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## Regularization-based multi-frame super-resolution: A systematic review

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## ABSTRACT

High-resolution is generally required and preferred for producing more detailed information inside the digital images; therefore, this leads to improve the pictorial information for human analysis and interpretation and to enhance the automatic machine perception. However, the real imaging systems may introduce some degradation or artifacts in the digital images. These distortions in the images are caused by a variety of factors such as blurring, aliasing, and noise, which may affect the resolution of imaging systems and produce low-resolution images. Numerous strategies like frequency and spatial domain approaches have been proposed in the literature. Spatial domain approaches are classified as one of the most popular approaches and split into interpolation-based approaches and regularization-based approaches. Nevertheless, these techniques still suffer from artifacts. Regularization-based approaches are a challenging in image super-resolution in the last decade. This paper attempts to investigate the current regularization-based super-resolution approaches which are commonly used for reconstructing the high-resolution image in the last decade. Furthermore, the focus is given on highlighting the strengths and limitations of these approaches aiming at determining their effectiveness and quality in reconstructing high-resolution images.

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## Contents

1. Introduction	755
2. Observation model	756
3. SR image reconstruction framework	757
4. Regularization-based SR approaches	757
4.1. Stochastic approaches	757
4.2. Deterministic approaches	760
4.3. Hybrid approaches	760
5. Discussion and analysis	761
6. Conclusion	761
References	761

## 1. Introduction

The process of reconstructing high-resolution (HR) images is one of the hottest research areas in the recent years in which a wide range of useful details are acquired from images. Super-resolution (SR) approaches are used in different domains to analyze and extract the essential information from the images (Yue et al., 2016). SR technologies are used in a wide range of applications to achieve the HR image and may be distinct in different

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applications. The HR image is generally required and preferred for producing more detailed information inside the digital images, therefore, this leads to the improvement of pictorial information for human analysis and interpretation and also for automatic machine perception (Köhler et al., 2016). Many applications of computer vision such as medical imaging, satellite imaging, pattern recognition, surveillance and forensic, astronomical imaging, and target detection are still in an urgent need for HR images. First, medical images are very beneficial for doctors to derive the important information and get the accurate diagnosis of the patient (Huangpeng et al., 2017; Yue et al., 2016). Second, satellite imaging applications such as remote sensing and LANDSAT provide multiple images of the same region where the use of SR techniques is to enhance the resolution of the target. Last, synthetic zooming of the area is another important application for surveillance and forensics to zoom objects in the image such as a criminal face or a car license plate (Yue et al., 2016).

In many real-life imaging systems, there are some artifacts such as blurring, aliasing, and noise that affect the image resolution (Huangpeng et al., 2017; Kumar and Diwakar, 2019; Yue et al., 2016). The blurring effects can appear within the image during the shooting process based on some factors such as scene movements, incorrect focusing, atmospheric confusion, and optical point spread function. Accordingly, it is much easier to remove the blur effects from the image accurately, if the shooting conditions at the time of getting the image are known. In addition, noise can be caused by a wide range of factors such as differences in detector sensitivity, visual defects, and environmental changes. There is no relationship amongst the pixels and the noise, because the noise is not spatially connected to the image. Also, down-sampling is a result of an inadequate spatial sampling that led to overlapping between high and low-frequency components (Begin and Ferrie, 2006; Huangpeng et al., 2017; Park et al., 2003; Yue et al., 2016).

As a result of these factors, many researchers develop various methods for producing a high-quality image based on SR image reconstruction approaches such as frequency domain approaches and interpolation-based approaches. However, the frequency domain approaches have many problems which prohibited researchers from advance development, especially in the case for the sensitivity of model errors and difficulty in dealing with more complex motion models (Begin and Ferrie, 2006; Hadhoud et al., 2004a; Papathanassiou and Petrou, 2005; Patanavijit, 2009; Yang and Huang, 2010; Yue et al., 2016). While the interpolation-based approaches regularly generate images with several drawbacks around the object's borders, consisting of zigzag, blurring, and aliasing edges (Yang and Huang, 2010). On the other hand, regularization approaches take advantage of the prior knowledge to fix the SR problem (Yue et al., 2016). Thus, the regularization approaches can be used as an attempt to stabilize the inversion

process and compensate for the absent information (El Mourabit et al., 2017; Yue et al., 2016). Additionally, they are used to represent a prior of the image, remove artifacts, and bring the prior information (Wang et al., 2017). The prior information generates a stable solution, improve the convergence rate, and include artificial constraints on the solution such as smoothness and edge-preserving (El Mourabit et al., 2017; Kiani and Drummond, 2017b; Long et al., 2017; Mohan, 2017; Wang et al., 2017). Therefore, regularization-based approaches are a challenging in SR image reconstruction (Hadhoud et al., 2004b). This research paper aims at present a comprehensive review of regularization-based multi-frame SR approaches for the last decade. A summary of most of the well-known works has been reported to identifying the limitations of those works. Some recommendations and future work directions have been drawn helping researchers to explore the unsolved problems related to image reconstruction.

