (1991). *Alge-tiles: A Hands-On Resource for Learning Algebra*. Ontario, Canada: Exclusive Educational Product.

Chappell, M. F., & Strutchens, M. E. (2001). Creating connections: promoting algebraic thinking with concrete models. *Mathematics Teaching in the Middle School*, *7*(1), 20-25.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research Methods in Education* (5th ed.). London & New York: Routledge Falmer.

Confrey, J. (1990). A review of research on students' conceptions. In C. Cazden (Ed.), *Review of Research in Education* (Vol. 16), (pp. 3-56). Washington, DC: American Educational Research Association.

Cooper, B., & Dunne, M. (2000). *Assessing Children's Mathematical Knowledge: Social Class, Sex and Problem-Solving*. Buckingham: Open University Press.

Creswell, J. W. (2013). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. Sage Publications.

D'Souza, S. M., & Wood, L. N. (2003). *Tertiary Students' View about Group Work in Mathematics, Australia*. Retrieved March 20, 2017 from http://hdl.handle.net/10453/3054

Datuk Seri Mahdzir Khalid. (2016). New KSSM, KSSR curriculum to be implemented in 2017. *Bernama*. Kuala Lumpur.

De Bock, D., Verschaffel, L., Janssens, D., Van Dooren, W., & Claes, K. (2003). Do realistic contexts and graphical representations always have a beneficial impact on students' performance? Negative evidence from a study on modelling non-linear geometry problems. *Learning & Instruction*, *13*(4), 441-463.

Dean, K, A. (2007). The effects of visual mathematical instruction on the perception and achievement of elementary visual-spatial learners. *Proquest Educational Journal*, *22*, 6.

Dienes, Z. P. (1960). *Building Up Mathematics.* London: Hutchison Education.

Dienes, Z. P. (1971). An example of the passage from the concrete to the manipulation of formal systems. *Educational Studies in Mathematics*, *3*, 337-352.

Drew, C. J., Hardman, M. L., & Hosp, J. L. (2008). *Designing and Conducting Research in Education*. United States of America: Sage Publications, Inc.

Drews, D., Dudgeon, J., Lawton, F., & Surtees, L. (2014). *Children's Errors in Mathematics*. SAGE Publication: London.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*, 103-131.

Egodawatte, G (2011). *Secondary School Students' Misconception in Algebra* (Unpublished PhD thesis). University of Toronto, Canada.

Faudiah, N. F., Suryadi, D., & Turmudi. (2016). *Some Difficulties in Understanding Negative Numbers Faced by Students: A Qualitative Study Applied at Secondary Schools in Indonesia*. International Education Studies: Canada.

Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). A longitudinal study of gender differences in young children's mathematical thinking. *Educational Researcher*, *27*, 6-11.

Freeman, S., Haak, D., & Wenderoth, M. P. (2011). Increased course structure improves performance in introductory biology. *CBE Life Sci Educ*, *10*(2), 175-186.

Goins, K. B. (2001). *Comparing the Effect of Visual and Algebra Tile Manipulatives Methods on Student Skill and Understanding of Polynomial Multiplication* (Unpublished doctoral dissertation). University of South Carolina, Columbia.

Goldin, G., & Shteingold, N. (2001). Systems of representation and the development of mathematical concepts. In A. A. Cuoco, & F. R. Curcio (Eds.), *The Role of Representation in School Mathematics* (pp. 1-23). Boston, Virginia: NCTM.

Goldsmith, P. A. (2006). Learning to understand inequality and diversity: Getting students past ideologies. *Teaching Sociology*, *34*(3), 263-277.

Graeber, A., & Johnson, M. (1991). *Insights into Secondary School Students' Understanding of Mathematics*. College Park, University of Maryland, MD.

Granger, C. (2008). Rasch analysis is important to understand and use for measurement. *Rasch Measurement Transactions*, *21*(3), 1122-1123.

Guillermo, M. J. (2005*). Pictorial Mathematics: An Engaging Visual Approach to the Teaching and Learning of Mathematics*. Etiwanda, CA: Meaningful Learning.

Hart, K. M., Brown, M. L., Kuchemann, D. E., Kerslake, D., Ruddock, G., & McCartney, M. (1981). *Children's Misunderstanding of Mathematics*. John Murray: London.

Hayes, B. (1996). *Teaching for Understanding of Negative Number Concepts and Operations*. Retrieved from http://www.aare.edu.au/96pap/hayeb96054.txt

Hayes, B., & Stacey, K. (1998). *Teaching Negative Numbers using Integers Tiles*. Melbourne, Australia: Heinemann.

Homeschool Mathematics. (2003). *How to Teach Integers*. Retrieved January 5, 2017 from http://www.homeschoolmath.net/teaching/integers.php

Ijenkeli, O. E., Paul, A. I., & Vershima, A. M. (2012). Impact of career-related instruction on mathematics achievement of rural and urban students in Benue State, Nigeria. *Research Journal of Mathematics and Statistics*, *4*(2), 39-41.

Irwin, H. C. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education*, *32*(4), 399-420.

İşleyen, T. (2012). Investigation the activity for teaching multiplication in algebraic expressions with algebra tiles from the perspective of misconceptions. *Energy Education Science and Technology Part B: Social and Educational Studies*, *4*(2), 1039-1046.

Jensen, J. L., & Lawson, A. (2011) Effects of collaborative group composition and inquiry instruction on reasoning gains and achievement in undergraduate biology. *CBE Life Sci Educ*, *10*(1), 64-73.

Jones, B. D., Uribe-Fiórez, L. J., & Wilkins, J. L. M. (2011). Motivating mathematics students with manipulatives: Using self-determination theory to intrinsically motivate students.

Motivation and Disposition: Pathways to Learning Mathematics, *73*(special issue), 215-227.

Johnson, K. A. (1993). Manipulatives allow everyone to learn mathematics. *Contemporary Education*, *65*(1), 10-12.

Krejcie, R. V., & Morgan, D. M. (1970). Determining sample size for research activities. *Educational and Psychological Measurement*, *30*, 607-610.

Karakırık, E., & Aydın, E. (2011). *Matematik Nesneleri.* Retrieved from https://www.academia.edu/9634384/MATEMAT%C4%B0K_NESNELER%C4%B0

Larbi, E. (2011). *The Effect of Algebra Tiles on Students' Performance in Algebra at a Junior High School* (Unpublished master's thesis). University of Cape Coast, Ghana.

Leahey, E., & Guo, G. (2001). Gender differences in mathematical trajectories. *Social Forces*, *80*, 713-732.

Learning Maths. (2002). *Meanings and Models for Operations*. Retrieved May 1, 2016 from http://www.learner.org/channel/courses/learningmath/number/support/Img4.pdf

Leitze, A. R., & Kitt, N. A. (2000). Using homemade algebra tiles to develop algebra and prealgebra concepts. *Mathematics Teacher*, *93*(6), 462-520.

Lim, C. S. (2011). *Assessment in Malaysian School Mathematics: Issues and Concerns*. Paper presented at the meeting of The APEC-Tsukuba International Conference, Japan.

Longfield, J. (2009). Discrepant teaching events: Using an inquiry stance to address students' misconceptions. *International Journal of Teaching and Learning in Higher Education*, *21*(2), 266-271.

Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *15*(1), 10–21.

Madihah, K. (2006). *Using the Algebraic Tiles to Improve Students' Understanding of Operations on Integers*. Brunei Darussalam.

Magruder, R. L. (2012). *Solving Linear Equations: A Comparison of Concrete and Virtual Manipulatives in Middle School Mathematics* (Unpublished doctoral dissertation). University of Kentucky, Kentucky.

Makonye, J. P., & Fakude, F. (2016). A study of errors and misconceptions in the learning of addition and subtraction of directed numbers in Grade 8. *SAGE Open*, October-December, 1-10. DOI: 10.1177/2158244016671375

Marlow, B. A., & Page, M. L. (2005). *Creating and Sustaining the Constructivist*.

McEwan, E. K. (2000). *The Principal's Guide to Raising Math Achievement*. United States of America: Corwin Press, Inc.

MEB. (2012). *Malaysia Educational Blueprint*. Ministry of Education, Putrajaya, Malaysia.

Midgley, C., Feldlaufer, H., & Eccles, J. (1989). Change in teacher efficacy and student self- and task-related beliefs in mathematics during the transition to junior high school. *Journal of Educational Psychology*, *81*, 247-258.

Midgley, C., Kaplan, A., & Middleton, M. (2001). Performance-approach goals: Good for what, for whom, under what circumstances, and at what cost? *Journal of Educational Psychology*, *93*, 77-86.

Midgley, C., Maehr, M. L., Hruda, L. Z., Anderman, E., Anderman, L., Freeman, K. E., et al. (2000). *Manual for the Patterns of Adaptive Learning Scales: PALS* [manual]. Ann Arbor: University of Michigan. Retrieved from http://www.umich.edu/~pals/PALS%202000_V12Word97.pdf

Moses, B. E. (1977). *The Nature of Spatial Ability and Its Relationship to Mathematical Problem Solving* (Unpublished Ph.D dissertation). Indiana University, USA.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics: An International Journal*, *47*(2), 175-197.

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of mathematical representations. In W. J. Masalaski, & P. C. Elliott (Eds.), *Technology-Supported Mathematics Learning Environments* (pp. 17-34). Reston, VA: National Council of Teachers of Mathematics.

Musser, G. L, Peterson, B. E., Burger, W. F. (2014). *Mathematics for Elementary Teachers: A Contemporary Approach* (10^{th} ed.). Thousand Oaks, CA: Corwin Press.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA.

Neuville, E., & Croizet, J. (2007). Can salience of gender identity impair mathematics performance among 7-8 years old girls? The moderating role of task difficulty. *European Journal of Psychology of Education*, *22*, 307-316.

Okpube, N. M. (2016). Card games and algebra tic tacmatics on achievement of junior secondary ii students in algebraic expressions. *International Journal of Evaluation and Research in Education*, *5*(2), 93-100.

Owoeye, J. S., & Yara, P. O. (2011). School location and academic achievement of secondary school in Ekiti State, Nigeria. *Asian Social Science*, *7*(5).

Passy. (2012). Adding integers using zero pairs. *Passy's World of Mathematics*. Retrieved from http://passyworldofmathematics.com/adding-integers-using-zero-pairs/

Patterson, K. (2010). *Rural Schools: Federal Expenditures and State Perspectives*. DC: Council of State Governments.

Paul, R., & Elder, L. (2001). *Critical Thinking Concepts and Tools*. The Foundation for Critical Thinking.

Piaget, J. (1964). Cognitive development in children. *Journal of Research in Science Teaching*, *40*(2), 176-186.

Piaget, J. (2001). *The Psychological of Intelligence*. New York: Routledge Classics.

Picciotto, H., & Wah, A. (1993). A new algebra: Tool, themes, concepts. *Journal of Mathematical Behaviour*, *12*(1), 19-42.

Presmeg, N. (1997). Research on visualisation in learning and teaching mathematics. *Emergence from Psychology*, p. 2.

