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OSCILLATING SUPERSONIC DELTA WINGS WITH CURVED LEADING EDGES

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Abstract. In the present study Supersonic similitude has been used to obtain stability derivatives in pitch and roll of a delta wing for the attached shock case. A strip theory is used in which strips at different span-wise locations are independent. This combines with the similitude to give a piston theory. The present theory is valid only for attached shock case. Effects of wave reflection and viscosity have not been taken into account. Some of the results have been compared with those of Hui et al (1982), Ghosh (1984), and Liu and Hui (1977). Results have been obtained for supersonic flow of perfect gas over a wide range of Mach numbers, incidences and sweep angles. A good agreement is obtained with Hui et al in some special cases.

Keywords and Phrases : Supersonic , Delta wings , Curved leading edges , Stability derivatives
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NOMENCLATURE

A	$\gamma(\gamma+1)$
A_F, A_H	amplitude of full & half sine wave
AR	aspect ratio
B	$[4/(\gamma+1)]$ 2
C	chord length
$C_{m\alpha}$, C_{mq}	stiffness &damping derivative in pitch
C_{lp}	rolling moment derivative due to rate of roll
$C_{m\theta}$	Stiffness derivative in pitch
$C_{m\dot{\theta}}$	Damping derivative in pitch
L	rolling moment
M_∞, U_∞	Free stream Mach number and velocity
M_2	Mach number behind the shock
Mp	Piston Mach number
M_s	Shock Mach number
S_1^1	Similarity parameters in supersonic flow

a_∞	free stream sound velocity
b	Semi span
h	Non dimensional pivot position, x_0/c
k	π/c
m	Pitching momentum
P	Pressure on the wing surface
P_∞	Free stream pressure
q	Rate of pitch
t	time in second
U, V	velocity components in X, Y direction
X, Y, Z	body fixed reference system
X_0	pivot position for pitching oscillation
α	Incidence angle
β	Shock wave angle
α_o	Mean incident for an oscillating wing
γ	Specific heat ratio
δ	Inclination of characteristic lines
ε	Sweep angle
θ	Half wedge angle
ρ_∞	free stream density

1 Introductory Remarks

Ghosh (1981) has given a unified supersonic similitude for a wedge which establishes that the similitudinal surface, in which the motion is independent of other such surfaces, is normal to the bow shock, rather than to the wedge surface as in Ghosh (1971). For a quasi-wedge or an oscillating wedge, the bow shock makes a small departure from a certain pre-determined wedge shape; the similitudinal surface is shown to be orthogonal to the latter. Ghosh's (1981) analysis is reproduced here.

2 Analysis

Fig. 1 shows that upper half of a steady wedge with attached bow shock in rectilinear flight from right to left in stationary air, at time t. Dimensional analysis indicates that the flow is conical in nature, i.e., at a given instant $\frac{\partial}{\partial r} = 0$ where r is the distance along a ray from the apex. Hence the bow shock must coincide with array. The space-fixed co-ordinate system (x, y) is so chosen that the x-axis coincides with the bow shock at time t=0. Conicality of the flow implies that the instantaneous streamlines have the same slope where

Steady Wedge:

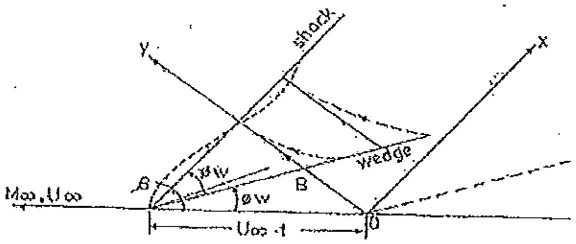

Fig. 1. shows the wedge at time t

they intersect a ray from the apex. Since the shock sets the fluid particles in motion normal to itself, the instantaneous streamlines intersect the shock at right angles. The dashed lines in fig. 1 are probable streamline shapes. We tentatively assume that the streamlines are straight shown by firm lines in Fig.1, if this leads to $\frac{\partial}{\partial r} = 0$ then it is a solution. Consider the plane flow on stream surface x=0. At time t the shock location on x=0 is

$$y_s = U_\infty t \sin\beta \tag{1}$$

And the body location can be shown to be

$$Y_b = OB = U_\infty t \sin\theta_w / \cos\theta_w \tag{2}$$

Since the flow in plane x=0 is independent of the flow in a neighboring parallel plane, it can be taken as a piston driven fluid motion where the piston Mach number$M_p = M_\infty \sin\theta_w / \cos(\beta - \theta_w)$ and the shock Mach number$M_s = M_\infty \sin\beta$. Since $M_p$is independent of t, pressure remains constant in this 1D space.

