Inverse Model Based Control for a Twin Rotor System

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***Abstract*— The use of active control technique has intensified in various control applications, particularly in the field of aircraft systems. A laboratory set-up system which resembles the behaviour of a helicopter, namely twin rotor multi-input multi-output system (TRMS) is used as an experimental rig in this research. This paper presents an investigation using inverse model control for the TRMS. The control techniques embraced in this work are direct inverse-model control, augmented PID with feedforward inverse-model control and augmented PID with feedback inverse-model control. Particle swarm optimization (PSO) method is used to tune the parameter of PID controller. To demonstrate the applicability of the methods, a simulated hovering motion of the TRMS, derived from experimental data is considered. The proposed inverse model based controller is shown to be capable of handling both systems dynamic as well as rigid body motion of the system, providing good overall system performance.**

Keywords; Twin rotor system, inverse model, particle swarm optimisation, PID

I. Introduction

The vast majority of conventional control techniques have been devised for linear time-invariant systems that are assumed to be completely known and well understood. In most practical instances, however, the system to be controlled is non-linear, time-varying and the basic physical processes in it are not completely known a priori. These types of model uncertainties are extremely difficult to manage, even with conventional adaptive techniques. The need for control methods for the aforementioned processes becomes very important. Therefore, this paper addresses an inverse model-based control of a TRMS which mimics the behaviour of a rotary wing air vehicle, helicopter.

The parametric model of the TRMS has previously been reported in [1], [2] showcases that the system is in a minimum phase. Therefore, the same system identification technique is employed to obtain the inverse model of the system. The aim from dynamic model inversion is to cancel the flexural effects of the controlled plant by constructing its inverse mapping and using it in the control law. This makes it feasible to employ linear control system tools for achieving the desired control objectives [3]. Inverse dynamics identification is defined as finding the inverse mapping of a system. It is useful to know the inverse dynamics of a plant in order to control it. Inverse models of dynamic system play a crucial role in many control strategies [4], [5].

Myriad methodologies have been scrutinized in terms or obtaining inverse model controller for example, Danai [6], uses feedforward network as the inverse model of the effect of the blade adjustment on helicopter vibrations. This method includes priory knowledge of the process by defining the initial coefficients of the internal model. Shuo and Jihong [5] on the other hand designed a dynamic inversion controller for both altitude loop and attitude loop from a simplified mathematical model derived from generic YAMAHA R-50 unmanned helicopter simulation model. Moreover, Rahideh *et al*. [3] have developed a model inversion control in combination with genetic algorithm to tune a proportional-derivative (PD) controller. A neural network element is then integrated with the feedback control system to compensate for model inversion error. However, a major problem with this kind of approach is, it needs a priori knowledge as well as the mathematical model of the system. Therefore, this work is done using a parametric approach utilizing the input and output data of the system in order to develop the inverse model of a TRMS.

II. Twin Rotor MIMO System

The twin-rotor multiple-input multiple-output (MIMO) system (TRMS) is a laboratory set-up developed by Feedback Instruments Limited [7] for control experiments. Its behaviour in certain aspects resembles that of a helicopter. For example, it possesses a strong cross-coupling between the collective (main rotor) and the tail rotor, like a helicopter. A schematic diagram of the TRMS used in this work is shown in Figure 1. It is driven by two DC motors. Its two propellers are perpendicular to each other and joined by a beam pivoted on its base that can rotate freely in the horizontal and vertical planes. The beam can thus be moved by changing the input voltage in order to control the rotational speed of the propellers. The system is equipped with a pendulum counterweight hanging from the beam, which is used for balancing the angular momentum in steady-state or with load.

The system is balanced in such a way that when the motors are switched off, the main rotor end of the beam is lowered. The controls of the system are the supply voltages of the motors. It is important to note that the geometrical shapes of the propellers are not symmetric. Accordingly, the system

This work was supported in part by the Ministry of Higher Education, Malaysia and the International Islamic University Malaysia, Kuala Lumpur, Malaysia.

behaviour in one direction is different from that in the other direction. Rotation of a propeller produces an angular momentum which, according to the law of conservation of angular momentum, is compensated by the remaining body of the TRMS beam. This results in interaction between the moment of inertia of the motors with propellers. This interaction directly influences the velocities of the beam in both planes. The measured signals are: position of the beam, which constitutes two position angles, and the angular velocities of the rotors. Angular velocities of the beam are software reconstructed by differentiating and filtering the measured position angles of the beam.