Provasnik, S., KewalRamani, A., Coleman, M. M., Gilberston, L, Herring, W., & Xie, Q. (2007). Status of education in rural America. *NCES 2007-040*. Washington: Institute of Education Sciences.

Quick Facts. (2014). *Quick Facts 2014: Malaysian Educational Statistics*. Putrajaya: EPRD.

Radatz, H. (1980). *Students' Errors in the Mathematical Learning Process: A Survey, for the Learning of Mathematics*. Retrieved from http://www.jstore.org/stable/40247696

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: a classroom study. *Journal of Computers in Mathematics and Science Teaching*, *24*(1), 5-25.

Research on the Benefits of Manipulatives. (2015, February 18). Retrieved from https://www.hand2mind.com/pdf/learning_place/research_math_manips.pdf

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: the case of decimal fractions. *Journal for Research in Mathematics Education*, *20*(1), 8-27.

Roselizawati, S., & Masitah, S. (2014). *Understanding Students' Mathematical Errors and Misconceptions: The Case of Year 11 Repeating Students*. Brunei Darussalam: International Scientific Publication and Consulting Services.

Rubin, R. J., Marcelino, J. M., Mortel, R., & Lapinid, M. R. (2014). *Activity-Based Teaching of Integer Concepts and Its Operations*. Paper presented at DLSU Research Congress 2014 De La Salle University, Manila, Philippines.

Sadler, J. T. (2012). *The Positives about Negatives: A Study of Errors and Misconceptions with Integer Operations in Adult Education* (Unpublished master's thesis). State University of New York.

Sarah, H. (2012). *Teaching Integers Operations: Work Mat, Coloured Counters and Number Lines*. Retrieved from mathequalslove.blogspot.my/2012/08/teaching-integer-operation-with.html

Saraswati, S. (2016). Supporting students' understanding of linear equations with one variable using algebra tiles. *Journal on Mathematics Education*, *7*(1), 21-32.

Savion, L. (2009). Clinging to discredited beliefs: the larger cognitive story. *Journal of the Scholarship of Teaching and Learning*, *9*, 81-92.

Schindler, M., & Hubmann, S. (2013). *About Students' Individual Concepts of Negative Integer – In Terms of the Order Relation*. Proceedings of Congress of European Research in Mathematics Education, 6 - 10 February, Turkey.

Schlosser, T. K. (2010). *Using Algebra Tiles to Aid Students in Factoring Polynomials* (Unpublished master's thesis). Central Connecticut State University, New Britain.

Schweitzer, D., Collin, M., & Baird, L. (2007). *A Visual Approach to Teaching Formal Access Models in Security*. Proceeding of The 11th Colloquium for Information Systems Security Education, Boston University.

Setyawati, R. D., & Indiati, I. (2018). *Analysis Misconception of Integers in Microteaching Activities*. 4th International Seminar of Mathematics, Science and Computer.

Sharp, J. M. (1995). *Results of Using Algebra Tiles as Meaningful Representations of Algebra Concepts*. Retrieved from http://www.eric.ed.gov/

Sibbald, T. (2009). The relationship between lesson study and self-efficacy. *School Science and Mathematics*, *109*(8), 450-460.

Silver, E., & Marshall, S. (1990). Mathematical and scientific problem solving: Finding, issues, and instructional applications. *Dimension of Thinking and Cognitive Instruction*. Hilldale (NJ): LEA.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, *26*, 9-15.

Skemp, R. R. (1976). *Relational Understanding and Instrumental Understanding, Mathematics Teaching*.

Small, M. (2013). *Eyes on Math: A Visual Approach to Teaching Math Concepts*. New York: Teachers College Press.

Smith, N. N. (2017). *Every Math Learner, Grades 6-12: A Doable Approach to Teaching with Learning Differences in Mind*. California: Corwin.

Sobol, A. J. (1998). *Formative and Summative Evaluation Study of Classroom Interactions and Student/Teacher Effects when Implementing Algebra Tiles Manipulatives with Junior*

High School Students (Unpublished doctoral dissertation). St. John's University, New York.

Steen, K., Brooks, D., & Lyon, T. (2006). The impact of virtual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Teaching*, 25(4), 373-391.

Stokes, S. (1999). *Visual Literacy in Teaching and Learning: A Literature Perspective*. Retrieved March, 7 2017 from ejite.isu.edu/Volume1No1/Stokes.html

Suh, J., & Moyer, P. S. (2007). Developing students' representation fluency using virtual and physical algebra balances. *Journal of Computers in Mathematics and Science Teaching*, 26(2), 155-173.

Tandiseru, S. R. (2015). The effectiveness of local culture-based mathematical heuristic-KR learning towards enhancing students' creative thinking skill. *Journal of Education and Practice*, *6*(12), 74-81.

Thornton, S. (2001). *A Picture is Worth a Thousand Words*. Retrieved February 15, 2017 from math.unipa.it/~grim/AThornton251.PDF

Trends in International Mathematics and Science Study. Retrieved August 2, 2015 from http://nces.ed.gov/timss/

Tunca, N. (2015). The regression level of constructivist learning environment characteristics on classroom environment characteristics supporting critical thinking. *Eurasian Journal of Educational Research*, *60*, 181-200.

Ural, A. (2016). *7th Grade Students' Understandings of Negative Integers*. Turkey: Macrothink Institute.

Vinogradova, N. (2007). Solving quadratic equations by completing squares. *Mathematics Teaching in the Middle School*, *12*(7), 403-405.

Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Cambridge, MA: Harvard University Press.

Waters, M. S. (2005). *A Mathematics Educator's Introduction to Rural Policy Issues*. OH: Appalachian Collaborative Centre for Learning, Assessment, and Instruction in Mathematics.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: an investigation of an explicit instruction model. *Learning Disabilities: Research & Practice*, *18*(2), 121-131.

Wright, B. D., & Stone, M. H. (1979). *Best Design Test: A Handbook for Rasch Measurement*. Chicago: Mesa Press.

Wood, D. A. (1960). *Test Construction: Development and Interpretation of Achievement Tests*. Columbus, OH: Charles E. Merrill Books, Inc.

Woolner, P. (2004). A comparison of a visual-spatial approach and a verbal approach to teaching mathematics. *Psychology of Mathematics Education*, 449-456.

Wubbels, T., Korthagen, F., & Broekman, H. (1997). Preparing teachers for realistic mathematics education. *Educational Studies in Mathematics*, *32*(1), 1-28.

Young, D. J. (2006). Rural and urban differences in student achievement in science and mathematics: a multilevel analysis. *An International Journal of Research, Policy and Practice*.

Yuliani, K., & Saragih, S. (2015). The development of learning devices based guided discovery model to improve understanding concept and critical thinking mathematically ability of students at Islamic junior high school of Medan. *Journal of Education and Practice*, *6*(24), 116-129.

APPENDIX A

TEST OF THE FOUR OPERATIONS OF INTEGERS

Name/*Nama*: ………………………… School/*Sekolah*: …….........................

Class/*Kelas*: ……………….................. UPSR Maths Grade/*Gred Matematik UPSR*: …..... Date/*Tarikh*: ………………………….

Answer **ALL** questions. You are given 30 minutes to answer the questions.
Sila jawab ***SEMUA*** *soalan. Anda diberi masa 30 minit untuk menjawab kesemua soalan.*

There are altogether 30 questions. **DO NOT** leave out any question.
Kertas ini mengandungi 30 soalan. ***JANGAN*** *tinggalkan soalan tanpa dijawab.*

Work out your answer and **all necessary work** in the space underneath each question. Please circle your answer in the column marked "Answer."
Jawab semua soalan dan ***tuliskan jalan kerja*** *ditempat yang disediakan. Bulatkan jawapan anda di ruang bertanda "Jawapan."*

Note: Calculators are not allowed.
Nota: Penggunaan kalkulator adalah tidak dibenarkan.

Question ***Soalan***	**Answer** ***Jawapan***
Example/*Contoh*: Simplify: $2 - 3 =$ *Permudahkan*: $2 - 3 =$	a) 1 b) -1 c) 5 d) -5
Simplify each of the following: *Permudahkan setiap soalan di bawah:*	
1) $2 + 6 =$	a) 4 b) -4 c) 8 d) -8
2) $6 + (-2) =$	a) 4 b) -4 c) 8

	d) -8
Question ***Soalan***	**Answer** ***Jawapan***
3) $2 + (-6)$	
4) $2 + (-2) =$	a) 0 b) 1 c) 4 d) -4
5) $-2 + 6 =$	a) 4 b) -4 c) 8 d) -8
6) $-6 + 2 =$	a) 4 b) -4 c) 8 d) -8
7) $-6 + 6 =$	a) 0 b) 1 c) 12 d) -12
8) $-2 + (-6) =$	a) 4 b) -4 c) 8 d) -8
9) $6 - 2$	a) 4 b) -4 c) 8 d) -8
10) $2 - 6$	a) 4 b) -4 c) 8

	d) -8
11) $2 - 2$	a) 0 b) 1 c) 4 d) -4
12) $-6 - 2$	a) 4 b) -4 c) 8 d) -8
13) $2 - (-6)$	a) 4 b) -4 c) 8 d) -8
14) $-2 - (-6)$	a) 4 b) -4 c) 8 d) -8
15) $6 - (-6)$	a) 0 b) 1 c) 12 d) -12
16) $-6 - (-2)$	a) 4 b) -4 c) 8 d) -8
17) 6×2	a) 8 b) -8 c) 12 d) -12
18) 2×-6	a) 8 b) -8 c) 12 d) -12

19) -2×6	a) 8 b) -8 c) 12 d) -12
20) -2×-6	a) 8 b) -8 c) 12 d) -12
21) -2×-2	a) 1 b) -1 c) 4 d) -4
22) 3×5	a) 8 b) -8 c) 15 d) -15
23) $3 \times (-5)$	(a) 8 (b) -8 (c) 15 (d) -15
24) -3×5	(a) 8 (b) -8 (c) 15 (d) -15
25) -3×-5	(a) 8 (b) -8 (c) 15 (d) -15
26) $6 \div 2$	a) 3 b) -3 c) 4 d) -4
27) $6 \div -2$	a) 3

	b) -3 c) 4 d) -4
28) $-6 \div 2$	a) 3 b) -3 c) 4 d) -4
29) $8 \div 4$	(a) 2 (b) -2 (c) 4 **(d)** -4
30) $8 \div (-4)$	(a) 2 (b) -2 (c) 4 (d) -4
31) $-8 \div 4$	(a) 2 (b) -2 (c) 4 (d) -4
32) $-8 \div -4$	(a) 2 (b) -2 (c) 4 (d) -4
33) $-6 \div -2$	a) 3 b) -3 c) 4 d) -4
34) Yesterday, Kamal had RM50 in his wallet. Then he worked at the coffee shop and made RM40 in tips. Which integer represents how much money is in Kamal's wallet now? *Semalam, Kamal mempunyai $RM50$ di dalam dompetnya. Kemudian beliau bekerja di kedai kopi dan mendapat tip sebanyak $RM40$. Integer manakah yang mewakili berapa banyak wang yang ada di dalam dompet Kamal sekarang?*	(a) RM10 (b) RM100 (c) RM90 (d) RM80