Therefore, $\partial p/\partial y = 0$. Since the streamlines are straight there is no centrifugal force; thus$\partial p/\partial x = 0$. Hence, $\partial p/\partial r = 0$. Thus the wedge flow is exactly equivalent to 1D piston motion normal to the shock. It can be shown that the relation between M_S&$M_P$yields the well known oblique shock relation giving the shock relation giving the shock angle in terms ofθ_w.

3 Quasi-wedge or Oscillating wedge:

Fig.1 shows the probable shape of the bow shock in dotted when the wedge is either oscillating or replaced by a quasi-wedge. The slope of the curved shock with x-axis remains small, say of orderϕ. Let the Mach number behind the shock, in body-fixed coordinate, beM_2. The characteristics make an angle $\delta = (\sin^{-1} 1/M_2 - \phi_W)$ with the x-axis. It can be shown that this angle remains small for fairly large values of θ_W even for moderate Mach numbers. For example for $M_\infty = 2, \theta_W = 15°$, we have $\delta = 13°$.

Again for M_∞=3, $\theta_W = 20°$ we get$\delta = 12.5°$. we stipulate (see Fig.1) that $\delta \leq 0.3$, and then $\phi \& \delta$ are of same order. since the gradient is normal to the characteristics we have,

$$\frac{\partial}{\partial x} = 0(\phi.\frac{\partial}{\partial y}) \tag{3}$$

Also the net perturbation introduced by the shock and Mach waves will chiefly be in the y-direction. Thus

$$u = 0(\phi.v) \tag{4}$$

Where u, v are velocity components in x, y directions. Eqs. (3) & (4) suggest transformations

$$u^1 = \phi^{-1}.u \& x^1 = \phi.x. \tag{5}$$

we apply these transformations to the equation of continuity to get

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y} = -\phi^2 \frac{\partial(\rho u^1)}{\partial x^1}.$$

Similarly, applying these transformations to the rest of the equations of motion and boundary conditions and neglecting terms of $0(\phi^2)$, we get equivalence with a 1D piston motion in y-direction and unified supersonic/hypersonic similitude.

Singh (1982) has utilized Ghosh's (1981) unified supersonic/hypersonic similitude for the calculation of unsteady moment derivatives in pitch for biconvex airfoils. Ghosh (1983) has extended the unified similitude to the case of a cone in supersonic flow.

4 Piston theory

In the present analysis Ghosh's (1981) unified supersonic/hypersonic similitude has been used in combination with a strip theory for a supersonic delta wing whose leading edge is curved. A thin strip of the wing, parallel to the centerline, can be considered independent of the Z dimension when the velocity component along the Z direction is small. This has been discussed by Ghosh's(1984). The strip theory combined with Ghosh's large incidence similitude leads to the piston analogy and pressure P on the surface can be directly related to equivalent piston mach no. M_P. In this case both M_P and flow deflections are permitted to be large. Hence light hill piston theory cannot be used but Ghosh's piston theory will be applicable.

$$\frac{P}{P_\infty} = 1 + AM_P^2 + AM_P(B + M_P^2)^{\frac{1}{2}} \tag{6}$$

Since strips at different span wise location are assumed independent of each other, the strip can be considered as a flat plate at an angle of attack. The angle of incidence is same as that of the wing. Angleϕ is the angle between the shock and the strip. A piston theory which has been used in eqn.(6) has been extended to supersonic flow. The Expression is given below

$$\frac{p}{p_\infty} = 1 + A(\frac{M_p}{\cos\phi})^2 + A(\frac{M_p}{\cos\phi})(B + (\frac{M_p}{\cos\phi})^2)^{\frac{1}{2}} \tag{7}$$

Where$p_\infty$is free stream pressure,$A = \frac{(\gamma+1)}{4}$, $B = (4/(\gamma+1)^2)$, γ is the specific heat ratio and M_p =the local piston Mach number normal to the wing surface.

