Estimation of Dynamic Conditional Correlations of Shariah-Compliant Stock Indices through the Application of Multivariate GARCH Approach

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Abstract: A major issue in both Islamic finance and conventional finance is whether the shocks to the volatilities in the asset returns are substitutes or complements in terms of taking risk. An understanding of how volatilities of and correlations between asset returns change over time including their directions (positive or negative) and size (stronger or weaker) is of crucial importance for both the domestic and international investors with a view to diversifying their portfolios for hedging against unforeseen risks. This study is the first attempt to advance the frontier of knowledge particularly in the fast growing field of Islamic Finance through the application of the recently developed Dynamic Multivariate GARCH approach. We analyze the daily returns of five Shariah-compliant stock indices (such as, FTSE Shariah China Index, FTSE Shariah India Index, FTSE Shariah USA index, FTSE Malaysia EMAS Shariah Index and Dow Jones Shariah Index) covering the period from 26 October 2007 to 9 March 2011. Our study is focused on investigating the following empirical questions: (i) Are the time-varying volatility parameters of these five Shariah-compliant stock indices significant and decaying? (ii) Are these dynamic parameters mean-reverting? (iii) Are these dynamic conditional volatilities of Shariah indices and dynamic conditional correlations between Shariah indices changing? Our findings based on the maximum likelihood estimates of dynamic conditional volatilities and dynamic conditional correlations tend to suggest: (i) the time-varying conditional volatility parameters of all these Shariah-compliant stock indices are highly significant with most of their estimates very close to unity implying a gradual decay in volatility (assuming both the Gaussian and ‘t’ distributions). Of the two distributions, however, the t-distribution appears to be more appropriate in capturing the fat-tailed nature of the distributions of asset returns (ii) a test of ‘no mean-reversion of volatility parameters’ of all these Shariah indices is rejected in all cases with the results showing a slow but significant mean reverting volatility of all Shariah indices excepting FTSE Shariah China index which decays faster than others after any shock to its volatility and finally (iii) dynamic conditional volatilities and conditional correlations of all these Shariah indices are not constant but are changing and time-varying. There is relatively low and even at times negative dynamic conditional correlation between FTSE Shariah China index and FTSE Shariah USA index with strong policy implications for the domestic and international investors in their portfolio diversification for hedging against unforeseen risks.

Key words: Dynamic Conditional Correlations, Multivariate GARCH, Shariah-Compliant Stock

INTRODUCTION

Shariah is a Divine Law which governs the practical aspect of a Muslim's daily life. In commerce, it can determine business style and indicate a desire to comply with 'halal' and ethical investing. Shariah-compliant investing is growing rapidly as an alternative investment class for all investors, both Muslim and non-Muslim, for its foundation in ethical business practices, social responsibility and fiscal conservatism. While Islamic clients may be mandated to invest only in a Shariah-compliant manner, other investors do so for the benefits they derive, including greater stability of returns, transparency and diversification.

The modern Shariah scholars have provided general rules for Shariah complaint investors to evaluate or screen whether a particular company is halal (lawful) or haram (unlawful) for investment (Wilson, 2004; Derigs and Marzban, 2008).

Qualitative screens: this screening process focuses on the activity of a company that is used as the main principle in Islamic investment criteria. For a company that does not comply with Shariah principles, for example, a company involves in production of alcohol for drinking, gambling, entertainment, and riba-based financial institutions, then, investment in this type of company is prohibited.

Quantitative screens: this screening process refers to three financial parameters of a company, namely:

(1) Debt/equity ratio. If a company’s debt financing is more than 33 percent of its capital, then it is not permissible for investment.

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(2) Interest-related income. If interest-related income of a company is more than 10 percent of its total income, then it is not permissible for investment. This income, however, should not come from its main business activities but from placing its surplus funds in investments that could yield interest income (Abdul Rahman et al., 2010).

(3) Monetary assets. This parameter refers to the composition of account receivables and liquid assets (cash at banks and marketable securities) compared to total assets. Various minimums have been set for the ratio of non-liquid assets (assets that are not in the form of money) necessary to make an investment permissible. Some set this minimum at 51 percent while a few cite 33 percent as an acceptable ratio of non-liquid assets to total assets.

A major issue in both Islamic finance and conventional finance is whether the shocks to the volatilities in the asset returns are substitutes or complements in terms of taking risk. In modern portfolio theory, the main theme advocates investors to diversify their assets across national borders, as long as returns to stock in these other markets are less than perfectly correlated with the domestic market. It is well established that greater diversification benefits exist the less correlated the markets are. Generally, there are two popular measures of diversification benefits: gain in expected returns and reduction in risk. An understanding of how volatilities of and correlations between asset returns change over time including their directions (positive or negative) and size (stronger or weaker) is of crucial importance for both the domestic and international investors with a view to diversifying their portfolios for hedging against unforeseen risks. Lower international correlation across stock markets is the starting place of global portfolio diversification strategy (Solnik, 1974). If correlations between stock returns are high, a loss in one stock is likely to be accompanied with another loss in other stock markets as well. Therefore, benefits of diversification are higher if the correlation between the stock returns is low or negative.