35) In January, the average temperature in Moscow and New York are $-8°C$ and $-5°C$ respectively. What is the difference between the temperature in Moscow and the temperature in New York? *Pada January, suhu tetap di Moscow and New York ialah* $-8°C$ *dan* $-5°C$. *Apakah perbezaan suhu di antara Moscow dan New York?*	a) 3 b) -3 c) 13 d) -13
36) Mary has RM250 in her account. She drew out RM50 twice from her account. How much is the balance in her account now? *Mary mempunyai RM*250 *didalam akaunnya. Beliau mengeluarkan RM50 sebanyak dua kali. Berapakah baki di dalam akaun beliau sekarang?*	a) 150 b) 100 c) -100 d) -150
37) Lisa owed mother RM30 to buy top-up for her telephone. After 2 weeks, she found that she needed to top-up for more credit. She borrowed another RM20 from her mother. How much does she owe her mother now, if owing money is considered negative? *Lisa telah meminjam sebanyak RM30 daripada ibu untuk menambah nilai kredit telefon. Selepas 2 minggu, beliau masih memerlukan menambahkan nilai kredit. Kemudian, beliau meminjam RM20 lagi daripada ibu. Berapakah nilai duit yang telah Lisa pinjam, jika meminjam duit itu dikira negatif?*	(a) RM50 (b) –RM50 (c) RM10 (d) −RM10
38) At the beginning of a draught season, the water level at the dam was 1cm above the critical level $(+1)$ and the water level now shows 3cm below the critical level (-3). How many cm of water did the dam lost? *Pada permulaan musim kemarau, paras air di empangan adalah* $1cm$ *diatas paras kritikal* (+1) *dan paras air sekarang adalah* $3cm$ *dibawah paras kritikal* (-3). *Berapa cm kah air yang surut di empangan tersebut?*	(a) $4cm$ (b) $-4cm$ (c) $2cm$ (d) $-2cm$
	a) $200m$

39) A submarine dove $300m$ below sea level (-300m). It then rose $100m$ dari from that position. At what depth is the submarine now written in integer? *Sebuah kapal selam telah menyelam sedalam* $300m$ *dibawah paras laut* ($-300m$). *Kemudian ia naik* $100m$ *daripada posisi tersebut. Berapakah paras kedalaman kapal selam tersebut sekarang dalam integer?*	b) $-200m$ c) $400m$ d) $-400m$
40) It was a very freaky weather day. The temperature started out at 9°C in the morning and went to -13°C at noon. How much is the change in temperature for that period of the day? *Cuaca hari ini sangat pelik. Suhu bermula pada* $9°$ *C pada waktu pagi dan turun sehingga* $-13\ °$ *C pada waktu tengahari. Berapakah perbezaan perubahan suhu pada hari tersebut?*	a) 22°C b) -22°C c) 4°C d) -4°C

End of paper

APPENDIX B

INTERVIEW GUIDE FOR TEACHERS

1. How many lessons did you teach on integers? Did you combine any lessons among the four operations of integers?
2. What method of teaching did you use in teaching the four operations of integers?
3. Do you think the method you used is effective in teaching your students?
4. Did you identify the common errors made by the students?
5. On which operations do your students have the most problems?
6. In what ways do your students usually make the mistake(s)?
7. How do you handle students who are struggling and having problems with integers?
8. What proportion of your students do you think grasped most of the main integers ideas that you dealt with in your lessons on integers?
9. What proportion of the Form 1 students whom you teach have reached a level of integers understanding?
10. Do you agree that "getting the students to understand integers is less important as compared to the students knowing how to get the correct answers? Give reasons why you say so.
11. What proportion of the students do you expect to pass and be able to answer questions in integers at the present time? Can you explain your students' performances on the EIIT?

APPENDIX C

INTERVIEW GUIDE FOR STUDENTS

1. What do you understand about integers?
2. Do you understand what your mathematics teacher has taught on integers? What method did she use?
3. When was the last time you learn integers?
4. Tell me how you solved questions 1, 4 (selecting question from the EIIT which most students got wrong)? [Note: Probe on this question well].
5. Among the four operations of integers, which operation did you find most difficult to solve? Why?
6. If you have any problems about integers, would you ask your teacher? Tell me how your teacher helped you.
7. Do you always discuss about the problems of integers with your friends? Describe how the discussion went.
8. Does your mathematics teacher provide enough notes, handouts, examples and exercises on integers?
9. If you have any problems on integers, what do you normally do? Does it really help you to solve your problem?
10. Do you feel interested in learning integers? Tell me about your thoughts.
11. Do you consider integers an easy topic or a difficult one? Why do you say that?

APPENDIX D

LESSON PLANS

Subject: Mathematics **Lesson Title:** Integers **Subtopic:** Addition of integers
Date:5/1/18 **Grade:** Form 1 Ibnu Sina **Time:** 8.00 am – 9.00 am
Duration of lesson: 2 periods (1 hour) **Number of students:** 30 students

---

Learning Outcome: At the end of the lesson, students should be able to:

1. Use and apply the rule of adding integers using algebraic tiles.

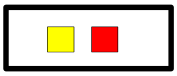
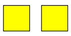
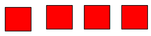
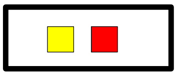
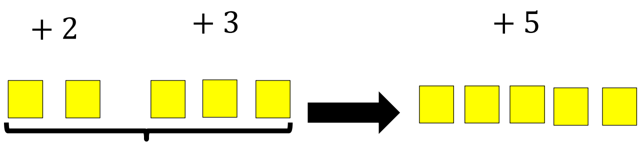
Pre-requisite Concepts and Skills:

Pre-requisite (5 minutes):

1. Review with the students on what they understand about integers and their knowledge on integers.
2. Review on line number, that is, as we go to the right side the numbers get bigger and vice versa.
3. Sign of a number is determined by the symbol in front of that number. For instance, +1 (this number has a positive sign and we pronounce it as positive one or just one), -1 (this number has a negative sign and we pronounce it as negative one).
4. Most of the time, the positive sign of a number can be hidden (not written down), for instance, number 3 can be written as +3 but it is understood when we write it as 2.
5. Explain to the students between binary operations of plus and minus and the unary operators that are positive and negative.

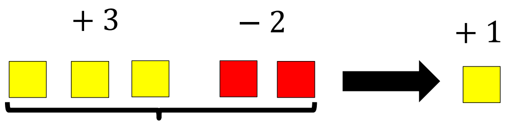
Material and Resources: Pencil, paper, whiteboard, marker-pen, yellow and red tiles.

Methodology:

Teacher's Activities	**Students' Activities**	**Time**
Set Induction: • Introduction to algebraic tiles. The teacher will need a small yellow square to represent $+1$ and a small red square (the flip-side) to represent -1.  • Explain to the students that +2 means drawing two yellow tiles  and -4 means drawing  four red tiles. • Explain to the students that addition means **combine or putting** in the positive or negative chips into the jar. • Explain about Zero Pairs because they are additive inverses of each other. When put together, they model zero. Do not use "cancel out" for zeroes. Use zero pairs or add up to zero. 	• Students listen to the teacher's explanation and respond to teacher's questions.	5 mins
Lesson Development: 1. Explain to the students how to add positive integers, add positive and negative integers together, and add negative integers with negative integers and negative integers with positive integers using the algebraic tiles. **Example 1:** $\mathbf{2 + 3 =}$ Addition means **COMBINE**. So, combining $2 + 3$ will be represented in the model below:  Therefore, combining 2 and 3 yellow tiles results in 5 yellow tiles, meaning $2 + 3 = 5$	• Students listen to the teacher's explanation and respond to teacher's questions.	15 mins

Therefore, we will write: $2 + 3 = 5$

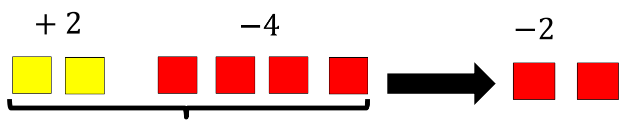
Example 2: $(+3) + (-2) =$



Make 2 zeroes to get 1 positive
Therefore, combining 3 yellow tiles with 2 red tiles will give the result of 1 yellow tile. It means, $(+3) + (-2) = +1$

Therefore, $3 + (-2) = +1$

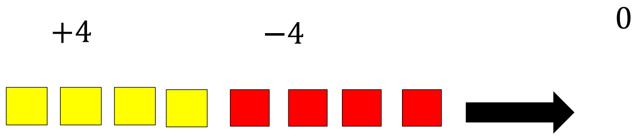
Example 3: $(+2) + (-4) = -2$



Make 2 zeroes to get 2 negatives.
Combining 2 yellow tiles with 4 red tiles gives the result of 2 red tiles. It means, $(+2) + (-4) = -2$

Therefore, $2 + (-4) = -2$

Example 4: $4 + (-4) =$

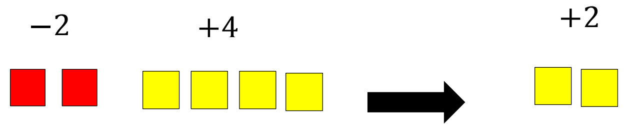


Make 4 zero pairs.
Combining 4 yellow tiles with 4 red tiles gives the result of 0. It means, $4 + (-4) = 0$

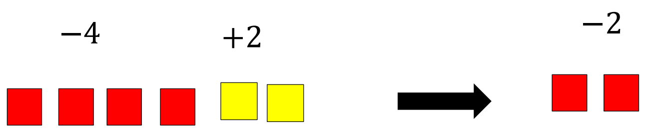
<table>
<tr>
<td>

Therefore, $4+(-4)=0$

Example 5: $-2+4$

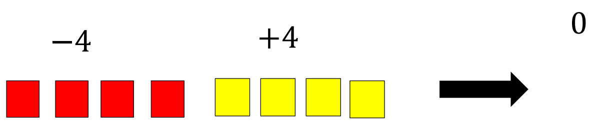


Therefore, : -2 $+4=+2$
Example 6: $-4+2$



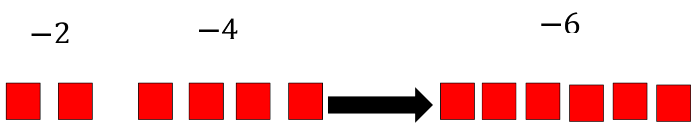
Therefore, $-4+2=-2$

Example 7: -4 $+4$



Therefore, $-4+4=\mathbf{0}$

Example 8: $-2+(-4)$



Combining 2 red tiles with 4 red tiles gives the result of 6 red tiles. It means, $(-2)+(-4)=-6$

Therefore, $-2+(-4)=-6$

</td>
<td></td>
<td></td>
</tr>
<tr>
<td>

Consolidation

The teacher asks students to do exercises on addition of integers using algebraic tiles.

</td>
<td>

- The students do the exercises.