5 Pitching moment derivatives

Let the mean incidence be $\alpha_0$for the wing oscillating in pitch with small frequency and amplitude about an axisx_0. The piston velocity and hence pressure on the windward surface remains constant on a span wise strip of length 2z at x. The pressure on the lee surface is assumed Zero. Therefore the nose up moment is

$$m = -2\int_0^c p.z(x - x_0)dx \tag{8}$$

The stiffness derivative is non-dimensionalized by dividing with the product of dynamic pressure, wing area and chord length.

$$-C_{m\alpha} = \frac{2}{\rho\infty U\infty^2 C^3\{cot\epsilon - \frac{4A_H}{\pi}\}} \left(\frac{-\partial m}{\partial \alpha}\right)_{\alpha=\alpha_0, q=0} \tag{9}$$

From Eq. 8 and 9

$$-C_{m\alpha} = \frac{4Ap_\infty M_\infty \cos\alpha_o F(S_1^1)}{\cos\phi \rho_\infty U_\infty^2 c^3\{cot\epsilon - \frac{4A_H}{\pi}\}} \int_q^c (x cot\epsilon - a_F \sin 2kx - a_H \sin kx)(x - x_0)dx \tag{10}$$

Solving the above equation we get

$$-C_{m\alpha} = \frac{\sin\alpha_o \cos\alpha_o f(S_1^1)}{\cos^2\phi\{cot\epsilon - \frac{4A_H}{\pi}\}}[(\frac{2}{3} - h)\cot\epsilon + \frac{1}{\pi}\{\frac{A_F}{2} + A_H(2h-1)\}] \tag{11}$$

Where,

$$f(S_1^1) = \frac{(\gamma+1)}{2s_1^1}F(S_1^1) = \frac{(\gamma+1)}{2s_1^1}[2s_1^1 + (B + 2s_1^{1^2})/(B + s_1^{1^2})^{\frac{1}{2}}] \tag{12}$$

where

$$S_1^1 = \frac{M_\infty \sin\alpha_o}{\cos\phi} \tag{13}$$

6 Damping derivative

The damping derivative is non-dimensionalised by dividing with the product of dynamic pressure, wing area, chord length and characteristic time factor $\left(\frac{c}{U_\infty}\right)$

$$-C_{m_q} = \frac{2}{\rho_\infty U_\infty c^4\{cot\epsilon - \frac{4A_H}{\pi}\}}\left(-\frac{\partial m}{\partial q}\right)_{\alpha=\alpha_o q=0} \tag{14}$$

Since m is given by integration to find $(\frac{\partial m}{\partial q})$ differentiation within the integration is necessary.

$$\left(\frac{\partial p}{\partial q}\right)_{\alpha=\alpha_o, q=o} = \frac{Ap_\infty(x - x_0)}{a_\infty \cos\phi}F(S_1^1) \tag{15}$$

Solving we get

$$-C_{mq} = \frac{\sin\alpha_o f(S_1^1)}{\cos^2\phi\{cot\epsilon - \frac{4A_H}{\pi}\}}\left[(h^2 - \frac{4}{3}h + \frac{1}{2})\cot\epsilon - \frac{1}{\pi}\{(2h-1)A_F + 2A_H(2h^2 - 2h - \frac{4}{\pi^2} + 1\}\right] \tag{16}$$

7 Rolling Moment derivative due to rate of roll

Let the rate of roll be p and rolling moment be L, defined according to the right hand system of reference.

$$L = 2\int_0^c \left(\int_0^{Z=f(x)} p.zdz \right) dx \tag{17}$$

The piston Mach number is given by

$$M_p = M_\infty \sin\alpha - \frac{z}{a_\infty}\bar{p} \tag{18}$$

The roll damping derivative is non-dimensionalzed by dividing with the product of dynamic pressure, wing area, and span and characteristic time factor $\frac{C}{U_\infty}$

$$-C_{lP} = \frac{1}{\rho_\infty U_\infty C^3 b\{cot\epsilon - \frac{4A_H}{\pi}\}} \left(\frac{-\partial L}{\partial p}\right)_{\substack{\alpha=\alpha_0 \\ \bar{p}=0}} \tag{19}$$

