

## Development of Mathematical Model for the Prediction of Microwave Signal Attenuation due to Duststorm

Zain Elabdin, Md. Rafiqul Islam, Othman O. Khalifa, Hany Essam A Raouf,  
Momoh Jimoh E Salami

*Wireless Communication and Signal Processing Research Group  
Kulliyah of Engineering, International Islamic University Malaysia  
Jalan Gombak, 53100 Kuala Lumpur  
E-mail: rafiq@iiu.edu.my*

### Abstract

*Signal attenuation caused by duststorm is one of the major problems in utilization of microwave bands for terrestrial and space communication especially at desert and semi desert area. This paper presented a mathematical model developed to predict the microwave signal attenuation due to duststorm. The proposed model enables the convenient calculation of the microwave signal path attenuation which relates visibility, frequency, particle size and complex permittivity of duststorm. The predicted values from the mathematical model are compared with the measured values observed in Sudan and Saudi Arabia shows relatively close agreement.*

### I. INTRODUCTION

Signal attenuation is an important parameter in telecommunications applications because of its importance in determining signal strength as a function of distance. A major cause of this phenomenon is atmospheric particles which can seriously limit the performance of telecommunication system especially at microwave level [1].

The effect of precipitating particles on signal attenuation received considerable attention especially at high frequencies [2]. As most of the work done in this area was carried out in Europe and USA, a significant amount of research has done in developing models to quantify the impact of rain and snow attenuation on communications systems operating in the microwave region [3,4,5,6,7].

In contrast of that, little work has been done to investigate the impact of duststorm on the same propagation paths [8, 9, 10]. Rapid development in telecommunication technology and increasing competition in the sector has extended service to new

locations and environments especially in Africa and Asia.

Duststorms occur in many parts of the world, especially in the African Sahara, Middle East and arid parts of Asia for a significant percentage of time as shown in Figure 1 and Table 1. During the storm, dust particles raised high enough above the earth's surface to lie within the path of microwave radio links causing a loss of signal energy and resulting in service interruption [11].

**TABLE 1. THE MEAN NUMBER OF HOURS WITH VISIBILITY REDUCED TO 5 KM OR LESS DUE TO DUST DURING THE 5 YEARS PERIOD 1970-74 IN 10 METEOROLOGICAL STATIONS IN AFRICAN SAHARA [12].**

No	Stations African	No. of Hrs /yrs	% per year
1	Bilma	2048	% 23.4
2	Atar	1493	% 17
3	Zinder	1450	% 16.6
4	Maine Soroa	1231	% 14
5	Agadez	1198	% 13.7
6	Adrar	731	% 8.3
7	Nguigmi	530	% 6
8	Tombouctou	473	% 5.4
9	Gao	453	% 5
10	Faya Largeau	431	% 4.9

### II. THEORY OF SINGLE-PARTICLE SCATTERING

Microwaves suffer absorption and scattering by the atmosphere especially at higher frequencies where the scattering effects become more severe. The knowledge of these scattering characteristics is essential to design reliable communications.

The basic theory underling mathematical model for attenuation is the theory for single particle scattering. The propagation effects may be modeled by volumetric integration of scattering by individual particles. When an object is illuminated by a wave, a part of the incident power is scattered out and another part is

absorbed by the object. The characteristics of these two phenomena, scattering and absorption, can be expressed most conveniently by assuming an incident plane wave.



Figure 1. Duststorm approaching Khartoum, Sudan, in April 30, 2007[13].

#### A Signal Attenuation due to Duststorm

The methods to predict the signal attenuation due to rain effects can be applied for duststorm because the general model for scattering in sand and dust particle populations is essentially the same as that for a population of hydrometeors; both of them are discrete random medium [14].

The signal attenuation due to duststorm is estimated generally by solving the forward scattering amplitude function of a single particle [15]. The solution may be carried out using the Rayleigh approximation or Mie solutions. The method depends largely on the wave number and particle radius [16].