</td>
<td>30 mins</td>
</tr>
</table>

<u>Exercises</u> Find the value of: 1. $5+3=$ 2. $5+(-3)=$ 3. $(-5)+3=$ 4. $-5+(-3)=$ 5. $4+6=$ 6. $4+(-6)=$ 7. $-4+6=$ 8. $-4+(-6)=$		
Evaluation & Closure • Back review with the students on addition of integers using algebraic tiles. • Teacher concludes the lesson on the topic that has been learned: 1. How to compute an adding of integers questions using algebraic tiles.	• Students pay attention to teacher's explanation.	5 mins

Subject: Mathematics **Lesson Title:** Integers **Subtopic:** Subtraction of integers
Date:5/1/18 **Grade:** Form 1 Ibnu Sina **Time:** 8.00 am – 9.00 am
Duration of lesson: 2 periods (1 hour) **Number of students:** 30 students

---

Learning Outcomes: At the end of the lesson, students should be able to:

2. Use and apply the rule of subtracting integers using algebraic tiles.

Pre-requisite Concepts and Skills:

Pre-requisite (5 minutes):

6. Review with the students on addition of integers using algebraic tiles.
7. Review with the students on two same numbers with different signs will give the answer zero (cancel out each other).

Material and Resources: Pencil, paper, whiteboard, marker-pen, yellow and red tiles.

Methodology:

<table>
<tr><th>Teacher's Activities</th><th>Students' Activities</th><th>Time</th></tr>
<tr><td>

Set Induction:

- Explain to the students that subtraction can be interpreted as "take away."
- Subtraction operation can also be thought as a "flipped to the opposite side."

</td><td>

- Students listen to the teacher's explanation and respond to teacher's questions.

</td><td>5 mins</td></tr>
<tr><td>

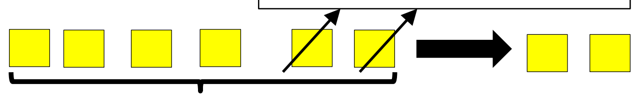
Lesson Development:

2. Explain the students how to subtract positive integers, subtract positive and negative integers, negative integers with positive integers and negative integers with negative integers using the algebraic tiles.

Example 1: $+4-(+2)=$

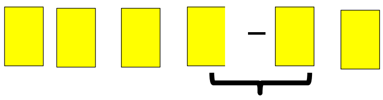
<u>Method 1: Take away</u>

Taking away 2 yellow tiles



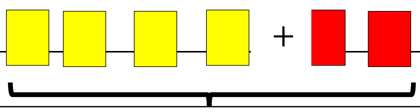
No need to add zero pair since all are yellow tiles.
2 yellow tiles are taken away from 4 yellow tiles.
Therefore, $(+4)-(+2) = +2$

<u>Method 2: Flipped to the opposite</u>



Subtraction sign makes the 2 yellow tiles after the operation is flipped to the opposite side of values. Therefore, for this case, 2 yellow tiles after operation becomes 2 red tiles. Meanwhile, the sign is also flipped to the other side of the operation.

Therefore, we will write $\mathbf{4-2=2}$



</td><td>

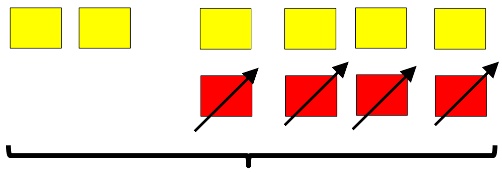
- Students listen to the teacher's explanation and response to teacher's questions.

</td><td>15 mins</td></tr>
</table>

Therefore, $(+4)-(+2) = +2$

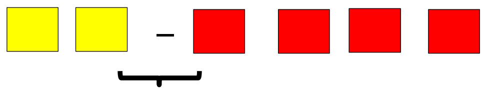
Example 2: $(+2) - (-4) =$

<u>Method 1: "Take away"</u>

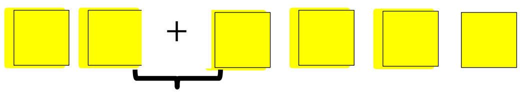


Add four pair-zeroes, and then take away four red tiles in order to get six yellow tiles. Therefore, $(+2) - (-4) = +6$

<u>Method 2: Flipped to the opposite</u>



Subtraction sign makes the 4 red tiles after the operation is flipped to the opposite side of values. Therefore, for this case, 4 red tiles after operation becomes 4 yellow tiles. Meanwhile, the sign is also flipped to the other side of the operation.

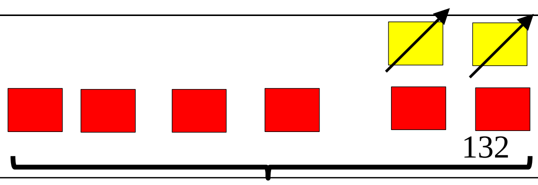


Therefore, $(+2) - (-4) = +6$

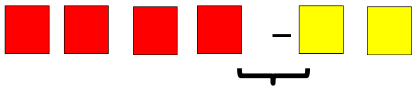
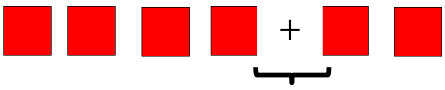
Therefore, we will write $(+2) - (-4) = +6$

Example 3: $(-4) - (+2) =$

<u>Method 1: Take away</u>



Add two pair-zeroes, and then take away two red tiles in order to get six red tiles. Therefore,

Method 2: Flipped to the opposite  Subtraction sign makes the 2 yellow tiles after the operation is flipped to the opposite side of values. Therefore, for this case, 2 yellow tiles after operation becomes 2 red tiles. Meanwhile, the sign is also flipped to the other sides of the operation.  Therefore, $(-4) + (-2) = -6$		
Consolidation The teacher asks students to do exercises on subtraction of integers using algebraic tiles. **Exercises** Find the value of: 1. $6 - 4 =$ 2. $6 - (-4) =$ 3. $-6 - 4 =$ 4. $-6 - (-4) =$ 5. $3 - 7 =$ 6. $3 - (-7) =$ 7. $-3 - 7 =$ 8. $-3 - (-7) =$	• The students do the exercises.	30 mins

Evaluation & Closure		
• Back review with the students on subtraction of integers using algebraic tiles. • Teacher concludes the lesson on the topic that has been learned. 2. How to compute a subtracting of integers questions using algebraic tiles.	• Students pay attention to teacher’s explanation.	5 mins

Subject: Mathematics **Lesson Title:** Integers **Subtopic:** Multiplication of integers
Date: 5/1/18 **Grade:** Form 1 Ibnu Sina **Time:** 8.00 am – 9.00 am
Duration of lesson: 2 periods (1 hour) **Number of students:** 30 students

---

Learning Outcomes: At the end of the lesson, students should be able to:

3. Use and apply the rule of multiplication integers using algebraic tiles.

Pre-requisite Concepts and Skills:

Pre-requisite (5 minutes):

8. Review with the students on addition and subtraction of integers using algebraic tiles.

Material and Resources: Pencil, paper, whiteboard, marker-pen, yellow and red tiles.

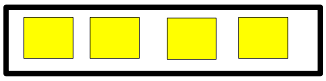
Methodology:

Teacher's Activities	**Students' Activities**	**Time**
Set Induction: • Explain to the students that 2×4 has the same value with 4×2 but it means something different. 2×4 means two groups of four and 4×2 means four groups of two. • Explain to the students that the algebraic tiles allow them to clearly distinguish between 2×-4 and -4×2. • Integer multiplication builds on whole number multiplication. • Explain to the students that the multiplier serves as the "counter" of sets needed. • Explain to the students the use the algebra tiles to model the multiplication. Identify the multiplier or counter. **REMEMBER: Groups must be in a positive manner.**	• Students listen to the teacher's explanation and respond to teacher's questions.	5 mins
Lesson Development: 3. Explain to the students how to multiply positive integers, multiply positive and negative integers together and multiply negative integers with negative integers and	• Students listen to the teacher's explanation and respond to teacher's questions.	15 mins

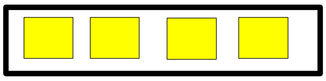
negative integers with positive integers using the jar method.

Example 1: $(+2) \times (+4) =$

Positive Group 1



Positive Group 2



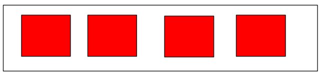
Two groups of four positives.
Therefore,
$(+2) \times (+4) = +8$

The counter indicates how many rows to make. It has this meaning if it is positive.

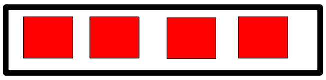
Therefore, $(+2) \times (+4) = +8$

Example 2: $(+2) \times (-4)$

Positive Group 1



Positive Group 2

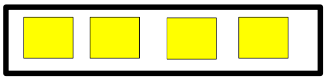


Two groups of four negatives.
Therefore,
$(+2) \times (-4) =$
-8

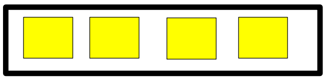
Therefore, $(+2) \times (-4) = -8$

Example 3: $(-2) \times (+4) =$

Negative Group 1



Negative Group 2

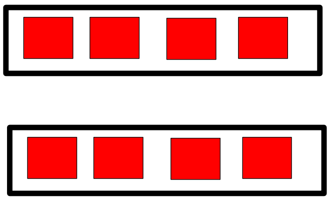


Two negative groups of four positive.

When groups are negative, flipped to the opposite will play the roles. After doing the multiplication, the result must be opposite to the transformation to the other side of value. For this case, two groups are negative. In order to make the groups positive, the tiles must be flipped to the other side. Therefore, the result is as follows:

Positive Group 1

Positive Group 2



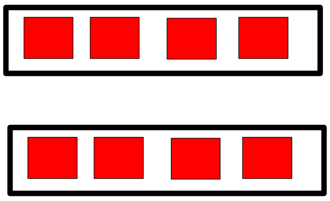
Two groups of four negatives. Therefore, $(+2) \times (-4) = -8$

Therefore, $\mathbf{(+2) \times (-4) = -8}$

Example 4: $\mathbf{(-2) \times (-4) =}$

Negative Group 1

Negative Group 2

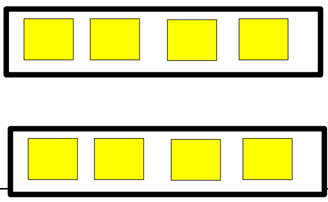


Two negative groups of four negative.