Where span =2b
Solving the above equation we get

$$-C_{lP} = \frac{\sin\alpha_o f(S_1^1)}{(\cos^2\phi)(cot^2\epsilon - \frac{4A_H cot\epsilon}{\pi})}$$

$$\left[\frac{\cot^3\epsilon}{12} + cot^2\epsilon\left(\frac{A_F}{2\pi} - \frac{A_H}{\pi^3}\right)(\pi^2-4) + \frac{1}{4}cot\epsilon(A_F^2 + A_H^2) - \frac{4}{9\pi}A_H^3 - \frac{16A_F A_H}{9\pi^2} - \frac{16A_F^2 A_H}{15}\right] \tag{20}$$

where

$$(S_1^1) = \frac{M_\infty \sin\alpha_0}{\cos\phi}$$

8 RESULTS AND DISCUSSIONS

Figure 2 below shows the region of validity for the present theory in Supersonic flow regimes.

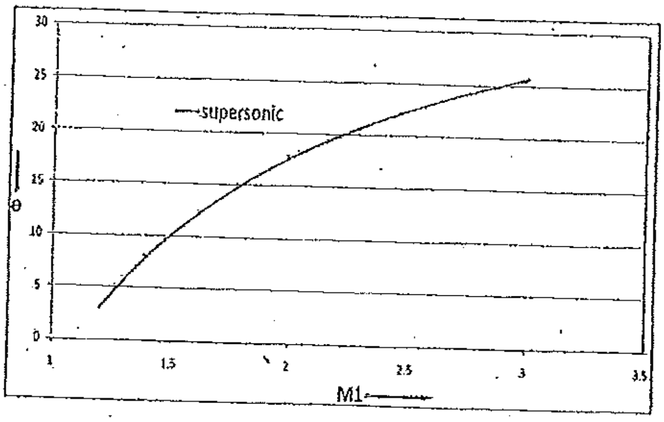

Fig. 2. Boundary of Validity for the Unified Supersonic Similitude

<u>Constant values:</u> Mach no, M=3 Incident angle, θ_0=10 Specific ratio, γ=1.4

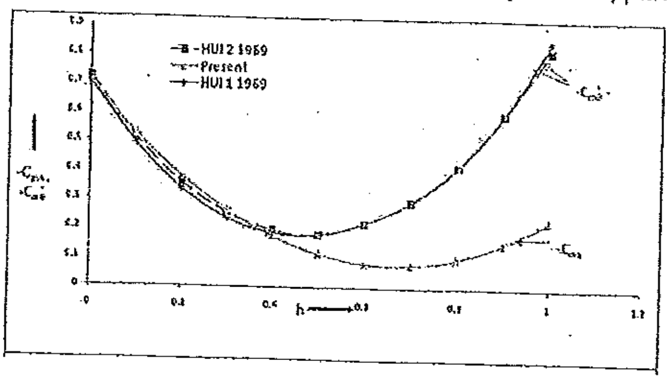

Fig. 3. Variation of Damping Derivative of a Wing with Pivot position

Constant values: Mach no, $M=2.47$, $\alpha_0=6°51'$, Specific ratio, $\gamma=1.4$

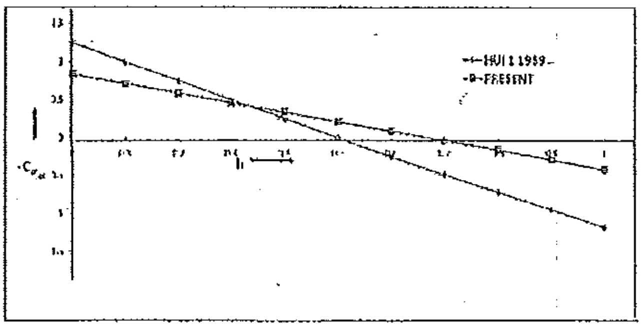

Fig. 4. Variation of Stiffness Derivative of a Wing with Pivot position

Figure 3 and 4 shows the variation of stiffness and damping derivatives with pivot position in supersonic flows. Stiffness and damping derivatives in pitch calculated by the present theory have been compared with Liu and Hui (1969). The stiffness derivative shows good agreement.