The rest of this paper is organized as follows. Section 2 illustrates observation model so that it reflects the HR image into the observed LR images. Section 3 describes the SR image reconstruction framework. Different regularization-based multi-frame SR approaches are represented in Section 4. A detailed discussion is offered in Section 5, and the paper is concluded in Section 6.

## 2. Observation model

An observation model identifies the true manner where the observed LR images are acquired. The image acquisition procedure is usually met with a collection of degradation factors such as optical diffraction, comparative motion, down-sampling, and system noise. Generally, most methods assume that the procedure of image acquisition consists of warping, blurring, down-sampling, and noise degradations as shown in Fig. 1, and the observation model is definitely simulated the following:

$$y_k = DB_k M_k X + n_k \quad (1)$$

where  $k$  is LR images that participated in the reconstruction process and  $X$  is the original image that degraded by warping ( $M$ ), blurring ( $B$ ), down-sampling ( $D$ ), and additive noise ( $n$ ). After the model is well-known, it may be used to inverse the process in order to retrieve the HR image from a various of LR images. Therefore, it can be said that the observation model is inverted in order that the problem requires a prior information from the HR image to get a reliable and suitable solution. (Park et al., 2003; Protter et al., 2009). Most authors are assuming that all LR images are collected in the similar environmental conditions and using the same sensor. Therefore, the observation model can be rewritten as:

$$Y_k = DBM_k X + n_k \quad (2)$$

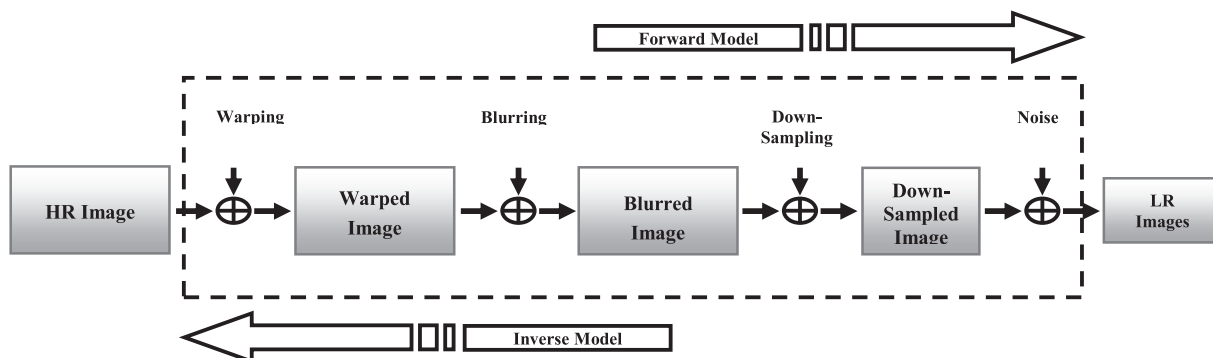


Fig. 1. The observation model employed in most SR techniques.

### 3. SR image reconstruction framework

This section explains the detail steps of image reconstruction process. Basically, the SR image reconstruction approach consists of three phases as elaborated in Fig. 2. These phases are called image registration, fusion, and reconstruction. First, the image registration process is used to estimate the motion information among LR images with sub-pixel shift and calculate and refine transformation parameters. Therefore, image registration is an essential component of SR image reconstruction because (El-Gamal et al., 2016):

- Information about the motion shifts between LR images presents the important constraints to assist in the SR solution.
- Incorrect motion estimation triggers objectionable artifacts in the HR image.
- The level of resolution improvement that can easily be accomplished based on the sub-pixel accuracy for the displacement details.

Then, the image fusion phase is used to fuse the registered images into a single image and interpolate the composed image into the HR grid (El Mourabit et al., 2017). Finally, the image reconstruction phase is used to restore the final SR image without any distortions (Bahy et al., 2014).

### 4. Regularization-based SR approaches

As we discussed earlier, the main purpose of SR image reconstruction is to generate a powerful HR image dependent on a few LR images that are captured through the exact same scene. In this paper, we focus on multi-frames SR reconstruction based on regularization approaches. We categorize the regularization approaches into three classes, namely: stochastic, deterministic, and hybrid approaches. Stochastic approaches use random variables in the form of probability distributions to provide stable estimates effectively and distinguish between possible solutions by utilizing a priori image model. While, deterministic approaches do not use any random variables but it can be formulated by choosing a variable to minimize the Lagrangian and solve the inverse problem by using the prior information about the solution which can be used to make the problem well posed. Lastly, hybrid approaches employ a combination of stochastic and deterministic approaches. We first review the basic problem identified by researchers in their research work. Second, we present their methodology to solve the problem. Finally, we discuss the strengths and limitations of each method to determine its effectiveness and quality in reconstructing HR images. Therefore, the reviewed approaches are summarized in the following subsections as depicted in Fig. 3. Also, some major techniques with a discussion of strengths and limitations are described in Table 1, Tables 2, and 3.