When groups are negative, flipped to the opposite will play the roles. After doing the multiplication, the result must be opposite to the transformation to the other side of value. For this case, two groups are negative. In order to make the groups positive, the tiles must be flipped to the other side. Therefore, the result is as follows:

Positive Group 1

Positive Group 2



Two groups of four positives. Therefore, $(-2) \times (-4) = +8$

Therefore, $(-2) \times (-4) = +8$		
Consolidation The teacher asks students to do exercises on multiplication of integers using algebraic tiles. **<u>Exercises</u>** Find the value of: 1. $4 \times 5 =$ 2. $4 \times (-5) =$ 3. $-4 \times 5 =$ 4. $-4 \times (-5) =$ 5. $3 \times 7 =$ 6. $3 \times (-7) =$ 7. $-3 \times 7 =$ 8. $-3 \times (-7) =$	• The students do the exercises.	30 mins
Evaluation & Closure • Back review back with the students on multiplication of integers using algebraic tiles. • Teacher concludes the lesson on the topic that has been learned: 3. How to compute multiplication of integers questions using algebraic tiles.	• Students pay attention to teacher's explanation.	5 mins

Subject: Mathematics **Lesson Title:** Integers **Subtopic:** Division of integers
Date: 5/1/18 **Grade:** Form 1 Ibnu Sina **Time:** 8.00 am – 9.00 am
Duration of lesson: 2 periods (1 hour) **Number of students:** 30 students

---

Learning Outcomes: At the end of the lesson, students should be able to:

4. Use and apply the rule of division integers using algebraic tiles.

Pre-requisite Concepts and Skills:

Pre-requisite (5 minutes):

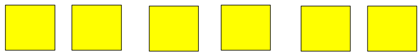
9. Review with the students on addition, subtraction and multiplication of integers using algebraic tiles.

Material and Resources: Pencil, paper, whiteboard, marker-pen, yellow and red tiles.

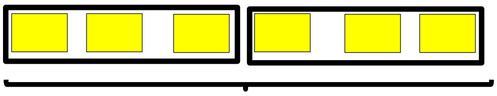
Methodology:

Teacher's Activities	Students' Activities	Time
Set Induction: • Explain to the students that the algebraic tiles can also be used to illustrate the division of integers. However, there are a few restrictions in integer division. If you represent the division by$(\frac{dividend}{divisor} = quotient)$, then the divisor should not be zero and the quotient should be an integer. • Explain to the students that like multiplication, division relies on the concept of a counter. • Tell students that divisor serves as counter since it indicates the number of rows to create. • Identify the divisor or counter.	• Students listen to the teacher's explanation and respond to teacher's questions.	5 mins
Lesson Development: 4. Explain to the students how to divide positive integers, divide positive and negative integers together and divide negative integers with negative integers and negative integers with positive integers using the jar method.	• Students listen to the teacher's explanation and respond to teacher's questions.	15 mins

Example 1: $\left.{(+\mathbf{6})}\middle/{(+\mathbf{2})}\right. =$



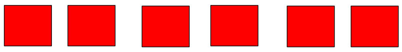
There are six yellow tiles. This case indicates six positive members that need to be divided into two positive groups. Therefore, the result will be drawn as:



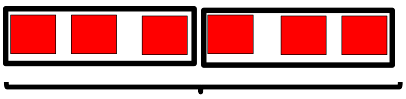
The result shows 3 members of positive tiles in two positive groups. Therefore,

$$\left.{(+6)}\middle/{(+2)}\right. = +3$$

Example 2: $\left.{(-\mathbf{6})}\middle/{(+\mathbf{2})}\right. =$



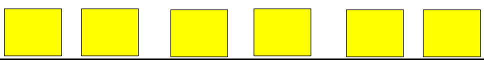
There are six red tiles. This case indicates six negative members that need to be divided into two positive groups. Therefore, the result will be drawn as:



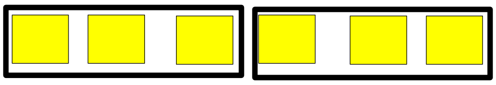
The result shows 3 members of negative tiles in two positive groups. Therefore,

$$\left.{(-6)}\middle/{(+2)}\right. = -3$$

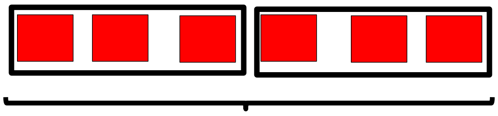
Example 2: $\left.{(+\mathbf{6})}\middle/{(-\mathbf{2})}\right. =$



There are six yellow tiles. This case indicates six positive members that need to be divided into two negative groups. Therefore, the result will be drawn as:



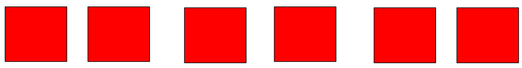
However, the result will be different to the case of $\mathbf{(+6)}/\mathbf{(+2)} =$ since the groups are no longer positive. Therefore, flipped to the opposite must be implied to ensure that the groups will become positive. Hence,



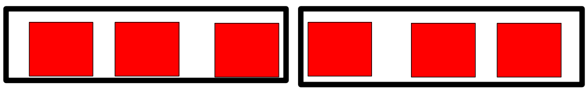
The result shows 3 members of negative tiles in two negative groups. Therefore,

$$(+6)/(-2) = -3$$

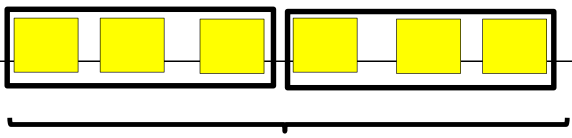
Example 4: $\mathbf{(-6)}/\mathbf{(-2)} =$



There are six red tiles. This case indicates six negative members that need to be divided into two negative groups. Therefore, the result will be drawn as:



However, the result will be different to the case of $\mathbf{(-6)}/\mathbf{(+2)} =$ since the groups are no longer positive. Therefore, flipped to the opposite must be implied to ensure that the groups will become positive. Hence,



The result shows 3 members of positive tiles in two negative groups. Therefore,

$$(-6)/(-2) = +3$$

Consolidation The teacher asks students to do exercises on division of integers using the jar method. **<u>Exercises</u>** Find the value of: 1. $\mathbf{6/3} =$ 2. $\mathbf{6/(-3)} =$ 3. $\mathbf{-6/3} =$ 4. $\mathbf{-6/(-3)} =$ 5. $\mathbf{8/2} =$ 6. $\mathbf{8/-2} =$ 7. $\mathbf{-8/2} =$ 8. $\mathbf{-8/-2} =$	• The students do the exercises.	30 mins
Evaluation & Closure • Back review with the students on division of integers using the jar method. • Teacher concludes the lesson on the topic that has been learned: 4. How to compute division of integers questions using jar method.	• Students pay attention to teacher's explanation.	5 mins

APPENDIX E

OBSERVATION CHECKLIST

Teacher ____________________________ Date __________ Prd/Class ________

ENVIRONMENT

Layout

• flexible, moveable	
• attractive, inviting, clean	
• safe and orderly	
• other	

Visual

• graphic organizers	
• flowcharts	
• samples of proficient work	
• essential questions posted	
• other	

Student work displayed

• current	
• varied	
• respects confidentiality	
• other	

Print rich environment

• variety of books	
• content specific print	
• other print media	
• books & media are current	
• multi-cultural materials	
• other	

Variety of instructional materials

• manipulatives	
• models	
• audio, video tapes	
• computer	
• other	

Classroom management

• rules/procedures posted	
• evidence of daily procedures	
• reinforces rules/procedures	
• other	

STUDENT ENGAGEMENT

Active engagement

• discussions	
• students on task	
• minimum of teacher lecture	
• student movement	
• manipulatives	
• directed by teacher	
• interest/excitement	
• other	

Student talk

• student initiated	
• balance of teacher/student talk	
• student/student talk	
• other	

Positive reinforcement

• genuine praise	
• respect for student	
• high expectations	
• other	

Student grouping

• whole class	
• groups of 4 or more	
• duo/trio	
• individual	
• other	

Group activity

• discussion	
• problem-solving	
• peer editing	
• study groups	
• writing/sharing	
• other	

Comments:

VARIED INSTRUCTIONAL STRATEGIES

Teacher activity

• lecture	
• discussion leader	
• modeling	
• monitoring/adjusting	
• formal assessment	
• informal assessment	
• other	

Authentic problems & questions

• problem solving activities	
• reflect core content/curriculum guide	
• real life connections	
• student self-assessment	
• experimental/hand-on learning	
• other	

Instruction/Orientation

• direct instruction	
• independent work	
• cooperative learning	
• other	

Choice

• teacher-initiated	
• student-initiated	
• other	

Learning Strategies

• addressing MI (verbal, logical-mathematical spatial, kinesthetic, musical, interpersonal, intrapersonal)	
• use of Marzano Strategies (Identifying Similarities & Differences; Summarizing & Note Taking; Reinforcing Effort & Providing Recognition; Homework & Practice; Nonlinguistic Representations; Cooperative Learning; Setting Goals & Providing Feedback; Generating & Testing Hypothesis; Activating Prior Knowledge; and Teaching Specific Types of Knowledge)	
• project-based learning	
• higher level questioning strategies	
• teacher acting as coach/facilitator	
• independent inquiry/research	
• sustained writing/reading	
• other	

APPENDIX F

PERMISSION FROM THE MINISTRY OF EDUCATION

KEMENTERIAN PENDIDIKAN MALAYSIA
MINISTRY OF EDUCATION MALAYSIA
BAHAGIAN PERANCANGAN DAN PENYELIDIKAN DASAR PENDIDIKAN
EDUCATIONAL PLANNING AND RESEARCH DIVISION
ARAS 1-4, BLOK E8
KOMPLEKS KERAJAAN PARCEL E
PUSAT PENTADBIRAN KERAJAAN PERSEKUTUAN
62604 PUTRAJAYA

KEMENTERIAN PENDIDIKAN MALAYSIA

Telefon : 03-8884 6500
Faks : 03-8884 6439
Laman Web : www.moe.gov.my

Ruj. Kami : KPM.600-3/2/3 Jld 34 (94)
Tarikh : 10 Januari 2017

Zulmaryan binti Embong
K.P.:871215115248

Lot 8094 Kampung Pulau Kudat
21700 Kuala Berang
Terengganu

Tuan,

KELULUSAN UNTUK MENJALANKAN KAJIAN DI SEKOLAH, INSTITUT PENDIDIKAN GURU, JABATAN PENDIDIKAN NEGERI DAN BAHAGIAN DI BAWAH KEMENTERIAN PENDIDIKAN MALAYSIA

Perkara di atas adalah dirujuk.

2. Sukacita dimaklumkan bahawa permohonan tuan untuk menjalankan kajian seperti di bawah telah diluluskan.

"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method"

3. Kelulusan ini adalah berdasarkan kepada kertas cadangan penyelidikan dan instrumen kajian yang dikemukakan oleh tuan kepada Bahagian ini. Walau bagaimanapun kelulusan ini bergantung kepada kebenaran Jabatan Pendidikan Negeri dan Pengetua / Guru Besar yang berkenaan.

4. Surat kelulusan ini sah digunakan bermula dari **10 Januari 2017 hingga 30 Jun 2017.**

5. Tuan juga mesti menyerahkan senaskhah laporan akhir kajian dalam bentuk *hardcopy* bersama salinan *softcopy* berformat Pdf di dalam CD kepada Bahagian ini. Tuan diingatkan supaya mendapat kebenaran terlebih dahulu daripada Bahagian ini sekiranya sebahagian atau sepenuhnya dapatan kajian tersebut hendak dibentangkan di mana-mana forum, seminar atau diumumkan kepada media massa.

Sekian untuk makluman dan tindakan tuan selanjutnya. Terima kasih.