Figure 5 and 6 shows the variation of stiffness and damping derivatives with pivot position in supersonic flows. Stiffness and damping derivatives in pitch calculated by the present theory have been compared with Liu and Hui (1969). The stiffness derivative shows good agreement. The difference in the damping derivative is attributed to the present theory being a quasi-steady one whereas Liu and Hui (1969) give an unsteady theory which predicts $C_{m\theta}$

Constant values: Mach = 1.75 Incident angle, α_0=6°51, Specific ratio,γ=1.4

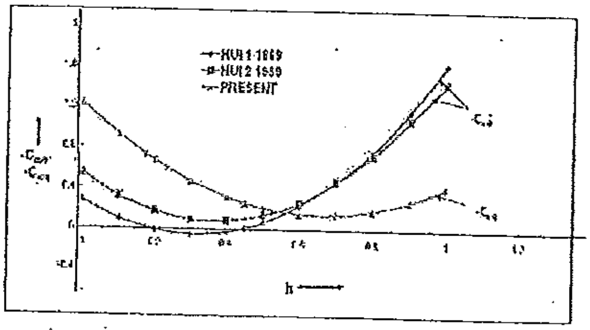

Fig. 5. Comparison of damping Derivative of a Wing

const. values: Mach no, M=1.7, Specific ratio, γ=1.4

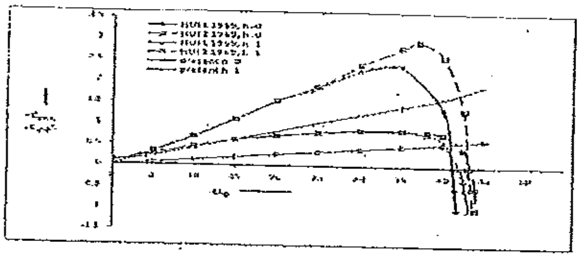

Fig. 6. Comparison of Stability Derivative of a wing in supersonic flow

Constant values: Mach no. M=4, Incident angle, α_0=15°, Specific ratio, γ=1.4

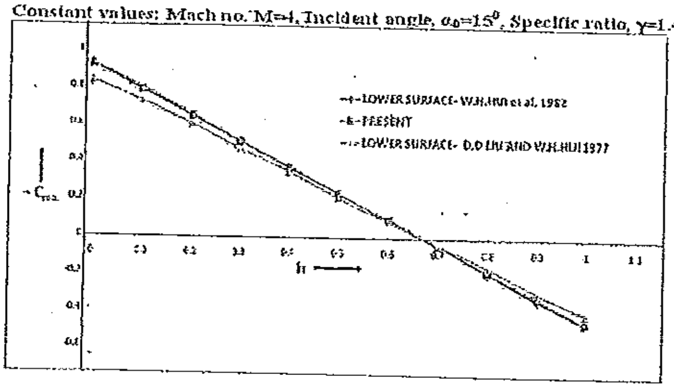

Fig. 7. Comparison of Stiffness Derivative with Theory of Liu and Hui for Triangular Wing

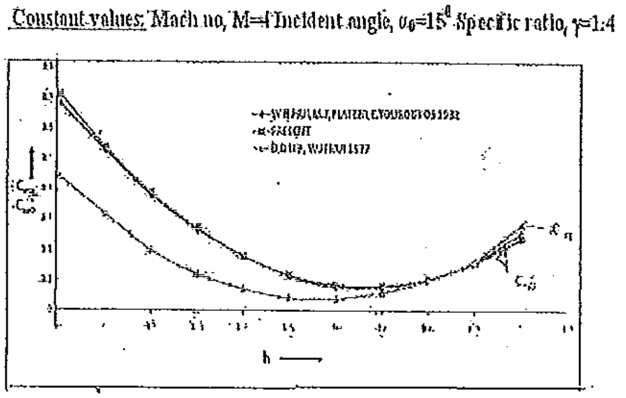

Fig. 8. Comparison of Damping Derivative of Triangular Wing

Figure 7 and 8 shows the variation of stiffness and damping derivatives with pivot position in supersonic flows. Stiffness and damping derivatives in pitch calculated by the present theory have been compared with Liu and Hui (1977). The stiffness derivative shows good agreement. The present work invokes strip theory arguments. Hui et al (1982) also use strip theory arguments whereby the flow at any span wise station is considered equivalent to an oscillating flat plate flow; this is calculated by perturbing the known steady flat plate flow (oblique shock solution) which serves as the 'basic flow' for the theory. For a pitching wing the mean incidence is the same for all 'strips' (irrespective of span wise location) and hence there is a single 'basic flow' which Hui et al have utilized to obtain closed form expression for stiffness and damping derivatives. They have not calculated the roll damping derivative. For a rolling wing the 'strips' are at different incidences and there is no single 'basic flow'; hence it is doubtful whether approach can be extended to yield a closed form expression for roll damping derivative. Their theory is valid for supersonic as well as hypersonic flows; whereas the present theory also gives closed form expressions for Stiffness and damping derivatives in pitch as well as roll damping derivative.