#### 4.1. Stochastic approaches

The total variation (TV) regularization model is very weak in the processing of flat-image regions. In addition, it is not able to achieve the automatic balancing of the different regions inside the image and suffer from artifacts (Yuan et al., 2012). Therefore,

Yuan et al. (2012) propose a spatial weighted TV (SWTV) method to overcome the limitations of the TV regularization model. They take into consideration the distribution of spatial details in various regions in the image. The idea of their work replies on employing the difference curvature instead of the image gradient to recognize the spatial feature for every pixel. Furthermore, the difference curvature is used to extract the information to determine the weighted parameter and restrict the TV model at every pixel. Finally, the SWTV is optimized by the Majorization-Minimization (MM) algorithm (Yuan et al., 2012). The SWTV method reduces artifacts in the flat-image regions and preserves the edge details. However, it often causes an exchange among preserving of the edge details and avoiding the effects of the staircase in soft-regions. Moreover, the work in (Ren et al., 2013) offers a fractional order TV regularization to take care of the texture information in the image and eliminate artifacts from the TV model. Zhang et al. (2012) propose a new algorithm to manage the coarseness of resolution for a hyperspectral image. This method is based on a maximum a posteriori (MAP) and Principal component analysis (PCA) algorithms (Zhang et al., 2012). PCA is used simultaneously in the motion estimation stage to decrease the computational cost and enhance the motion accuracy in the image reconstruction stage to eliminate the noise. However, a hyperspectral image has a high dimension of information that causes computationally intense for image processing. Moreover, this method supposes that the blurring prior is known and this hypothesis is not easy to estimate the blurring.

Non-local means (NL-means) regularization filter indicates preferable effectiveness to remove the noise and protect the edge in image regions than the TV regularization model (Kim and Byun, 2013). However, the image properties do not show well, if the noise and edge are presented in each region within the image. This leads to a disappearance of the edge in the image region with a huge edge and it also has a small noise. Furthermore, the noise is not removed in the image region with a small edge and also it has a lot of noise (Kim and Byun, 2013). Therefore, Kim and Byun (Kim and Byun, 2013) propose a regularization method based on an edge-adaptive NL-means filter to enhance the efficiency of NL-means filter to be able to remove the noise and protect the edge in image regions. However, this method can't estimate the regularization parameter automatically and its computational cost is very high. Panagiotopoulou (Panagiotopoulou, 2013) presents a novel SR method based on the combination of the Var-norm and the bilateral total variation (BTV) regularization to automatically renew the weights. This method is proposed to overcome the limitations of the  $L_1$ -norm and  $L_2$ -norm. While the  $L_1$ -norm and  $L_2$ -norm give all measurements one fixed weight and each measurement a double of the measurement value respectively. Because MAP is used to adjust the regularization parameter manually and ignore the local spatial adaptive characteristics in the images, Shao et al. (2013) propose a spatially adaptive Laplacian Markov Random Field (MRF) prior base on a Bayesian framework. This prior has the ability to protect image components, decrease staircases in the smooth regions, and automatically adjust the regularization parameter. However, the information contained within LR images is too restricted to apply this method. Tikhonov regularization approach is used to eliminate the noise but it loses a few information from the image-regions (Zeng and Lu, 2013). Zeng and Lu

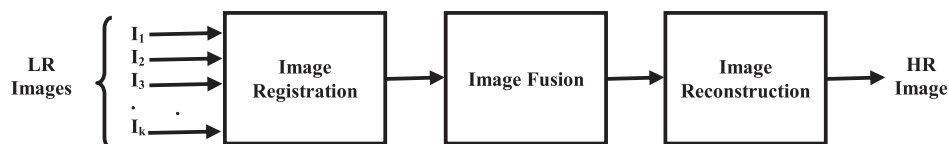


Fig. 2. SR image reconstruction processes.