"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(DR SHAMSUDIN BIN MOHAMAD)
Ketua Unit
Sektor Penyelidikan dan Penilaian
b.p. Pengarah
Bahagian Perancangan dan Penyelidikan Dasar PendidikanKementerian Pendidikan Malaysia

QUALITY SYSTEM
SIRIM
STANDARDS MALAYSIA
CERTIFIED TO ISO 9001:2008
CERT. NO: AR 3166

s.k

1. Pengarah
   Jabatan Pendidikan Negeri Terengganu

2. Pengarah
   Jabatan Pendidikan Negeri Kedah

3. Pengarah
   Jabatan Pendidikan Negeri Selangor

4. Pengarah
   Jabatan Pendidikan Negeri Johor

5. Pengarah
   Jabatan Pendidikan Wilayah Persekutuan Kuala Lumpur

KEMENTERIAN PENDIDIKAN MALAYSIA
MINISTRY OF EDUCATION MALAYSIA
BAHAGIAN PERANCANGAN DAN PENYELIDIKAN DASAR PENDIDIKAN
EDUCATIONAL PLANNING AND RESEARCH DIVISION
ARAS 1-4, BLOK E8
KOMPLEKS KERAJAAN PARCEL E
PUSAT PENTADBIRAN KERAJAAN PERSEKUTUAN
62604 PUTRAJAYA

KEMENTERIAN PENDIDIKAN MALAYSIA

Telefon : 03-8884 6500
Faks : 03-8884 6439
Laman Web : www.moe.gov.my

Ruj. Kami : KPM.600-3/2/3 Jld 34 (94)

Tarikh : 10 Oktober 2017

Zulmaryan binti Embong
K.P.:871215115248

Lot 8094 Kampung Pulau Kudat
21700 Kuala Berang
Terengganu

Tuan,

KELULUSAN UNTUK MENJALANKAN KAJIAN DI SEKOLAH, INSTITUT PENDIDIKAN GURU, JABATAN PENDIDIKAN NEGERI DAN BAHAGIAN DI BAWAH KEMENTERIAN PENDIDIKAN MALAYSIA

Perkara di atas adalah dirujuk.

2. Sukacita dimaklumkan bahawa permohonan tuan untuk menjalankan kajian seperti di bawah telah diluluskan.

"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method"

3. Kelulusan ini adalah berdasarkan kepada kertas cadangan penyelidikan dan instrumen kajian yang dikemukakan oleh tuan kepada Bahagian ini. Walau bagaimanapun kelulusan ini bergantung kepada kebenaran Jabatan Pendidikan Negeri dan Pengetua / Guru Besar yang berkenaan.

4. Surat kelulusan ini sah digunakan bermula dari **10 Oktober 2017 hingga 30 April 2018.**

5. Tuan juga mesti menyerahkan senaskhah laporan akhir kajian dalam bentuk *hardcopy* bersama salinan *softcopy* berformat Pdf di dalam CD kepada Bahagian ini. Tuan diingatkan supaya mendapat kebenaran terlebih dahulu daripada Bahagian ini sekiranya sebahagian atau sepenuhnya dapatan kajian tersebut hendak dibentangkan di mana-mana forum, seminar atau diumumkan kepada media massa.

Sekian untuk makluman dan tindakan tuan selanjutnya. Terima kasih.

"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(Dr. ROSLI BIN ISMAIL)
Ketua Sektor
Sektor Penyelidikan dan Penilaian
b.p. Pengarah
Bahagian Perancangan dan Penyelidikan Dasar Pendidikan
Kementerian Pendidikan Malaysia

APPENDIX G

PERMISSION FROM THE STATE EDUCATION DEPARTMENT

جابتن ڤنديديقن نڬري ترڠڬانو

JABATAN PENDIDIKAN NEGERI TERENGGANU
Jalan Bukit Kecil,
20604 Kuala Terengganu,
Terengganu Darul Iman.

Tel. : 09- 621 3000/3001
Faks. (Fax) : 09- 622 7207/09- 623 8415
Laman Web : jpnterengganu.moe.gov.my

Ruj. Kami : P.T.06030-30(86)
Tarikh : 2 Februari 2017

Zulmaryan binti Embong,
Lot 8094 Kampung Pulau Kudat,
21700 Kuala Berang, Hulu Terengganu,
Terengganu.

Tuan,

KELULUSAN UNTUK MENJALANKAN KAJIAN DI SEKOLAH-SEKOLAH DI BAWAH JABATAN PENDIDIKAN NEGERI TERENGGANU

Dengan hormatnya dimaklumkan bahawa surat tuan adalah dirujuk.

2. Untuk makluman, permohonan tuan untuk menjalankan kajian bertajuk **Analysing Students' Error and Improving Students' Performance in Operations on Integers Using Jar Method** adalah diluluskan.

3. Surat kelulusan ini sah digunakan bermula dari **10 Januari 2017 hingga 30 Jun 2017**.

4. Sehubungan dengan itu, tuan diminta mengemukakan ke jabatan ini senaskah hasil kajian tersebut dalam bentuk elektronik berformat *Pdf* di dalam *CD* setelah selesai kajian kelak. Tuan juga perlu memohon kebenaran serta mengadakan perbincangan dengan pihak pengurusan sekolah yang terlibat agar proses pengajaran dan pembelajaran tidak terganggu.

Sekian, terima kasih.

"Memperkasakan Transformasi Pendidikan"
"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(HAMID BIN YAHYA)
Penolong Pendaftar Institusi Pendidikan,
Jabatan Pendidikan Negeri Terengganu,
b.p. Ketua Pendaftar Institusi Pendidikan,
Kementerian Pendidikan Malaysia.

s.k.

Pengarah,
Bahagian Perancangan dan Penyelidikan Dasar Pendidikan,
Kementerian Pendidikan Malaysia.

Pengarah Pendidikan Negeri Terengganu

(Sila catatkan rujukan Jabatan ini apabila berhubung)

CERTIFIED TO ISO 9001:2008
CERT. NO.: AR 4166

CERTIFIED TO ISO 9001:2008
CERT. NO.: AR 4166

جابتن ڤنديديقن نݢري ترڠݢانو

JABATAN PENDIDIKAN NEGERI TERENGGANU
Jalan Bukit Kecil,
20604 Kuala Terengganu,
Terengganu Darul Iman.

Tel. : 09-621 3000/3001
Faks (Fax) : 09-622 7207/09-623 8415
Laman Web : jpnterengganu.moe.gov.my

Ruj. Kami : P.T.06030-33(4)
Tarikh : 22 November 2017

Zulmaryan binti Embong,
NO. E211, Jalan SBC 12,
Batu Caves,
Taman Sri Batu Caves,
68100 Selangor.

Tuan,

KELULUSAN UNTUK MENJALANKAN KAJIAN DI SEKOLAH-SEKOLAH DI BAWAH JABATAN PENDIDIKAN NEGERI TERENGGANU

Dengan hormatnya dimaklumkan bahawa surat tuan adalah dirujuk.

2. Untuk makluman, permohonan tuan untuk menjalankan kajian bertajuk **"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Methord"** adalah diluluskan.

3. Surat kelulusan ini sah digunakan bermula dari **10 Oktober 2017 hingga 30 April 2018**.

4. Sehubungan dengan itu, tuan diminta mengemukakan ke jabatan ini senaskah hasil kajian tersebut dalam bentuk elektronik berformat *Pdf* di dalam *CD* setelah selesai kajian kelak. Tuan juga perlu memohon kebenaran serta mengadakan perbincangan dengan pihak pengurusan sekolah yang terlibat agar proses pengajaran dan pembelajaran tidak terganggu.

Sekian, terima kasih.

"Memperkasakan Transformasi Pendidikan"
"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(HAMID BIN YAHYA)
Penolong Pendaftar Institusi Pendidikan,
Jabatan Pendidikan Negeri Terengganu,
b.p. Ketua Pendaftar Institusi Pendidikan,
Kementerian Pendidikan Malaysia.

s.k.

Pengarah,
Bahagian Perancangan dan Penyelidikan Dasar Pendidikan,

Pengarah Pendidikan Negeri Terengganu

MZA/surat lulus kajian/nor

(Sila catatkan rujukan Jabatan ini apabila berhubung)

CERTIFIED TO ISO 9001:2008 CERTIFIED TO ISO 9001:2008

Visit Beautiful Terengganu Malaysia 2017

JABATAN PENDIDIKAN SELANGOR
Jalan Jambu Bol 4/3E, Seksyen 4,
40604 SHAH ALAM
SELANGOR DARUL EHSAN,
MALAYSIA

Tel : 03 - 5518 6500
Faks : 03 - 5510 2133
Laman Web : http//jpnselangor.moe.gov.my

Rujukan Kami : JPNS.PPN 600-1/56 JLD.67(44)
Tarikh : 13/02/2017

ZULMARYAN BINTI EMBONG
NO.6, LOT 3727
JALAN TEMBUSU, KAMPUNG MELAYU SUBANG
40150 SHAH ALAM
SELANGOR

Tuan,

ANALYSING STUDENTS ERRORS AND IMPROVING STUDENTS PERFORMANCE IN OPERATIONS OF INTEGERS USING JAR METHOD

Perkara di atas dengan segala hormatnya dirujuk.

2. Jabatan ini tiada halangan untuk pihak tuan menjalankan kajian/penyelidikan tersebut di sekolah-sekolah dalam Negeri Selangor seperti yang dinyatakan dalam surat permohonan.

3. Pihak tuan diingatkan agar mendapat persetujuan daripada Pengetua/Guru Besar supaya beliau dapat bekerjasama dan seterusnya memastikan bahawa penyelidikan dijalankan hanya bertujuan seperti yang dipohon. Kajian/Penyelidikan yang dijalankan juga tidak mengganggu perjalanan sekolah serta tiada sebarang unsur paksaan.

4. **Surat kelulusan ini sah digunakan bermula dari 10 Januari 2017 hingga 30 Jun 2017.**

5. Tuan juga diminta menghantar senaskah hasil kajian ke Unit Perhubungan dan Pendaftaran Jabatan Pendidikan Selangor sebaik selesai penyelidikan/kajian.

Sekian, terima kasih.

"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(NOR FARIDAH BINTI A. BAKAR)
Penolong Pendaftar Institusi Pendidikan dan Guru
Jabatan Pendidikan Selangor
b.p. Ketua Pendaftar Institusi Pendidikan dan Guru
Kementerian Pendidikan Malaysia

s.k: - Fail

" Jabatan Pendidikan Selangor Terbilang "

JABATAN PENDIDIKAN SELANGOR
Jalan Jambu Bol 4/3E, Seksyen 4,
40604 SHAH ALAM
SELANGOR DARUL EHSAN,
MALAYSIA

Tel : 03 - 5518 6500
Faks : 03 - 5510 2133
Laman Web : http//jpnselangor.moe.gov.my

Rujukan Kami : JPNS.PPN 600-1/49 JLD.75(46)
Tarikh : 23/11/2017

ZULMARYAN BINTI EMBONG
LOT 8094 KAMPUNG PULAU KUDAT
21700 KUALA BERANG
SELANGOR

Tuan,

" ANALYSING STUDENTS' ERRORS AND IMPROVING STUDENTS' PERFORMANCE IN OPERATIONS OF INTEGERS USING JAR METHOD "

Perkara di atas dengan segala hormatnya dirujuk.