The attenuation of electromagnetic radiation ( $A_T$ ) over a path of extent  $L$  through precipitating particles may be written as [5]:

$$A_T (dB) = \int_0^L A_p dx \quad (1)$$

Where  $A_p$  (dB/km) is the specific attenuation characterizing the precipitating particles.

Several authors used the following expression to calculate the attenuation due to rain [3, 5, 21]:

$$A_p = 4.343 \times 10^3 \int_{a_{\min}}^{a_{\max}} \sigma_t(a) \cdot N(a) da \quad [dB/km]. \quad (2)$$

Where  $N(a)da$  are particles number per unit volume of air with particles radius between  $r$  and  $r+dr$ ,  $\sigma_t$  is the total attenuation cross section efficiency factors of particle of radius  $a$ .

This can be expressed here for duststorm as:

$$A_T (dB) = \int_0^L A_d dx \quad (3)$$

Where  $A_d$  (dB/km) is the specific attenuation characterizing the duststorm which can be expression as:

$$A_d = 4.343 \times 10^3 \int_{a_{\min}}^{a_{\max}} \sigma_t(a) \cdot N(a) da \quad [dB/km]. \quad (4)$$

Where  $N(a)da$  are particles number per unit volume of air with dust particles radius between  $r$  and  $r+dr$ ,  $\sigma_t$  is the total attenuation cross section efficiency factors of dust particle of radius  $a$ .

#### B Dependence on Visibility

To calculate the attenuation by the above equations requires data for the number of particles of dust  $N$ , which is difficult to measure accurately.

On the other hand, statistical information on duststorm visibility is available. Goldhirsh [15] express the visibility in term of the particle density as:

$$V (km) = \frac{5.5 \times 10^{-4}}{N a_e^2} \quad (5)$$

Where the units of  $N$  are particles /m<sup>3</sup> and  $a_e$  is the equivalent particle radius in meters.

By solving  $N$  in the above formula we can express the particle density in term of the visibility and the radius as:

$$N = \frac{5.5 \times 10^{-4}}{V a_e^2} \quad (6)$$

### III. ANALYTICAL MODELS FOR SCATTERING

Two models give an analytical solution for the scattering of a plane wave by a spherical particle, Rayleigh approximation and Mie solution. Rayleigh approximation loses its reliability as size of the dust particles approaches the operating wavelength or vice-versa, because Rayleigh Formula is based on the assumption that  $a \ll \lambda$ , where ( $a$ ) is the dust particle radius and  $\lambda$  is the operating wavelength in meters. This is the reasons for difficulty to use it for frequencies higher than 37 GHz [17].

In contrast to Rayleigh scattering Mie solutions to scattering embraces all possible ratios of diameter to wavelength and do not depend upon any such limitation and can be utilized to predict attenuation in microwave wave band with high reliability especially at higher frequencies.

Chu [14] developed a formula to predict signal attenuation caused by dust particles using Rayleigh approximation, so a new formula can predict signal attenuation due to dust particles at higher frequencies is highly recommended for new telecommunication application.

Collin [18] presented an expression of the total cross-section efficiency factors ( $\sigma_t$ ) using Mie solutions as:

$$\sigma_t = \frac{\lambda^2}{2\pi} (ka)^3 \left( c_1 + c_2 (ka)^2 + c_3 (ka)^3 \right) \quad (7)$$

Where  $C_1$ ,  $C_2$  and  $C_3$  are constants whose values depend on real ( $\epsilon'$ ) and imaginary part ( $\epsilon''$ ) of the dielectric constant of the particles as:

$$C_1 = \frac{6\epsilon''}{(\epsilon' + 2)^2 + \epsilon''^2} \quad (8)$$

$$C_2 = \epsilon'' \left\{ \frac{6}{5} \frac{7\epsilon'^2 + 7\epsilon''^2 + 4\epsilon' - 20}{[(\epsilon' + 2)^2 + \epsilon''^2]^2} + \frac{1}{15} + \frac{5}{3[(2\epsilon' + 3)^2 + 4\epsilon''^2]} \right\} \quad (9)$$

$$C_3 = \frac{4}{3} \left\{ \frac{(\epsilon' - 1)^2 (\epsilon' + 2) + [2(\epsilon' - 1)(\epsilon' + 2) - 9] + \epsilon''^4}{[(\epsilon' + 2)^2 + \epsilon''^2]^2} \right\} \quad (10)$$