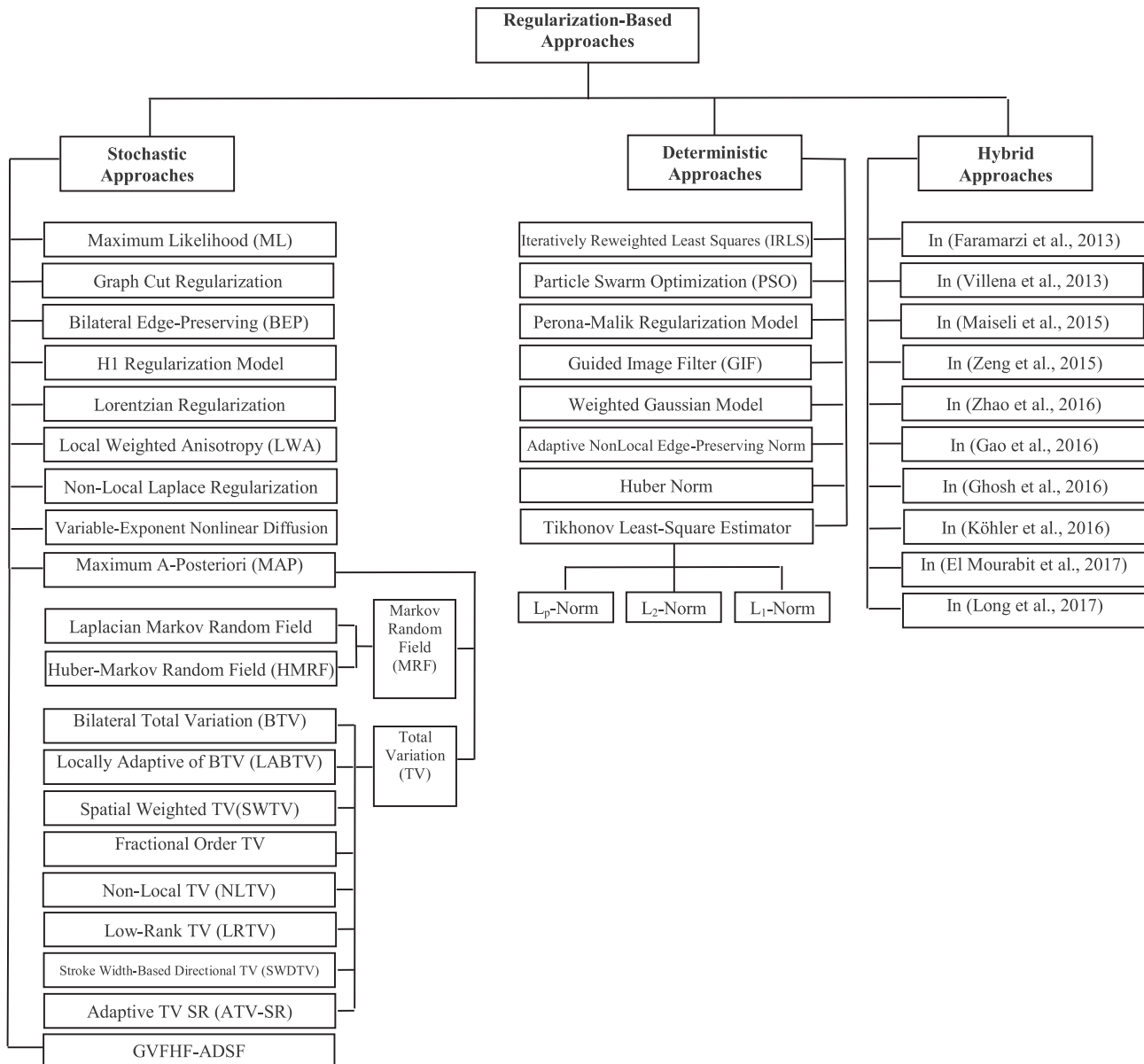


Fig. 3. The classification of regularization-based multi-frames SR approaches.

(2013) offer the non-local TV (NLTV) with a weighted data fidelity function to estimate correctly the registration parameters. This method can decrease the noise around edges and improve the image details. However, it avoids the local spatial adaptive characteristics of the images. Chen et al. (2014) propose a novel  $L_{\text{mix}}$  model in order to protect the edges during the noise removal process. This model combines both of the TV and the  $H^1$  models in accordance with updating the weighting parameters (Chen et al., 2014). Villena et al. (2014) suggest a new method based on the Bayesian framework. This method is used to register and rebuild the image by combining spatially adaptive and image filters (Villena et al., 2014). This method is used to preserve edges and textures in the image and automatically deduce all parameters. However, it presumes a simple transformation to the register LR images.