This study is the first attempt to advance the frontier of knowledge particularly in the fast growing field of Islamic Finance through the application of the recently-developed Dynamic Multivariate GARCH approach. Our study is focused on investigating the following empirical questions:

i. Are the time-varying volatility parameters of these five Shariah-compliant stock indices significant and decaying?
ii. Are these dynamic parameters mean-reverting?
iii. Are these dynamic conditional volatilities of Shariah indices and dynamic conditional correlations between Shariah indices changing?

The literature on multivariate volatility modeling is large and expanding. Pesaran and Pesaran (2007) provide a recent application of it on the futures markets such as currency futures, government bonds and equity index futures. A general class of such models is the multivariate generalized autoregressive conditional heteroscedastic (MGARCH) model (Engle and Kroner, 1995). This model can be used to estimate the dynamic conditional correlation (DCC), how to compute the VaR of a portfolio, and how to calculate forecasts of conditional volatilities and correlations. From a financial point of view, MGARCH model opens the door to better decision tools in various areas, such as asset pricing, portfolio selection, option pricing, and hedging and risk management. The most obvious application of MGARCH (multivariate GARCH) models is the study of the relations between the volatilities and co-volatilities of several markets. For example, is the volatility of a market leading the volatility of other markets?

This paper is organized as follows. Following this introduction, a brief description of the methodology and literature review is given. This is followed in Section 3 by a discussion on the empirical results. The concluding remarks with the policy implications, limitations of the study and possible future research are given at the end of the paper.

**Methodology and a Brief Literature Review:**

**2.1 Data:**

This paper employs daily stock price indices from 26 October 2007 to 9 March 2011 for five Shariah compliant stock indices, namely, FTSE Shariah China Index, FTSE Shariah India Index, FTSE Shariah USA Index, FTSE Bursa Malaysia EMAS Shariah Index and Dow Jones Shariah index. All the data are collected from DataStream at INCEIF. The indices are denominated in local currency units. Prior to the analysis, all stock price indices are transformed into logarithm form.

**2.2 Multivariate GARCH model and Dynamic Conditional Correlations (DCC):**

In a multivariate GARCH (p,q) model, conditional variance and covariance of each asset depend upon not only on its own past conditional variance and past squared innovations but also on the past squared innovations and past conditional variances of the other assets (Bollerslev et al. 1994). The multivariate GARCH model is used in this paper to estimate the Dynamic Conditional Correlations (DCC) for a portfolio composed of returns on five Shariah compliant stocks as mentioned above. The estimation of the Dynamic Conditional Correlations (DCC) has a lot of potentials.
Firstly, DCC allows for the analysis of time variation in both mean and variance equation. Whereas, Rolling Regressions and Kalman Filters are intended to examine time varying relationships entered only in the mean equation.

Secondly, DCC allows us to look at how correlations change over time. DCC approach follows ARCH model solution to modeling the evolving nature of volatility. Specifically, ARCH models estimate a weighted average of a variable’s entire history of volatility with more weight given to the recent past and less weight given to the long past observations. Similarly, the DCC model estimates a weighted average of correlations that incorporates the entire history of a relationship between variables.

Thirdly, the DCC approach allows series to have periods of positive, negative, or no correlation. Thus both direction and strength of the correlation can be considered. When two series move in the same direction, the correlation increases and is positive. When they move in the opposite directions, the correlation is decreased and may become negative.

Last but not least, the DCC approach allows asymmetries, meaning that the weights are different for positive and negative changes to a series, which is an insightful advantage of this model.

DCC estimation involves 2 steps:

(i) Univariate volatility parameters are estimated by using GARCH models for each of the variables. So if there are two variables, then two GARCH equations are estimated. Just as an example, 
\[ h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma \varepsilon_{t-2}^2 \mathbb{I}_{\varepsilon_{t-1}^2 > 0} \]
(GJR, 1993 Asymmetric GARCH equation).

Where \( \mathbb{I} \) is an indicator function in which it equals 1 when the standardized residuals of the series (\( \varepsilon_t \)) are positive and equals 0 otherwise. A negative value of ‘m’ implies that periods with negative residuals would be immediately followed by periods of higher variance compared to the periods of positive residuals. The equation for GARCH is estimated in step 1 (for each variable) to estimate the residual (\( \varepsilon_t \)).