#### IV. THE PREDICTED MODEL

By substituting Collin expression (Eq. 7) for the total cross-section efficiency factors ( $\sigma_t$ ) and the particle density expression in (Eq. 6),  $A_d$  (dB/km) may alternately be expressed as:

$$A_d = 4343 \int_{a_{\min}}^{a_{\max}} \left[ \frac{\lambda^2}{2\pi} (ka)^3 \left( c_1 + c_2 (ka)^2 + c_3 (ka)^3 \right) \cdot \frac{5.5 \times 10^{-4}}{Va_e^2} \right] da \quad (11)$$

A further approximation can be made in these calculations, assuming that every dust particle in a real storm may be replaced by an equivalent particle ( $a_e$ ) whose radius is the mean radius for all dust particles. By this assumption the value of equivalent particle radius ( $a_e$ ) consider as constant value and Eq. 11 may alternately be expressed as algebraic expression:

$$A_d = 4343 \times \left[ \frac{\lambda^2}{2\pi} (ka_e)^3 \left( c_1 + c_2 (ka_e)^2 + c_3 (ka_e)^3 \right) \cdot \frac{5.5 \times 10^{-4}}{Va_e^2} \right] \quad (12)$$

By substituting  $k = 2\pi / \lambda$  in the above expression became:

$$A_d = 4343 \cdot \left[ \frac{\lambda^2}{2\pi} \left( \frac{2\pi a}{\lambda} \right)^3 \left( c_1 + c_2 \left( \frac{2\pi a}{\lambda} \right)^2 + c_3 \left( \frac{2\pi a}{\lambda} \right)^3 \right) \cdot \frac{5.5 \times 10^{-4}}{Va_e^2} \right] \quad (13)$$

By substituting Eq. 8, 9 and 10 in Eq. 13 and after several algebraic calculations we can alternately express the specific attenuation due to duststorm  $A_d$  (dB/km) as:

$$A_d = \frac{a_e f}{V} \left( x + y a_e^2 f^2 + z a_e^3 f^3 \right) \text{ [dB/km]}. \quad (14)$$

Where  $a_e$  is the equivalent particle radius in meters,  $V$  is the visibility in kilometer and  $f$  is the frequency in GHz and  $x$ ,  $y$  and  $z$  are constants whose values depend on real ( $\epsilon'$ ) and imaginary part ( $\epsilon''$ ) of the dielectric constant of the particles as:

$$x = \frac{1886 \cdot \epsilon''}{(\epsilon' + 2)^2 + \epsilon''^2} \quad (15)$$

$$y = 137 \times 10^3 \cdot \epsilon'' \left\{ \frac{6}{5} \frac{7\epsilon'^2 + 7\epsilon''^2 + 4\epsilon' - 20}{[(\epsilon' + 2)^2 + \epsilon''^2]^2} + \frac{1}{15} + \frac{5}{3[(2\epsilon' + 3)^2 + 4\epsilon''^2]} \right\} \quad (16)$$

$$z = 379 \times 10^4 \left\{ \frac{(\epsilon' - 1)^2 (\epsilon' + 2) + [2(\epsilon' - 1)(\epsilon' + 2) - 9] + \epsilon''^4}{[(\epsilon' + 2)^2 + \epsilon''^2]^2} \right\} \quad (17)$$

#### V. DETERMINATION OF ATTENUATION AT DIFFERENT BANDS

A further simplification can be made in this general formula to more effective and become easier for using by microwave network designer and engineers. This can be achieved by subdivided the general formula into several formulas each one can deal with specific microwave band.