Due to the extreme lack of the resolution in the imaging systems, it may dampen the presence of aliases that affect the quality of the images (Wang et al., 2014). Wang et al. (2014) offer a new approach depending on the Bayesian framework and apply the

TV model to improve the image resolution. In this approach, the motion estimation is applied by using an effective approach and it is optimized by gradient descent algorithm (Wang et al., 2014). This method significantly increases the resolution as well as preventing blurry and noise from the image. However, the TV model has a heavy computational cost. Also, this method is unable to successfully recover the fine information. Gao and Qin (2015) propose a new approach based on locally weighted anisotropy regularization (LWAR) and successive regularization. LWAR is used to restrict the softness for the image reconstruction. Bregman iterative algorithm is utilized to improve the SR image (Gao and Qin, 2015). This approach is able to remove the noise and protect the edges in image regions. However, this approach can't estimate the motion parameters well. Shi et al. (2015) propose a method called the low-rank TV (LRTV) to incorporate all information from the images. This method is optimized by the alternating direction method of multipliers (ADMM) to efficiently restore the HR image (Shi et al., 2015). However, this model has a heavy computational cost related to the TV model. Zhang et al. (2015) propose a new method

**Table 1**  
Evaluation of different stochastic techniques and methods.

Approach	Strengths	Limitations
TV (Yuan et al., 2012)	TV protects edge information and prevents ringing results.	TV is very weak in the flat regions and suffers from artifacts.
Tikhonov (Zeng and Lu, 2013)	It eliminates the noise.	It loses a few information from the image-regions.
SWTV (Yuan et al., 2012)	SWTV reduces artifacts and preserves the edge information.	SWTV causes an exchange among preserving of the edge details and avoiding the effects of the staircase in soft-regions.
Edge-Adaptive NL-Means (Kim and Byun, 2013)	It removes the noise and protects the edge in image regions.	It can't estimate the regularization parameter automatically and has a high computational cost.
Var-norm + BTV (Panagiotopoulou, 2013)	It automatically renews the weights and removes the noise.	It still has artifacts.
Fractional Order TV (Ren et al., 2013)	It removes artifacts and staircase edges.	The shortcomings of the global TV model.
Spatially Adaptive Laplacian MRF (Shao et al., 2013)	It restores fine image details, preserves edges, and removes artifacts.	Information in LR images is too restricted to apply it.
NLTV (Zeng and Lu, 2013)	NLTV can decrease the noise around edges and improve the image details.	NLTV avoids the local spatial adaptive characteristics of the images.
Lmix(Chen et al., 2014)	Lmix protects the edges during the noise removal process.	The shortcomings of the global TV model.
LWAR + Successive Regularization (Gao and Qin, 2015)	It can remove the noise and protect the edges in image regions.	It can't estimate the motion parameters well.
LRTV (Shi et al., 2015)	LRTV enhances the details in the restored HR images.	LRTV has a heavy computational cost related to the TV model.
MAP-MRF (Zhang et al., 2015)	More robust to blurry images.	MRF may produce insufficient results.
SWDTV (Abedi and Kabir, 2016)	Foreground and background regions are smoothed.	SWDTV is unable to completely rebuild the text edge details and has a high computation cost.
TV + Low-Rank (Jun-Bao et al., 2016)	It restores more high-frequency details.	It can't make a balance between the edges and noises.
Non-Local Laplace + BTV (Laghrib et al., 2016)	It reduces the noises and motion outliers.	It has more staircases effects and a high computational cost.
SRT + HFET (Nayak and Patra, 2016)	It can preserve the image information and avoid the presence of artifacts.	It has a high computational cost.
GVFHF-ADSF (Huang et al., 2017)	It can effectively suppress noise and enhance edges.	It has a high computational cost.

**Table 2**  
Evaluation of different deterministic techniques and methods.

Approach	Strengths	Limitations
BEP + $L_1 + L_2$ (Zeng and Yang, 2013)	It preserves sharp edges well without producing visual artifacts.	BEP applies a constant scale variable for the entire image and dismisses the main characteristics of an image.
LARSR (Bahy et al., 2014)	LARSR outperforms on LABTV by finding the optimal values for the regularization parameters automatically.	The early convergence of PSO to the local minimum capturing.
In Maiseli et al. (2014)	It improves the edges and recovers a fine detail.	It includes some blurring in the HR image and has a low-contrast.
In Yadav et al. (2014)	It reduces the noise, protects the edges, and drops off the computational time.	It still suffers from artifacts.
In Yang et al. (2015)	It protects the edges and keeps the smoothness of image regions.	It still suffered from the noise.
In Shen et al. (2016)	It achieves a good balance between noise suppression and edge preservation.	It uses the $L_2$ norm that has an optimum solution if the white Gaussian distribution is used.
IRLS (Kiani and Drummond, 2017a)	It is very simple to understand and it reduces the ambiguity and the noise in the solution.	IRLS is not suitable for most practical situations.