2. Jabatan ini tiada halangan untuk pihak tuan menjalankan kajian/penyelidikan tersebut di sekolah-sekolah dalam Negeri Selangor seperti yang dinyatakan dalam surat permohonan.

3. Pihak tuan diingatkan agar mendapat persetujuan daripada Pengetua/Guru Besar supaya beliau dapat bekerjasama dan seterusnya memastikan bahawa penyelidikan dijalankan hanya bertujuan seperti yang dipohon. Kajian/Penyelidikan yang dijalankan juga tidak mengganggu perjalanan sekolah serta tiada sebarang unsur paksaan.

4. **Surat kelulusan ini sah digunakan bermula dari 10 Oktober 2017 hingga 30 April 2018.**

5. Tuan juga diminta menghantar senaskah hasil kajian ke Unit Perhubungan dan Pendaftaran Jabatan Pendidikan Selangor sebaik selesai penyelidikan/kajian.

Sekian, terima kasih.

"BERKHIDMAT UNTUK NEGARA"

Saya yang menurut perintah,

(NOR FARIDAH BINTI A.BAKAR)
Penolong Pendaftar Institusi Pendidikan dan Guru
Jabatan Pendidikan Selangor
b.p. Ketua Pendaftar Institusi Pendidikan dan Guru
Kementerian Pendidikan Malaysia

s.k: - Fail

" Jabatan Pendidikan Selangor Terbilang "

JABATAN PENDIDIKAN NEGERI KEDAH
KOMPLEKS PENDIDIKAN, JALAN STADIUM
05604 ALOR SETAR
KEDAH DARUL AMAN

Telefon : 04-740 4000
Faks : 04-740 4342
Laman Web : www.jpn.moe.gov.my/jpnkedah

"MUAFAKAT KEDAH"

Ruj Kami : JPK. SPS.UPP 600-1/1/2(94)
Tarikh : 12 Februari 2017

Zulmaryan binti Embong
Lot 8094, Kampung Pulau Kudat
21700 Kuala Berang
Terengganu

Tuan,

Kebenaran Untuk Menjalankan Kajian/ Soal Selidik di Jabatan Pendidikan Negeri / Pejabat Pendidikan Daerah dan Sekolah – Sekolah di Negeri Kedah Darul Aman

Saya dengan hormatnya diarah merujuk kepada perkara tersebut di atas.

2. Dimaklumkan bahawa permohonan tuan/puan untuk menjalankan kajian yang bertajuk **" Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method "** telah **diluluskan**.

3. Kelulusan ini adalah berdasarkan kepada apa yang terkandung di dalam cadangan penyelidikan yang tuan/puan kemukakan ke Kementerian Pendidikan Malaysia.Tuan/Puan dikehendaki mengemukakan senaskah laporan akhir kajian setelah selesai kelak dan diingatkan supaya mendapat kebenaran terlebih dahulu daripada Jabatan ini sekiranya sebahagian atau sepenuhnya dapatan kajian tersebut hendak dibentangkan di mana-mana forum, seminar atau diumumkan kepada media.

4. Kebenaran ini adalah tertakluk kepada persetujuan Pengetua sekolah berkenaan dan adalah **sah bermula dari 10 Januari 2016 hingga 30 Jun 2017.**

Sekian, terima kasih.

" BERKHIDMAT UNTUK NEGARA "
" MUAFAKAT KEDAH "
" PENDIDIKAN CEMERLANG KEDAH TERBILANG "

Saya yang menurut perintah,

(ABDULLAH BIN ABDULL MANAF)
Penolong Pengarah Kanan (Ketua Unit)
Unit Perhubungan dan Pendaftaran
Sektor Pengurusan Sekolah
b.p. Pengarah Pendidikan Negeri Kedah Darul Aman

"1 Malaysia: Rakyat Didahulukan, Pencapaian Diutamakan"
Sila catatkan rujukan Jabatan ini apabila berhubung

"MUAFAKAT KEDAH"

Ruj Kami : JPK. SPS.UPP 600-1/1/2 Jld 4 ( )
Tarikh : 16 November 2017

Zulmaryan binti Embong
Lot 8094 Kampung Pulau Kudat
21700 Kuala Berang
Terengganu

Tuan,

Kebenaran Untuk Menjalankan Kajian/ Soal Selidik di Jabatan Pendidikan Negeri / Pejabat Pendidikan Daerah dan Sekolah – Sekolah di Negeri Kedah Darul Aman

Saya dengan hormatnya diarah merujuk kepada perkara tersebut di atas.

2. Dimaklumkan bahawa permohonan tuan untuk menjalankan kajian yang bertajuk **"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method "** telah **diluluskan**.

3. Kelulusan ini adalah berdasarkan kepada apa yang terkandung di dalam cadangan penyelidikan yang tuan kemukakan ke Kementerian Pendidikan Malaysia. Tuan dikehendaki mengemukakan senaskhah laporan akhir kajian setelah selesai kelak dan diingatkan supaya mendapat kebenaran terlebih dahulu daripada Jabatan ini sekiranya sebahagian atau sepenuhnya dapatan kajian tersebut hendak dibentangkan di mana-mana forum, seminar atau diumumkan kepada media.

4. Kebenaran ini adalah tertakluk kepada persetujuan Pengetua sekolah berkenaan dan adalah **sah bermula dari 10 Oktober 2017 hingga 30 April 2018.**

Sekian, terima kasih.

" BERKHIDMAT UNTUK NEGARA "
" MUAFAKAT KEDAH "
" PENDIDIKAN CEMERLANG KEDAH TERBILANG "

Saya yang menurut perintah,

(ABDULLAH BIN ABDULL MANAF, BCK)
Penolong Pengarah Kanan (Ketua Unit)
Unit Perhubungan dan Pendaftaran
Sektor Pengurusan Sekolah
b.p. Pengarah Pendidikan Negeri Kedah Darul Aman

JABATAN PENDIDIKAN JOHOR
XXXXXXXXXXXXXXXXXXXXXXXX
JALAN TUN ABDUL RAZAK
80604 JOHOR BAHRU
JOHOR DARUL TA'ZIM

Telefon : 07-2310000
Faks : 07-2347132
Laman web : www.moe.gov.my/jpnjohor
Email : jpn.johor@moe.gov.my

UNIT PERHUBUNGAN DAN PENDAFTARAN
NO. TEL : 07-2311470 NO. FAKS : 07-2380539

Rujukan Kami : JPNJ.PP(31)/100-5/3/2/Jld.9(27)
Tarikh : 15 Februari 2017

Zulmaryan binti Embong
Lot 8094 Kampung Pulau Kudat
21700 Kuala berang
Terengganu

Tuan,

Kebenaran Untuk Menjalankan Kajian Di Sekolah-Sekolah, Institut Perguruan, Jabatan Pendidikan Negeri Dan Bahagian-Bahagian Di Bawah Kementerian Pendidikan Malaysia.

Dengan hormatnya surat daripada KPM Bil: KPM.600-3/2/3Jld.34(94) bertarikh 10.1.2017 berkaitan permohonan adalah dirujuk.

2. Sukacita dimaklumkan bahawa Jabatan ini tiada apa-apa halangan bagi membenarkan tuan menjalankan kajian ke sekolah-sekolah **Kerajaan dan Swasta Negeri Johor** bertajuk:

"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method"

3. Sehubungan dengan itu, tuan boleh berhubung terus dengan Pejabat Pendidikan Daerah berkenaan bagi mendapatkan maklumat dan tindakan selanjutnya.

4. Surat kelulusan ini sah digunakan bermula dari **10 Januari 2017 hingga 30 Jun 2017**

5. Sila bawa surat ini semasa membuat kajian dan tuan/puan hendaklah kemukakan ke jabatan ini senaskah laporan akhir kajian setelah selesai kelak.

Sekian, terima kasih.

"SEHATI SEJIWA "
"BERKHIDMAT UNTUK NEGARA "

Saya yang menurut perintah,

(AZHARI BIN SAMSURI)
Penolong Pendaftar Institusi Pendidikan dan Guru
Jabatan Pendidikan Negeri Johor
b.p. Ketua Pendaftar Institusi Pendidikan dan Guru
Kementerian Pendidikan Malaysia

azh ya

CERTIFIED TO ISO 9001 : 2008
CERT. NO. : AR4166

CERTIFIED TO ISO 9001 : 2008
CERT. NO. : AR4166

KEMENTERIAN PENDIDIKAN MALAYSIA

JALAN TUN ABDUL RAZAK,
80604, JOHOR BAHRU
JOHOR DARUL TA'ZIM
e-mel : jpn.johor@moe.gov.my

Telefon : 07-231 0000
Pengarah : 07-231 0198
Timbalan : 07-231 0199
Faks : 07-2384691
Laman web : http://jpn.moe.gov.my/jpnjohor

UNIT PERHUBUNGAN DAN PENDAFTARAN
NO. TEL : 07-2311474 NO. FAKS : 07-2380539

Rujukan Kami : JPNJ.PP(31)/100-5/3/2/Jld.12(43)
Tarikh : 19 November 2017

Zulmaryam binti Embong
No.E211, Jalan SBC 12
Taman Sri Batu Caves
68100 Batu Caves
Selangor

Tuan,

Kebenaran untuk Menjalankan Kajian di Sekolah-Sekolah, Institut Perguruan, Jabatan Pendidikan Negeri dan Bahagian-Bahagian di Bawah Kementerian Pendidikan Malaysia.

Dengan hormatnya surat daripada KPM Bil: KPM.600-3/2/3Jld34(94) bertarikh 10.10.2017 berkaitan permohonan adalah dirujuk.

2. Sukacita dimaklumkan bahawa jabatan ini tiada halangan bagi membenarkan tuan menjalankan kajian ke sekolah-sekolah **Kerajaan dan Swasta Negeri Johor** bertajuk:

"Analysing Students' Errors and Improving Students' Performance in Operations of Integers Using JAR Method"

3. Sehubungan dengan itu, tuan boleh berhubung terus dengan Pejabat Pendidikan Daerah berkenaan bagi mendapatkan maklumat dan tindakan selanjutnya.

4. Surat kelulusan ini sah digunakan bermula dari **10 Oktober 2017 hingga 30 April 2018**

5. Sila bawa surat ini semasa membuat kajian dan tuan/puan hendaklah kemukakan kepada jabatan ini senaskhah laporan akhir kajian setelah selesai kelak.

Sekian, terima kasih.