TABLE 2. LISTING OF SPECIFIC FORMULAS FOR PREDICTING SIGNAL ATTENUATION DUE TO DUSTSTORM AT VARIOUS FREQUENCY BANDS

Frequency Band(GHz)	$\epsilon', \epsilon''$	Specific Formulas [dB/km]
S-Band (2 – 4)	4.56, 0.25 [19]	$A_{dS} = 11 \cdot \frac{a_e f}{V} + 433 \cdot \frac{a_e^3 f^3}{V} + 2.46 \times 10^5 \frac{a_e^4 f^4}{V}$
X-Band (8 – 12)	5.73, 0.42[19]	$A_{dX} = 13 \cdot \frac{a_e f}{V} + 702 \cdot \frac{a_e^3 f^3}{V} + 2.5 \times 10^5 \frac{a_e^4 f^4}{V}$
Ku-Band (12–18)	5.5, 1.3 [20]	$A_{dku} = 42 \cdot \frac{a_e f}{V} + 2806 \cdot \frac{a_e^3 f^3}{V} + 2.4 \times 10^5 \frac{a_e^4 f^4}{V}$
K-Band (18–26.5)	5.1, 1.4 [20]	$A_{dk} = 50 \cdot \frac{a_e f}{V} + 3368 \cdot \frac{a_e^3 f^3}{V} + 2.38 \times 10^5 \frac{a_e^4 f^4}{V}$
Ka-Band (26.5–40)	4, 1.33 [20]	$A_{dka} = 66 \cdot \frac{a_e f}{V} + 3712 \cdot \frac{a_e^3 f^3}{V} + 2.2 \times 10^5 \frac{a_e^4 f^4}{V}$
W-Band (56–100)	3.5, 1.64 [20]	$A_{dW} = 94 \cdot \frac{a_e f}{V} + 6232 \cdot \frac{a_e^3 f^3}{V} + 2.1 \times 10^5 \frac{a_e^4 f^4}{V}$

By this technique we can determinate especial formula for each band have ability to calculate the signal attenuation due to dust particles at every microwave bands.

It is clear from the general formula that the signal attenuation due to dust particles depends on; visibility, frequency, dusts size and dielectric constant. The dielectric constant values of dust particles are important components in the determination of attenuation. A number of investigators have measured the dielectric constant of sand and dust samples.

A further approximation applied in the new subdivided formulas, assuming that a single value of the dielectric constant can be fitted for all frequencies range in specific microwave band.

Table 1 gives a list of specific formulas for predicting signal attenuation due to duststorm ( $A_d$ ) at various frequency bands. In Figures 2, 3, 4, 5, 6 and 7 plotted the signal attenuation (dB/km) versus frequency for four different values of visibility at particle radius equal to 50 $\mu$ m at S, X, Ku, K, Ka and W- bands respectively.

## VI. COMPARISON WITH MEASURED DATA

Measurement in Khartoum-Sudan on September 1, 2007 shows attenuation 0.67 dB per kilometer observed by the author on 15 km link at 13 GHz (Ku band). The duststorm produced a visibility smaller than 5 m. The predicted value found by the Ku-band specific attenuation formula is 0.55 dB per kilometer for the same visibility assuming dust particle radius equal to 50 $\mu$ m.

Several duststorms measured by Alhaider and Ali during 1987 in Saudi Arabia on 14 km microwave link at 40 GHz (Ka-band) [22]. Table 3 gives the result of

measurements and calculations for different value of visibility at 40 GHz. The predicted values found by the mathematical attenuation model using the Ka-band specific attenuation formula. Comparison between measured ( $A_m$ ) and calculated ( $A_c$ ) values appear in Figure 8 shows close agreement. Dust particle radius equal to 30 $\mu$ m consider in the calculation according to measurements in Riyadh area [23].

