

Periodic p -adic Gibbs Measures of q -State Potts Model on Cayley Trees I: The Chaos Implies the Vastness of the Set of p -Adic Gibbs Measures

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Abstract

We study the set of p -adic Gibbs measures of the q -state Potts model on the Cayley tree of order three. We prove the vastness of the set of the periodic p -adic Gibbs measures for such model by showing the chaotic behavior of the corresponding Potts–Bethe mapping over \mathbb{Q} for the prime numbers $p \equiv 1 \pmod{3}$. In fact, for $0 < |a - 1| < |a|^2 < 1$

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and we could not conclude the vastness of the set of the periodic p -adic Gibbs measures. In a forthcoming paper with the same title, we will treat the case $0 < |q|_p \leq |\theta - 1|_p < 1$ for all prime numbers p .

Keywords

p -adic Potts model p -adic Gibbs measure Phase transition Chaos

Mathematics Subject Classification

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Notes

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