based on the graph cuts technique. They drop all HR pixels on LR images and identify the LR pixels that fall within the impact region. Also, they utilize the maximum a posteriori Markov random field (MAP-MRF) to reduce the energy function, restore the HR image, and reduce the calculation cost. However, MRF may produce insufficient results. Zhao et al. (2015) propose a new method based on a Bayesian framework to protect the edges and decrease the noises in the image. They use an adaptive norm to regulate the relationship amongst the pixels (Zhao et al., 2015). However, this method is unable to eliminate the noise, specifically if the noise is strong. Abedi and Kabir (2016) propose the stroke width-based directional TV (SWDTV) regularization method for a document image SR. SWDTV is an updated version of the BTV to smooth the characters depending on the local width and direction. They use MAP method to reduce the integration of the regularization and data fidelity terms. However, this method is unable to completely rebuild the text edge details and it has a high computation cost. Chen et al. (2016) use a reasonable observation model to incorporate the absent details. Also, they use a Bayesian framework based on Kull-

back-Leibler (K-L) divergence to estimate the motion parameters and protect the edge details in the image (Chen et al., 2016).

The properties of natural images can't be fully mentioned by the TV regularization model (Jun-Bao et al., 2016). Therefore, Jun-Bao et al. (2016) combine both TV model with the low-rank model to produce a new method for SR and generate the HR image. This method is able to recover good quality HR image but it can't make a balance between the edges and noises. Laghrib et al. (2016) propose a new approach depends on a diffusion registration and mix a non-local Laplace regularization with a BTV model to reduce noises and motion outliers. However, it has more staircases effects and a high computational cost. Nayak and Patra (2016) propose a novel regularization based SR approach dependent on using structural regularization term (SRT) and high-frequency energy term (HFET). This approach can preserve the image information and avoid the presence of artifacts. However, it has a high computational cost. Huang et al. (2017) propose a new multi-frame SR approach by employing both image enhancement and denoising into the SR procedures. Firstly, they propose a new gradient vector flow hybrid

**Table 3**  
Evaluation of different hybrid techniques and methods.

Approach	Strengths	Limitations
AM + HMRF (Faramarzi et al., 2013)	It reduces artifacts and preserves the edge details.	HMRF can't illustrate the complicated relationships among neighboring pixels.
TV + $L_1$ + SAR (Villena et al., 2013)	It recovers image edges and smooths inner regions.	The weights have been identified empirically.
Variable-Exponent Nonlinear Diffusion (Maiseli et al., 2015)	It generates more resolute scenes and avoids blocking artifacts inherent in the conventional TV.	It has a more blurring in edges.
ATV-SR (Zeng et al., 2015)	It preserves edges and fine details while suppressing noises and avoids block effect.	It can't distinguish between the edges and noises within the image regions.
In Zhao et al. (2016)	It preserves the detail of an image while avoids artifacts.	NLTV can't smooth the image well.
In Gao et al. (2016)	It reduces the interference of noise.	It uses a low noise level.
In Ghosh et al. (2016)	It is simple and robust.	The computational resource is limited.
In Köhler et al. (2016)	It is easy to implement.	It uses a local selection of the sparsity parameter.
Perona-Malik model + Weickert filter (El Mourabit et al., 2017)	It is used to protect the edges and eliminate the noises.	It cannot achieve a good balance between preserving the edges and suppressing the noise.
envL1/TV (Long et al., 2017)	It reduces the noises.	It is not good to preserve the edges well.

field (GVFHF) to capture the object boundaries more accurately in images. The GVFHF uses both the gradient vector flow (GVF) and the gradient filed (GF). Secondly, they use the anisotropic diffusion shock filter (ADSF) to propose the GVFHF-ADSF for improving and denoising the reconstructed image. Lastly, the GVFHF-ADSF approach is employed as a regularization term and the steepest descent algorithm is adopted to solve the inverse SR problem. The GVFHF-ADSF approach can effectively suppress both Gaussian and salt-and-pepper noise and enhance edges of the reconstructed image. However, this approach needs to apply it on a variety of applications and has a high computational cost. Mohan (2017) proposes a method based on MAP framework to reduce a cost function. Mohan uses Lorentzian norm and U-curve approach to get on the regularization parameter which removes the artifacts and decreased the computational cost respectively. This method outperforms on artifacts and using Lorentzian norm is stronger than  $L_p$  norm. Wang et al. (2017) propose a fast-new approach derived from MRF regularization. Initially, they present an end-to-end SR approach for correcting the HR image that is caused by the reconstruction errors in LR space without analytical into HR space, where the computation cost is clearly reduced. In addition, they propose a new regularization term derived from MRF, in order to achieve the smoothness and preserve the edges at the same time (Wang et al., 2017).

