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Volume 24, Issue 1, 2018, Pages 131-150Ergodicity of  $p$ -majorizing quadratic stochastic operators (Article)Saburov, M.<sup>a,b</sup> [✉](#) [👤](#)<sup>a</sup>Department of Computational and Theoretical Sciences, International Islamic University Malaysia, Kuantan, Pahang, 25200, Malaysia<sup>b</sup>College of Engineering and Technology, American University of the Middle East, 250 St., Egaila, Kuwait

## Abstract

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A scrambling square stochastic matrix plays an important role in the theory of the classical Markov chain. One of the classical results states that a row-stochastic matrix is strongly ergodic if and only if its some power is a scrambling matrix. In this paper, we deal with the similar problem for a cubic stochastic matrix. We introduce a notion of  $p$ -majorizing quadratic stochastic operators and study the strong ergodicity of  $p$ -majorizing quadratic stochastic operators associated with scrambling, Sarymsakov, and Wolfowitz cubic stochastic matrices. © 2018 Polymat Ltd. All rights reserved.

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