Constant values: Mach No, M=4, Pivot Position, h=0.667, Specific ratio, γ=1.4

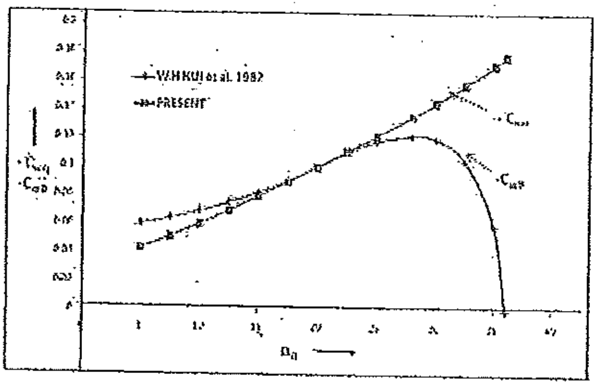

Fig. 9. Comparison of Damping in Pitch Derivative of Delta Wing with Angle Of Attack

Figure 9 represents damping derivative in pitch with angle of incidence. This present theory is in good agreement with Hui et al (1982) for angle of incidence up to thirty degrees and then there is no matching with the results of Hui et al (1982). This may be due to the detachment of the shock wave and stalling of the flow

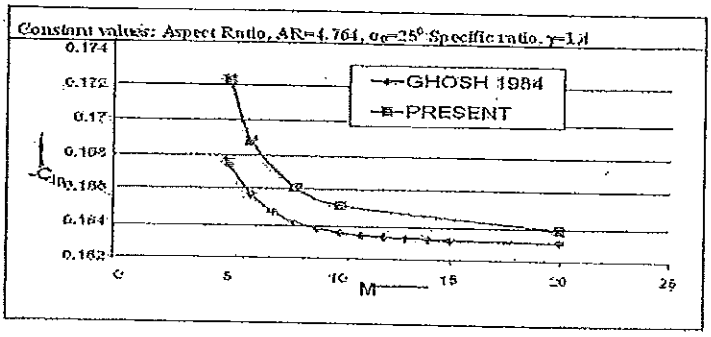

Fig. 10. Rolling Moment Derivative Vs Mach No.

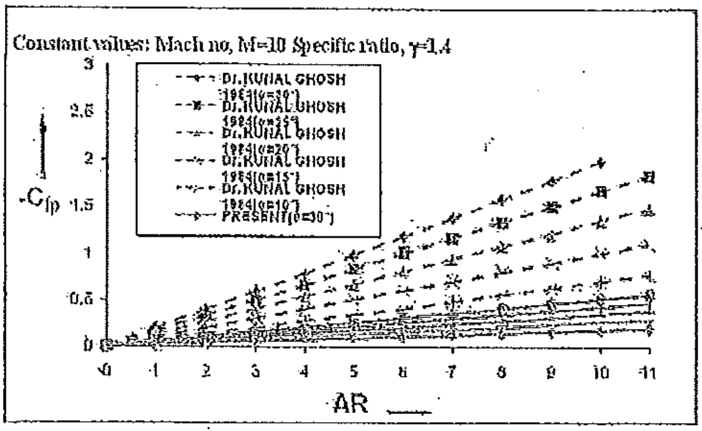

Fig. 11. Roll Damping Derivative Vs Aspect Ratio of Delta Wings

Figure 10 and 11 show the dependence of Roll damping derivative with Mach number and with the aspect ratio. The roll damping derivative decreases with Mach number initially then confirms the Mach number independence principle for large Mach numbers. Further, the roll damping derivative increases with aspect ratio of the wing. There was an error in the formulation in Ghosh (1984) for roll damping derivatives and hence in the present work the same has been corrected and implemented.

9 CONCLUSIONS

Present theory demonstrates its wide application range, in incidence, aspect ratio and the Mach number. The theory is valid only when the shock wave is attached with the wing. The effect of Lee surface has been neglected. The effects of viscosity and wave reflection are neglected. The present theory is simple and yet gives good results with remarkable computational ease.

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