#### 4.2. Deterministic approaches

BTV model generates artifacts during the processing of smooth-image regions (Zeng and Yang, 2013). Zeng and Yang (2013) propose a new SR approach constructed on the regularization framework. The main idea of their approach is to combine the benefits of  $L_1$  and  $L_2$  norms in both the fidelity and regularization terms. Also, they use a bilateral edge-preserving (BEP) regularization model to capture the relationship between two pixels (Zeng and Yang, 2013). This method preserves the edge details, reduces artifacts in the flat-image regions, and discovers the single optimal SR image. However, BEP regularization model applies a constant scale variable for the entire image and dismisses the main characteristics of the image. Bahy et al. (2014) propose a local adaptive regularized SR (LARSR) approach. LARSR doesn't use a fixed regularization parameter but uses automatically an adaptive one based on the particle swarm optimization (PSO) method. Therefore, the processing results prove that LARSR outperforms others reconstruction approaches. However, the primary difficulty occurs in this approach because of the early convergence of PSO to the local minimum capturing. Maiseli et al. (2014) use the low-pass filter for interpolating the unidentified pixel values. Then, the aliases

in the low-frequency components have been corrected. This approach integrates an improved adaptable Perona-Malik regularization model to improve the edges and recover a fine detail in the HR image. However, it includes some blurring in the HR image and has a low-contrast. Yadav et al. (2014) propose a new approach based on the guided image filter (GIF) method which uses the information from the colored image. This approach is used to reduce the noise, protect the edges, and drop off the computational time. However, GIF has a nice property of edge-preserving smoothing but the image still has a noise.

Due to the heavy computational cost and blurring effects of the MAP, Yang et al. (2015) propose a new method for SR image reconstruction based on the regularization framework. They develop a data fidelity term through incorporating both  $L_1$  and  $L_2$  norms depending on defining the residual weight parameters (RWP) and channel weight parameters (CWP). Also, the regularization term is developed by using the regional adaptive weight coefficients (RAWC) (Yang et al., 2015). This method is used well to protect the edges and keep the smoothness of image regions but it still suffered from the noise. Shen et al. (2016) propose a new approach to identify the optimum norms for the data fidelity and regularization terms. They approximate the data fidelity norm in a Gaussian case (Shen et al., 2016). They use a local adaptive norm for the regularization term to obtain a strong stability among noises reductions and edges protection. However, this approach uses the  $L_2$  norm that has an optimum solution if the white Gaussian distribution is used. Kiani and Drummond (2017a) suggest a simple approach based on an iteratively reweighted least squares (IRLS) to reduce objectivity function that involves a mix of m-estimator regularization terms. This approach is very simple to understand but it is not suitable for most practical situations.

#### 4.3. Hybrid approaches

Faramarzi et al. (Faramarzi et al., 2013) address that the unified blind approach is degraded by blurring, line aliasing, and noisy effects. As a result, they develop a new method derived from alternating minimization (AM) algorithm to construct the HR image. They use the Huber-Markov random field (HMRF) regularization model to exploit the nature smoothing of the HR image. An edge-emphasizing smoothing technique is used to estimate the blurring through improving strong smooth edges against step edges with filtration of the poor components. The blur estimation is applied in the filter domain instead of the pixel domain. Also, the  $L_2$  norm is applied in the frequency domain to allow high-speed non-iterative optimization (Faramarzi et al., 2013). This method reduces artifacts such as blurring and noise in image

regions and preserves the edge details. However, this method uses HMRF as a prior that is not able to illustrate the complicated relationships among neighboring pixels. Villena et al. (Villena et al., 2013) present a new approach based on merging the sparse and non-sparse priors to reconstruct the SR image. They merge TV and  $L_1$ -norm as a sparse prior with simultaneous autoregressive (SAR) as a non-sparse prior (Villena et al., 2013). This method is able to preserve the edge details in the image and avoid the over-smoothing of inner image-regions. However, it can be difficult to identify the ideal contribution to each of priors before the combination and the weights have been identified empirically. Motivated by the drawbacks of the TV model, Maiseli et al. (Maiseli et al., 2015) suggest a new approach based on a spatial regularization called the variable-exponent nonlinear diffusion. This approach uses a convolution operation with the Gaussian filter and eliminates the presence of artifacts. However, it has a more blurring in edges. In order to solve the problems of the TV model, Zeng et al. (Zeng et al., 2015) propose an adaptive TV SR (ATV-SR) model that uses a modern edge indicator. The spatial, gray, and gradient similarities are designed at the same time of constructing a robust trilateral tensor to observe the local pattern of the image (Zeng et al., 2015). ATV-SR incorporates both  $L_1$  and  $L_2$  norms as a prior model to protect the edges and eliminate the noises respectively. However, the modern edge indicator is not able to distinguish between the edges and noises within the image regions.