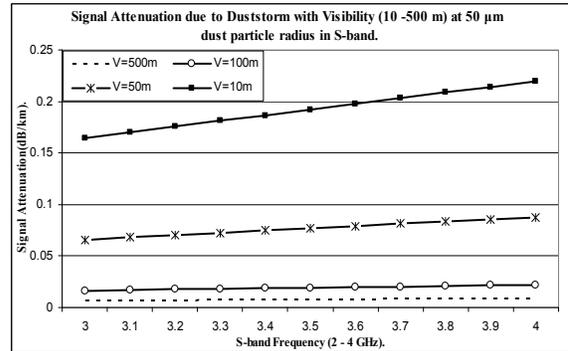


Figure 2. Signal attenuation (dB/km) Vs frequency at S-band

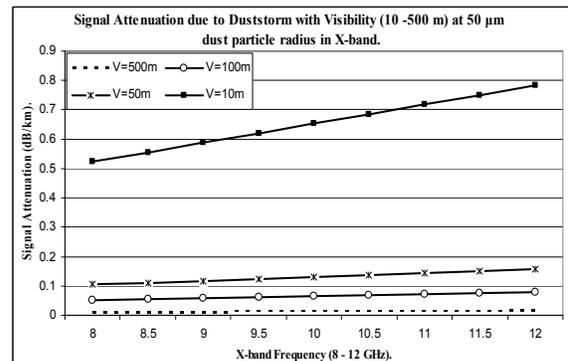


Figure 3. Signal attenuation (dB/km) Vs frequency at X-band

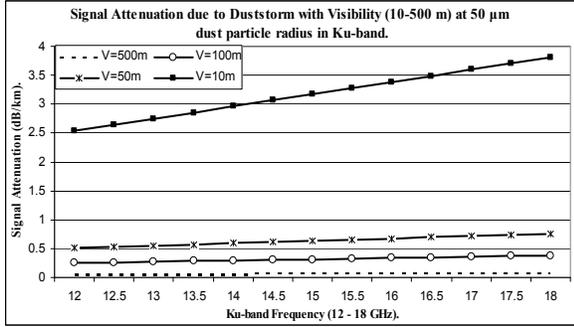


Figure 4. Signal attenuation (dB/km) Vs frequency at Ku-band

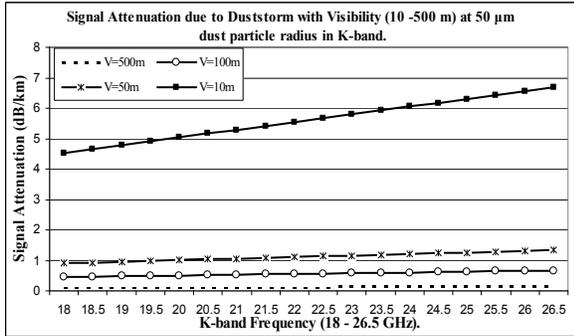


Figure 5. Signal attenuation (dB/km) Vs frequency at K-band

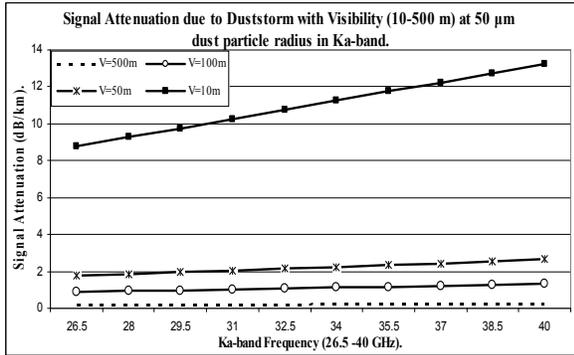


Figure 6. Signal attenuation (dB/km) Vs frequency at Ka-band

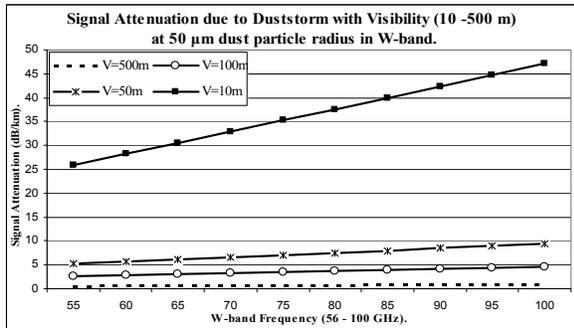


Figure 7. Signal attenuation (dB/km) Vs frequency at W-band

Data measured in Sudan and Saudi Arabia at different microwave links with different frequencies for several duststorms show close agreement to predicted values found by the mathematical attenuation model.

TABLE 3: RESULT OF MEASUREMENTS AND CALCULATIONS OF SIGNAL ATTENUATION DUE TO DUSTSTORM FOR DIFFERENT VALUE OF VISIBILITY AT 40 GHz

$A_c$ (dB/km)	$A_m$ (dB/km)	$V$ (km)
0.13	0.14	0.625
0.064	0.1	1.25
0.06	0.071	1.42
0.021	0.05	3.75
0.014	0.036	5.56

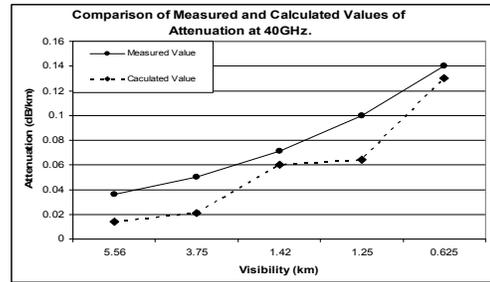


Figure 8. Comparison between Measured and Calculated values of Attenuation at 40 GHz

## VII. CONCLUSIONS

A model has been developed to deal with all possible ratios of dust particles diameter to wavelength and predict attenuation in microwave wave band with high reliability. In this model the term visibility ( $V$ ) is applied to denote the degree of duststorm density instate of total number of dust particles ( $N$ ).

The calculations showed that the attenuation is varying from 13 to 0.2dB/km at 40 GHz for dust particle radius equal to 50 $\mu$ m as the visibility between 10 to 500m. While at higher frequency of the band (100 GHz) the attenuation is varying from 47 to 2dB/km for dust particle radius equal to 50 $\mu$ m as the visibility between 10 to 500m.

These results can help expand the use of higher frequencies offering more bandwidth for new application especially at areas affected by duststorms. However additional comparisons between modeled and measured attenuation values are required especially at higher frequencies to reinforce the validity of the model.

