

On surjective second order non-linear Markov operators and associated nonlinear integral equations

Positivity

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Abstract

It was known that orthogonality preserving property and surjectivity of nonlinear Markov operators, acting on finite dimensional simplexes, are equivalent. It turns out that these notions are no longer equivalent when such kind of operators are considered over on infinite dimensional spaces. In the present paper, we find necessary and sufficient condition to be equivalent of these notions, for the second order nonlinear Markov operators. To do this, we fully describe all surjective second order nonlinear Markov operators acting on infinite dimensional simplex. As an application of this result, we provided some sufficient conditions for the existence of positive solutions of nonlinear integral equations whose domain are not compact.

Keywords

Non-linear Markov operator Quadratic stochastic operator
Orthogonality preserving Surjective Integral equation

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