Many details are missed within the image because of the ignoring of the sensor measurement and the model errors (Zhao et al., 2016). Zhao et al. (Zhao et al., 2016) use a reasonable observation model to incorporate the absent details. Also, they merge the adaptive non-local edge-preserving norm based on a Bayesian framework with non-local similarity to reduce the artifacts from the NLTV method. However, NLTV can't smooth the image well. Gao et al. (Gao et al., 2016) offer the SR approach by using multichannel blind deconvolution (MBD) to approximate the convolution kernel for LR images. Also, they use regularization term to generate the HR image (Gao et al., 2016). However, if an average low noise level is used, this approach can produce an excellent result. Ghosh et al. (Ghosh et al., 2016) propose a new approach dependent on an adaptive regularization. They employ both of Huber norm for maximum likelihood (ML) estimator and a directional Huber-Markov regularization (Ghosh et al., 2016). This approach is simple and robust but the computational resources are restricted.

Most techniques based on simplified estimated of image acquisition are adopted and demonstrated very little effectiveness in real-world applications (Köhler et al., 2016). Köhler et al. (Köhler et al., 2016) propose a new method based on a spatially adaptive Bayesian model and an iterative algorithm. They use a weighted Gaussian model and a weighted BTV to reflect on noises and utilize natural images sparsity respectively. But, they use a local selection of the sparsity parameter. El Mourabit et al. (El Mourabit et al., 2017) propose a new approach based on the variational framework. They use the benefits of Perona–Malik model in the smooth image-regions and use a non-linear tensor from Weickert filter. This approach is used to protect the edges and eliminate the noises. But, it cannot achieve a good balance between preserving the edges and suppressing the noise in the reconstructed HR image. Long et al. (Long et al., 2017) suggest a new approach called as envL1/TV model. This model based on the combination of the  $L_1$  and the  $L_2$  TV models. It is used to reduce the noises but it is not good to preserve the edges well.

## 5. Discussion and analysis

In general, the regularization approaches can be used as an attempt to stabilize the inversion process and compensate for the

absent information. Additionally, they are used to represent a prior of the image, remove artifacts, and bring the prior information. The prior information generates a stable solution, improves the convergence rate, and includes artificial constraints on the solution such as smoothness and edge-preserving.

From this review, many researchers implement various methods to produce a high-quality image based on the regularization approaches. These reviewed approaches are actually sensitive to deviations among the supposed and actual model. From the comparisons in Table 1, Tables 2, and 3, most of them are still suffering from an imbalance between the edges preservation and the noise suppression inside the reconstructed HR image. In which, if the noise is completely eliminated from the reconstructed HR image, this leads to smoothness in the edges such as methods in (Abedi and Kabir, 2016; Faramarzi et al., 2013; Köhler et al., 2016; Laghrib et al., 2016; Wang et al., 2017; Wang et al., 2014; Zeng and Lu, 2013; Zeng et al., 2015). On the other side, if the edges are preserved well in the reconstructed HR image, this leads to suffering the image from the noise such as methods in (El Mourabit et al., 2017; Gao et al., 2016; Jun-Bao et al., 2016; Maiseli et al., 2014; Maiseli et al., 2015; Nayak and Patra, 2016; Ren et al., 2013; Shen et al., 2016; Shi et al., 2015; Villena et al., 2014; Wang et al., 2017; Yadav et al., 2014; Yang et al., 2015; Yuan et al., 2012; Zeng and Yang, 2013; Zhao et al., 2015; Zhao et al., 2016).

In addition, the regularization parameter selection represents a challenge when treating the ill-posed inverse problems. The regularization parameter is often manually selected by testing a set of values and then selecting the optimum parameter that is compatible with the best results which it is examined through quantitative indicators or visual inspection such as methods in (Kim and Byun, 2013; Villena et al., 2013). Where, if large values are chosen for the regularization parameter, they generally result in a smoother solution that is used for noise suppression but leads to edge smoothing. However, if small values are chosen for the regularization parameter, the edge is preserved well but the noise cannot be completely suppressed. However, this process takes a long time and self-process. Therefore, there are a large number of different strategies that are developed to make adaptive estimates of the regularization parameter such as methods in (El Mourabit et al., 2017; Mohan, 2017; Panagiotopoulou, 2013; Villena et al., 2014; Zeng and Lu, 2013).

## 6. Conclusion

In this paper, we presented and examined the current regularization-based SR approaches for reconstructing the HR image for the last decade. The focus is given on examining the strengths and the weakness of the solutions designed for image reconstruction aiming at determine its effectiveness and quality in reconstructing HR images. In the last decade, researchers spend a lot of effort to reconstruct the HR image so that the noise is reduced and the edges are preserved. However, most of these approaches are still suffering from an imbalance between the edges preservation and the noise suppression inside the reconstructed HR image. Therefore, regularization-based approaches are still a challenging in SR image reconstruction.

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