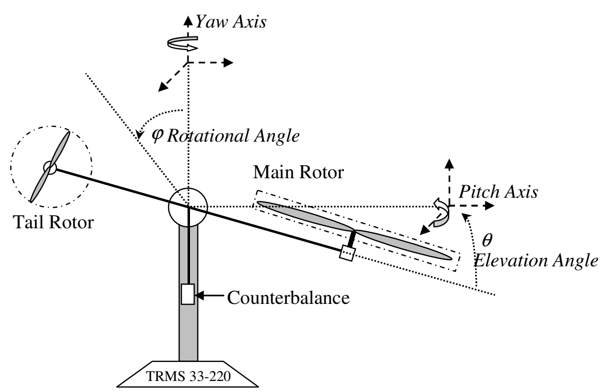

Figure 1. Twin rotor multi input multi output system

III. PARTICLE SWARM OPTIMISATION

Particle swarm optimization [8] is a population-based evolutionary optimization method, inspired by the collective behaviours of birds and flocks. The PSO algorithm is similar to evolutionary computation in producing a random population initially and generating the next population based on current cost, but it does not need reproduction or mutation to produce the next generation. Thus, PSO is faster in finding solutions compared to other evolutionary computation technique.

Dynamic spread factor PSO (SFPSO) is employed in this paper [1], [9]. The algorithm is found highly effective in improving major issues in basic PSO that are premature convergence and preservation of diversity. As originally developed, inertia weight, w is decreased linearly from 0.9 to 0.4 during a run. Suitable selection of the inertia weight provides a balance between global and local exploration and exploitation and results in less iteration on average to find a sufficiently optimal solution. The mathematical representation of SFPSO is given as

$$
\begin{aligned}
x_{id_{new}} &= x_{id} + v_{id_{new}} \\
v_{id_{new}} &= (w * v_{id}) + c_1(rand_1(p_{id} - x_{id})) \\
&\quad + c_2(rand_2(p_{gd} - x_{id}))
\end{aligned} \tag{1}
$$

where w, c_1 and c_2 are given by

$$
\begin{aligned}
w &= \exp(-iter/(spread\_factor \times \max\_iteration)) \\
spread\_factor &= 0.5(spread + deviation) \\
c_1 &= 2(1 - iter/\max\_iteration) \\
c_2 &= 2
\end{aligned} \tag{2}
$$

where x_{id} and v_{id} represent the position vector and velocity vector of the ith particle in the d-dimensional search space respectively. The first part of velocity vector equation in (1) represents the previous velocity, which provides the necessary momentum for particles to roam across the search space. The second part, known as 'cognitive' component, represents the personal thinking of each particle. The cognitive component encourages the particles to move towards their own best positions found so far. The third part is known as the 'social' component, which represents the collaborative effect of the particles, in finding the global optimal solution. The social component always pulls the particles towards the global best particle found so far. In order for particles to keep exploring the search space, it is imperative that they must know their whereabouts and relative distances from each other. The spread factor in SFPSO algorithm measures the distribution of particles in the search space as well as the precision and accuracy of the particles with respect to global optimum. Therefore, when all the particles move within the vicinity of global optimum, both the dynamic SF and hence the inertia weight will drop in value drastically. This will not only force all the particles to converge, but also allow the algorithm to achieve extremely high precision.

Throughout this work, SFPSO is used in search for parameter estimation for both the value of inverse model as well as the value of kp, ki and kd parameters for the PID controllers. The SFPSO process is best explained in Figure 2.