"SEHATI SEJIWA "
"BERKHIDMAT UNTUK NEGARA "

Saya yang menurut perintah,

(MOHD HASSIM BIN SUDIMAN)
Penolong Pendaftar Institusi Pendidikan dan Guru
Jabatan Pendidikan Negeri Johor
b.p. Ketua Pendaftar Institusi Pendidikan dan Guru
Kementerian Pendidikan Malaysia

kajian/azh-ys/17

CERTIFICATE TO ISO 9001 : 2008
CERT. NO. : AR4166

CERTIFICATE TO ISO 9001 : 2008
CERT. NO. : AR4166

APPENDIX H

RELIABILITY RESULT FOR PILOT STUDY

```
-------------------------------------------------------------------------------
| PERSON      65 INPUT      62 MEASURED              INFIT          OUTFIT    |
|          TOTAL      COUNT      MEASURE  REALSE     IMNSQ   ZSTD  OMNSQ   ZSTD|
| MEAN      29.9       39.8         1.75     .53        .97     -.1   1.07    .1|
| S.D.       5.6        1.5         1.23     .21        .30     1.2    .80   1.1|
| REAL RMSE    .57 TRUE SD    1.09  SEPARATION  1.92  PERSON RELIABILITY  .79|
|-----------------------------------------------------------------------------|
| ITEM      40 INPUT      40 MEASURED                INFIT          OUTFIT    |
|          TOTAL      COUNT      MEASURE  REALSE     IMNSQ   ZSTD  OMNSQ   ZSTD|
| MEAN      46.3       61.7          .00     .42       1.00      .0   1.06    .0|
| S.D.      12.4         .5         1.49     .15        .12      .7    .56   1.0|
| REAL RMSE    .45 TRUE SD    1.41  SEPARATION  3.13  ITEM   RELIABILITY  .91|
-------------------------------------------------------------------------------
```

APPENDIX I

VALIDITY RESULT FOR PILOT STUDY

```
 PERSON - MAP - ITEM
       <more>|<rare>
 4         X  +
           X T|
              |
              |
           X  |  Q38
   XXXXXXX    |
            | |  Q35
              |  Q40
 3        XX  +T
             S|
          XX  |
              |
     XXXXXX   |
              |
      XXXXX   |
 2            +
      XXXXX   |
        XXX   |  Q13
             M|
          XX  |S Q14
      XXXXX   |
              |  Q12     Q16
          XX  |  Q15
 1       XXX  +  Q21     Q33
         XXX  |  Q20     Q25
              |  Q32
          XX S|  Q3      Q37     Q8
          XX  |  Q2
          XX  |  Q7
          XX  |  Q4
           X  |  Q5
 0        XX  +M Q6
          XX  |  Q39
              |
           X  |  Q28
             T|  Q30
              |  Q27
              |  Q10     Q31
              |
-1            +  Q19
              |
              |  Q26     Q36
              |
              |S Q1      Q24     Q34     Q9
              |
```

APPENDIX J

USED OBSERVATION CHECKLIST

Teacher	Collaborative Learning	Active Learning	Creative and Critical Thinking	Multiple Representation
1	• Pair • Whole class • Discussion	• Monitoring • Students self-assessment • Informal assessment	• Reinforce rules/procedure • Problem solving • Real life connection	• Yes (number line + analogy) • Direct instruction
2	• Pair • Whole class • Teacher-centred	• Monitoring • Students self-assessment	• Reinforce rules/procedure • Problem solving	• No (only number line) • Direct instruction • lecture
3	• Pair • Whole class • Teacher-centred	• Monitoring • Students self-assessment	• Reinforce rules/procedure • Problem solving • Manipulatives	• Yes (number line + analogy) • Direct instruction
4	• Groups of 4 • Whole class • Discussion •	• Discussion leader • Monitoring • Students self-assessment • Teacher acting as facilitator	• Reinforce rules/procedure • Problem solving	• No (only number line) • lecture
5	• Whole class • Teacher-centred	• Informal assessment	• Reinforce rules/procedure • Problem solving activities	• Yes (number line + analogy) • Lecture only • Direct instruction
6	• Individual • Pair • Whole class • Teacher-centred • Student-student talk	• Monitoring • Respect, praise students	• Reinforce rules/procedure • Real life connection (lift, boat)	• Yes (number line + analogy) • Lecture
7	• Pair • Whole class • Student-student talk • Discussion • Direct instruction	• Student movement • Monitoring • Informal assessment	• Reinforce rules/procedure • Real life connection	• No (only number line) • Lecture •
8	• Groups of 4 • Whole class • Teacher-centred • Student-student talk	• Monitoring • Discussion leader • Respect, praise students	• Reinforce rules/procedure • Real life connection	• No (only number line) • lecture

APPENDIX K

CLASSROOM OBSERVATION REPORT

Example 1: Classroom Observation Report

Teacher's Name: Cikgu X

Room: 1 Cekal

Date/Time of Observation: 8 January 2018

Syllabus: Integers (Subtraction)

Cikgu X has been a Discipline Teacher for five years. He has been teaching mathematics for 13 years. This is the third time he has taught 1 Cekal on integers. Previously, he introduced the concept of integers and the addition operation of integers. Today, he continues the lesson on the subtraction operation of integers using the number line approach. He added more discussions regarding the subtraction operation with the students than he previously did.

On this day, 40 students were present. 18 students were male and 22 were female. Class began at 10.30 and ended at 11.30 after the school recess time. The class began with a lecture reviewing how to use a number line method to solve the addition of integers. Then, the teacher continued the lecture by talking about subtraction using a number line method. After 20 minutes of lecture, he transitioned into solving subtraction questions. He wrote three questions on the whiteboard. The students were divided into pairs and tasked to work on the number line rules and procedures to solve the problems. Later, the teacher and students answered the questions together.

In my opinion, Cikgu X demonstrated that he met all of the following standards for excellence in teaching the operations of integers. Firstly, he was thorough and current with respect to command of the subject matter as he was most familiar with the topic. In addition, the teaching techniques and methodologies were excellent. He managed to explain the number line method in a good manner. Secondly, Cikgu X has the ability to organise course materials and to communicate the number line method effectively. His teaching also entails respect for students, effective response to student questions, and the timely evaluation of and feedback to students. Thirdly, he has the capacity to relate the lesson on integers to other fields of knowledge such as the concept of the thermometer, lift and river.

Although Cikgu X has a very good way to explain integers, not every student was able to grasp the method. For those with basic knowledge on integers, they were able to get the ideas within all these steps and procedures. However, those who have difficulties to understand abstract ideas may not be able to understand the whole lesson. In addition, he did not give students the space to do their own thinking and creating, and he did not promote creative and critical thinking in his lesson. The students were merely required to follow the rules and procedures to get the answers. For him, as long as the students get the answers right, then that is a sign that they understood the lesson. In addition, Ciksu X did not challenge students with difficult questions while helping them to understand the concept in a bigger picture. He only focused on the textbook exercise and did not provide enough amount of time for discussion. He simply asked students for the answers by addressing the whole class without checking on the steps in which students obtained the answers.

As a conclusion, Cikgu X had to deal with environmental challenges: a hot-packed classroom with 40 students, students who were still adapting to secondary school life, and class after recess time where the students just had their food. He dealt with these calmly and with a sense of humour that was also apparent at some of the times. However, there are a few things Cikgu X might have done differently such as calling on the students who spoke less during the discussion to solve the problems in front of the class. This is to minimise students from feeling left out from the discussion.

APPENDIX L

TEST ON THE FOUR OPERATIONS OF INTEGERS
(RECOMMENDED VERSION)

Name/*Nama*: ………………………… School/*Sekolah*: …….........................

Class/*Kelas*: ……………….................. UPSR Maths Grade/*Gred Matematik UPSR*: …..... Date/*Tarikh*: ………………………….

Answer **ALL** questions. You are given 30 minutes to answer the questions.
*Sila jawab **SEMUA** soalan. Anda diberi masa 30 minit untuk menjawab kesemua soalan.*

There are altogether 30 questions. **<u>DO NOT</u>** leave out any question.
*Kertas ini mengandungi 30 soalan. **JANGAN** tinggalkan soalan tanpa dijawab.*

Work out your answer and **<u>all necessary work</u>** in the space underneath each question. Please circle your answer in the column marked "Answer."
*Jawab semua soalan dan **tuliskan jalan kerja** di tempat yang disediakan. Bulatkan jawapan anda di ruang bertanda "Jawapan."*

Note: Calculators are not allowed.
Nota: Penggunaan kalkulator adalah tidak dibenarkan.

Question ***Soalan***	**Answer** ***Jawapan***
Example/*Contoh*: Simplify: $2 - 3 =$ *Permudahkan*: $2 - 3 =$	e) 1 f) -1 g) 5 h) -5
Simplify each of the following: *Permudahkan setiap soalan di bawah:*	
1) $2 + 6 =$	e) 4 f) -4 g) 8 h) -8
2) $6 + (-2) =$	e) 4 f) -4

	g) 8 h) -8
Question ***Soalan***	**Answer** ***Jawapan***
3) $2 + (-6)$	
4) $2 + (-2) =$	e) 0 f) 1 g) 4 h) -4
5) $-2 + 6 =$	e) 4 f) -4 g) 8 h) -8
6) $-6 + 2 =$	e) 4 f) -4 g) 8 h) -8
7) $-6 + 6 =$	e) 0 f) 1 g) 12 h) -12
8) $-2 + (-6) =$	e) 4 f) -4 g) 8 h) -8
9) $6 - 2$	e) 4 f) -4 g) 8 h) -8
10) $2 - 6$	e) 4 f) -4 g) 8 h) -8

11) $2 - 2$	e) 0 f) 1 g) 4 h) -4
12) $-6 - 2$	e) 4 f) -4 g) 8 h) -8
13) $2 - (-6)$	e) 4 f) -4 g) 8 h) -8
14) $-2 - (-6)$	e) 4 f) -4 g) 8 h) -8
15) $6 - (-6)$	e) 0 f) 1 g) 12 h) -12
16) $-6 - (-2)$	e) 4 f) -4 g) 8 h) -8
17) 6×2	e) 8 f) -8 g) 12 h) -12
18) 2×-6	e) 8 f) -8 g) 12 h) -12

19) -2×6	e) 8 f) -8 g) 12 h) -12
20) -2×-6	e) 8 f) -8 g) 12 h) -12
21) -2×-2	e) 1 f) -1 g) 4 h) -4
22) 3×5	e) 8 f) -8 g) 15 h) -15
23) $3 \times (-5)$	(e) 8 (f) -8 (g) 15 (h) -15
24) -3×5	(e) 8 (f) -8 (g) 15 (h) -15
25) $6 \div 2$	e) 3 f) -3 g) 4 h) -4
26) $6 \div -2$	e) 3 f) -3 g) 4 h) -4
27) $-6 \div 2$	e) 3

	f) -3 g) 4 h) -4
28) $8 \div 4$	(e) 2 (f) -2 (g) 4 (h) -4
29) $8 \div (-4)$	(e) 2 (f) -2 (g) 4 (h) -4
30) $-8 \div 4$	(e) 2 (f) -2 (g) 4 (h) -4
31) $-8 \div -4$	(e) 2 (f) -2 (g) 4 (h) -4
32) $-6 \div -2$	e) 3 f) -3 g) 4 h) -4

End of paper