## REFERENCES

- [1] R. K. Crane, Propagation Phenomena Affecting Satellite Communication Systems Operating in the Centimeter and Millimeter Wavelength Bands, Proc. IEEE, 59(2), 173-88, 1971.
- [2] K. Williams and R. Greeley, Radar Attenuation by Sand: Laboratory Measurements of Radar Transmission, IEEE Transactions on Geoscience and Remote Sensing, vol. 39, No. 11, Nov 2001.
- [3] R. L. Olsen et al., The  $\alpha$ B relation in the calculation of rain attenuation, IEEE Trans. On Antennas and Propagation, vol. AP-26, no. 2, Mar. 1978.
- [4] L. Li, P. Kooi, M. Leong, T. Yeo and M. Gao, Microwave Attenuation by Realistically Distorted Raindrops: Part I-Theory, IEEE Transactions on Antennas and Propagation, v. 43, No. 8, Aug 1995.
- [5] R. R. Rogers, Statistical Rainstorm Models: Their Theoretical and Physical Foundations, IEEE Trans. On Antennas and Propagation, July 1976.
- [6] R. K. Crane, Prediction of Attenuation by Rain, IEEE Trans. Commun., 29, 1717-33.1980.
- [7] Y.M. Jain and P.A. Watson, Attenuation in Melting Snow on Microwave and Millimeter Wave Terrestrial Radio Link. Electron. Lett., 21(2), 68-9 (1985).
- [8] S K Srivastava and B R Vishwakarma, Cross-polarization and Attenuation of Microwave/Millimeter Wave Propagation in Storm Layer Containing Sand, Silt and Clay as Dust Constituents. IE(I) Journal-ET V.84, Jul 2003.
- [9] Y. Ruike, W. Zhensen, and Y. Jinguang, The Study Of MMW and MW Attenuation Considering Multiple Scattering Effect in Sand and Dust Storms at Slant Paths, International Journal of Infrared and Millimeter Waves, Vol. 24, No. 8, Aug 2003.
- [10] Zhong Yu, Zongren Peng, Peng Liu, and Xinqiao Wu, The Influence of Charged Sand Particles on the External Insulation Performance of Composite Insulators in Sandstorm Condition. IEEE 8th International Conference on properties and applications of Dielectric Materials, June 2006.
- [11] Yingxia Xu, Jiying Huang, and Yingle Li, The Effect of Attenuation Induced By Sand and Dust Storms on Ka-Band Electromagnetic Wave Propagation a long Earth-Space Paths. International Journal of Infrared and Millimeter Waves, Vol. 23, No. 11, November 2002.
- [12] G. N'tchayi Mbourou, J. J. Bertrand and S. E. Nicholson, The Diurnal and Seasonal Cycles of Wind-Borne Dust over Africa North of the Equator, Journal Of Applied Meteorology, vol. 36, pp 868-882, 1997.
- [13] [http://www.dailymail.co.uk/pages/live/articles/news/big\\_cloud\\_of\\_dust\\_hangs\\_over\\_Khartoum\\_the\\_Daily\\_Mail.html](http://www.dailymail.co.uk/pages/live/articles/news/big_cloud_of_dust_hangs_over_Khartoum_the_Daily_Mail.html)
- [14] T. S. Chu, "Effects of sandstorms on microwave propagation." Bell Sys. Tech. J. v. 58, no. 2, Feb. 1979.
- [15] J. Goldhirsh, "Attenuation and Backscatter from a Derived Two-Dimensional Duststorm Model" IEEE Trans. Antennas Propagation, v. 49, no. 12, pp. 1703-1711, 2001.
- [16] A. Ishimaru, Wave Propagation and Scattering in random media (IEEE, N.Y. 1997).
- [17] B.R. Vishvakarma and C.S. Rai, Limitations of Rayleigh Scattering in The Prediction of Millimeter Wave Attenuation in Sand and Dust Storms, Geoscience and Remote Sensing Symposium, IEEE Intr., 1993.
- [18] Collin R., E., Antenna & radiowave propagation (McGraw-hill International Edition, Singapore, 1985).
- [19] S. I. Ghobrial and S. M. Sharief, Microwave attenuation and cross polarization in dust storms, IEEE Trans. Antennas Propagat., v. AP-35, pp. 418-425, Apr. 1987.
- [20] Y. Ruike, W. Zhensen and Y. Jinguang, The study of MMW and MW attenuation considering multiple scattering effect in sand and dust storms at slant paths, International Journal of Infrared and Millimeter Waves, Vol. 24, No. 8, August 2003.
- [21] Oguchi, Electromagnetic Wave propagation and Scattering in hydrometeors, Proceedings of The IEEE, vol.71, No.9, Sep 1983.
- [22] Alhaider, M.A. Ali, A.A., Experimental studies on Millimeterwave and Infrared propagation in Arid Land: The Effect of Sand Storms, Sixth International Conference on Antennas and Propagation ICAP, 1989.
- [23] Ahmed et al., Airborne Dust size Analysis for Tropospheric Propagation of Millimetricwaves into Duststorms, IEEE Transactions on Geoscience and Remote Sensing, vol. GE-25, No. 5, Sep 1987.