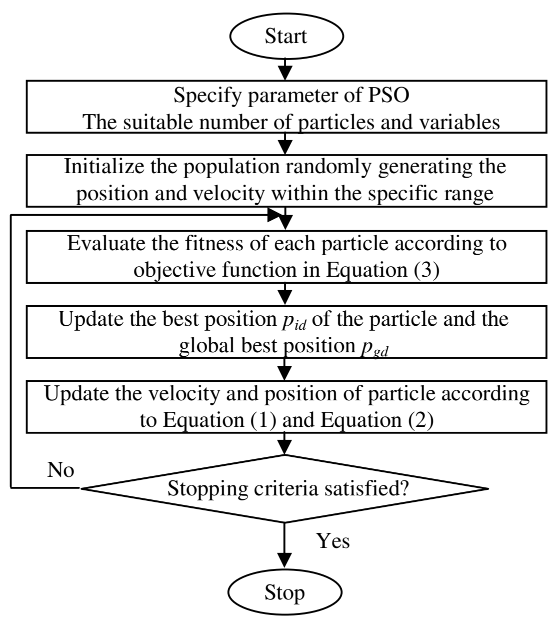

Figure 2. The flowchart of SFPSO algorithm

IV. INVERSE MODEL IDENTIFICATION

In this section, the identification of the inverse model of a TRMS is discussed. The forward parametric model of the system that mimic well the TRMS behaviour has been obtained in the previous work using the same SFPSO optimization approach [1], [2]. Dynamic model inversion is a popular method to achieve feedback linearization because it is easy to apply and its physical meaning is clear [10]. The inverse model is designed to have an accurate inverted model replicating the system model. The input of the inverted model is pitch angle of rotor and the output is the main voltage to the system. Figure 3 depicts the schematic diagram of the inverse model identification technique. SFPSO is used to obtain the inverse model parameters of the system. The characteristics of the SFPSO used in the inverse modeling identification are shown in Table I. The difference between the predicted and actual input is recorded as error, $\varepsilon(t) = u(t) - \hat{u}(t)$, which in turn is used to form the objective function of the optimization process. Mean squared error (MSE) is used in this work as the objective function and given as

$$f(x) = \frac{1}{s}\sum_{t=1}^{s}(u(t) - \hat{u}(t))^2 \qquad (3)$$

where $s = 1000$

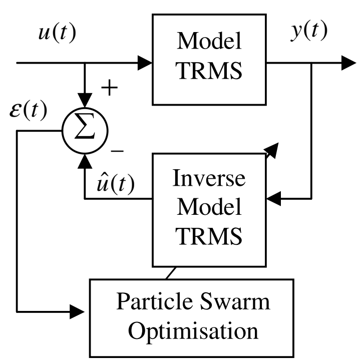

Figure 3. Schematic diagram of the inverse model identification

The obtained inverse model of the system is then set in cascade with the parametric model of TRMS as shown in Figure 4. The simplest approach for controller design is a completely open loop control strategy, in which the controller is the inverse of the process. For this open loop inverse model control, the input to the inverted model is the desired value of pitch angle, $\alpha_{ref}(t)$, the output of the inverted model, that is the input to the plant, is the voltage of the main rotor, $u(t)$, and the output of the plant is the pitch angle, $\alpha(t)$. The output response of the open loop control in Figure 5 shows that the inverse model has reduced system vibration to some extent, around the set point (rigid-body motion). Therefore, it is evident that the inverse model technique can be used to enhance the tracking characteristics of the system.

TABLE I. THE CHARACTERISTIC OF THE SFPSO COEFFICIENTS FOR INVERSE MODEL IDENTIFICATION

Characteristics	
1. Number of generations	500 generations
2. Number of particles	30 particles
3. Number of variables	
• Inverse model	8 variables

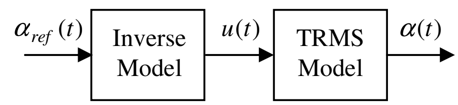

Figure 4. The open loop inverse model control

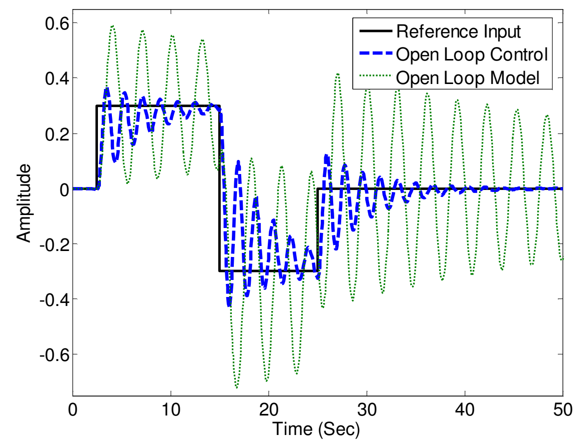

Figure 5. The open loop response of model and inverse control

V. INVERSE-MODEL BASED CONTROL APPROACHES

A. *Direct inverse-model control*

After the promising results from the open loop inverse control, direct inverse control is developed to further investigate the inverse model in closed loop (Figure 6). The TRMS inverse model will have some inaccuracies which may lead to deviations between the reference input and the system output. However, it has been noticed that these are relatively small where the output response started to follow the desired input response even though there is still some overshoot and fluctuation before the system meet the settling time. The result of using direct inverse control is illustrated in Figure 7.

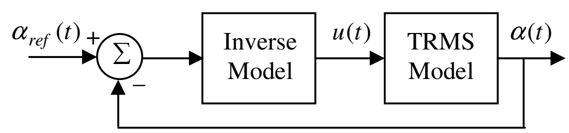

Figure 6. The block diagram of a direct inverse control

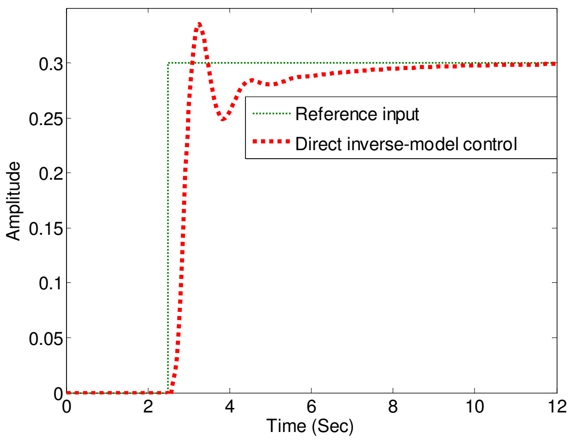

Figure 7. Closed loop response of the direct inverse-model control to a reference input

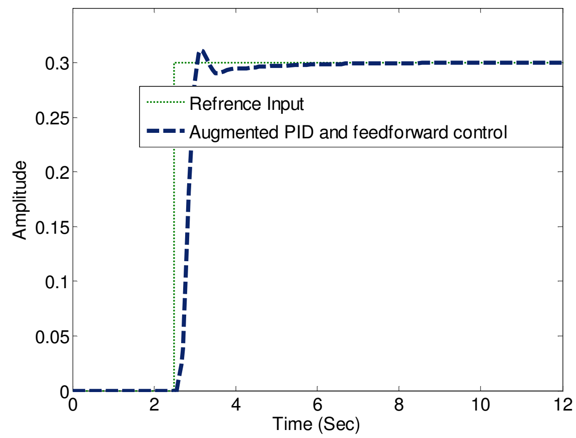

Figure 9. Closed loop response of the augmented PID and feedfforward inverse control to a reference input

B. Augmented PID and feedforward inverse control

Fixed stabilising controller has been proposed in [11], [12]. This scheme has first been applied to the control of robot arm trajectory where a proportional controller with gain was used as the stabilizing feedback controller. In this work, the augmented PID and feedforward inverse controller is developed for control of rigid body motion of the TRMS system to enhance the tracking performance of the TRMS in hovering position as shown in Figure 8. The total input that enters the plant is the sum of the feedback control signal and the feedforward control signal which is calculated from the inverse dynamic model. The model uses the desired pitch angle as the input and the output is the main voltage.

Figure 9 shows that the output response of an augmented PID and feedforward inverse controller is better than using direct inverse-model control. The value of overshoot has been reduced by 14% and the settling time also has been reduced for up to 6 second. The optimum values of PID parameters are obtained using SFPSO. The characteristics of the SFPSO and the optimum PID parameter values are listed in Table II and Table III.

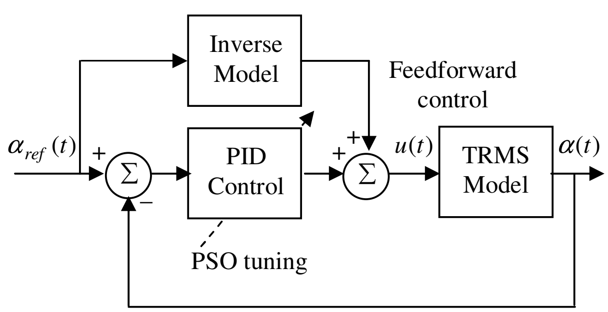

Figure 8. The block diagram of a stabilising PID controller

C. Augmented PID and feedback inverse control

In the absence of model inversion error, the combination of feedback inverse control loop together with a highpass filter design is applied to augment the vertical attitude control system, see Figure 10. The cutoff frequency value for both high pass filter and low pass filter is set to be 0.15Hz. From previous study [1], [2], the main dominant mode of the TRMS is in between 0-1Hz which is 0.349Hz.

The control structure in Figure 10 comprises into two control loops. The first loop (Loop 1) will correspond to a standard PID controller, with a low pass filter (LPF) located at the feed back loop. The main purpose of the LPF is to reject frequencies above 0.15Hz including the main resonance mode of the system so that the first loop will solely concentrate on control of the rigid body motion of the system.

The second loop (Loop 2) is a negative feedback loop where the inverse model is used as a controller to augment the hovering control of the system albeit of using the PID feedback controller alone. The high pass filter (HPF) will reject the low-frequency (rigid-body) dynamics of the system. Thus, this loop will control the flexible motion dynamics (vibrations) of the system.

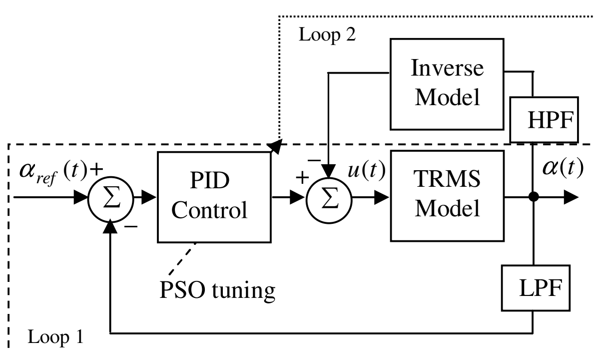

Figure 10. The block diagram of a specialised controller

Figure 11 shows the comparison of using the entire three inverse-model based controllers endeavor in this work. It is evident from the output response that the direct inverse-model control has a poor performance and the augmented PID and feedforward inverse controller improves the performance. However, the third controller which is the augmented PID and feedback inverse controller scheme gives superior performance with no overshoot and faster settling time, and hence leads to perfect response.

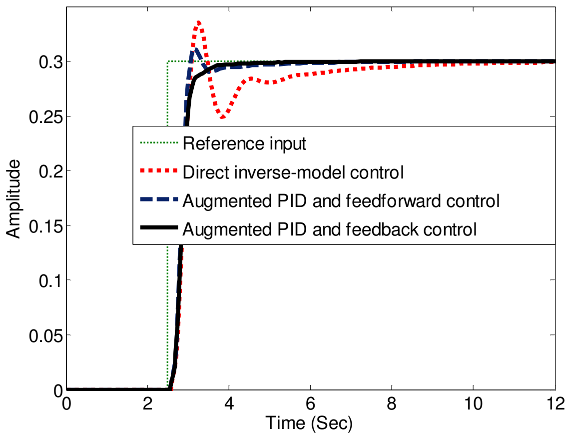

Figure 11. Closed loop response of the control system to a reference input

TABLE II. THE CHARACTERISTIC OF THE SFPSO COEFFICIENTS FOR PID CONTROLLERS

Characteristics	
4. Number of generations	200 generations
5. Number of particles	30 particles
6. Number of variables	
• PID controllers	3 variables

TABLE III. PID PARAMETER VALUE FOR AUGMENTED PID AND FEEDFORWARD CONTROL AND AUGMENTED PID AND FEEDBACK CONTROL

Augmented PID and feedforward control			Augmented PID and feedback control		
Kp	*Ki*	*Kd*	*Kp*	*Ki*	*Kd*
12.36	7.58	5.23	1.73	1.21	0.47

VI. CONCLUSION

In this work, a scrutinized investigation on parametric inverse model approach has been developed and applied to a TRMS in simulation environment in terms of its 1 DOF hovering motion. An inverse model control approach with PID in feedforward and feedback configurations has been developed. It has been demonstrated that the inverse model control has great potential in controlling the flexible dynamics of the system.

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