(ii) The standardized residuals (\( \varepsilon_t \)) from the first step are used as inputs for estimating a time-varying correlation matrix (by estimating DCC equation parameters).

\[ H_t = D_t R_t D_t \]

Here:
\[ H_t : \text{Conditional covariance matrix} \]
\[ D_t : \text{Diagonal matrix of conditional time varying standardized residuals (} \varepsilon_t \text{) that are obtained from the univariate GARCH models (on-diagonal elements or variance or volatility component)} \]
\[ R_t : \text{Time varying correlation matrix (off-diagonal elements)} \]

The likelihood of the DCC estimator is written as:
\[ L = -0.5 \sum_{t=1}^{T} \left( (\varepsilon_t^2 + 2 \log(|D_t|) + \log(|R_t|) + \varepsilon_t \varepsilon_t^* R_t^{-1} \varepsilon_t) \right) \]

(a) In the first step, only the volatility component (\( D_t \)) is maximized; i.e. the log likelihood is reduced to the sum of the log likelihood of univariate GARCH equations.

(b) In the second step, correlation component (\( R_t \)) is maximized (conditional on the estimated \( D_t \)) with elements \( \varepsilon_t \) from step 1. This step gives the DCC parameters, \( \alpha \) and \( \beta \),

\[ R_t = (1 - \alpha - \beta) \bar{R} + \alpha \varepsilon_{t-1} \varepsilon_{t-1}^* + \beta R_{t-1} \]

(DCC equation)

If \( \alpha = \beta = 0 \), then \( R_t \) is simply \( \bar{R} \) and CCC model is sufficient. The models have GARCH-type dynamics for both the conditional correlations and the conditional variances. The time-varying conditional variances can be interpreted as a measure of uncertainty and thus give us insight into what causes movement in the variance.

The two-step estimation of the likelihood function is consistent, albeit inefficient (Engle and Sheppard, 2001). The DCC allows asymmetries, meaning the weights are different for positive and negative changes to a series. The asymmetries are in the variances (not in the correlations) (Cappiello, Engle and Shephard, 2003).

Conditional correlation is a forecast of the correlation that would be appropriate next period conditional on this period’s data. Therefore the uncertainty in this forecast (assuming correctly specified model) is simply due to only parameter uncertainty.

The main merit of DCC in relation to other time-varying estimating methods (such as, rolling regressions and Kalman filters and their variants such as, Flexible Least squares) is that it accounts for changes in both the mean and variances of the time series (unlike the above methods which account for only the time-varying changes in the mean). In other words, DCC allows for changes both in the first moment (mean) and the second moment (variance). Understanding how correlations and volatility change over time and when they would be strong or weak is a persuasive motivation for the use of DCC models particularly in the financial markets. The DCC modeling allows us to pinpoint changes (both when they occur and how) in the interdependence between series.
The dynamic conditional correlations (DCC) enable a determination of whether the shocks to the volatilities in the forward and futures returns of various maturities are substitutes or complements in terms of taking risk. Such empirical estimates are crucial for deciding whether or not to hedge against unforeseen circumstances, as well as for dynamic option pricing.

This recently-developed technique was also employed by Lanza et al. (2006) to estimate the dynamic conditional correlations in the daily returns on West Texas Intermediate oil forward and future prices. They found that from 1985 to 2004, the DCC can vary dramatically in contrast to the common view that the volatility of futures price returns at different maturities are perfectly correlated. In general, the dynamic volatilities in the returns in the WTI oil forward and future prices could be either independent or interdependent over time.

The DCC estimates of the conditional correlations between the volatilities of forward and futures returns were always statistically significant. Their results indicate that the assumption of constant conditional correlations (CCC) (between returns at different maturities) was not supported empirically (because DCC between the forward and futures returns varied dramatically). Only in the case of the dynamic volatilities of the 3-months futures returns and 6-months future returns were the range of variation (between the max and min) relatively narrow, namely (0.832, 0.996). In general, the dynamic volatilities in the returns in the WTI forward and futures prices could be either independent or interdependent over time.

In the case of DCC between forward 1-month and futures 1-month, the max is 0.998 implying that forward one month and futures one month returns would have the same risk. However, the min is -0.291 implying that shocks to either of them are not perfect substitute in terms of risk.

Bollerslev (1990) assumed that the conditional variance for each return, \( h_i \) (i=1, …, m) follows a univariate GARCH process, that is, CCC specification:

\[
h_t = \alpha_i + \sum_{j=1}^p \beta_j h_{t-j} + \sum_{j=1}^q a_j \varepsilon_{t-j}^2 (CCC \ model)\]

Where \( a_j \) represents the ARCH effects or short-run persistence of shocks to return \( j \) and \( \beta_j \) represents the GARCH effects, or contribution of shocks to return \( i \) to long-run persistence.

CCC specification above assumes independence of the conditional variances across returns and does not accommodate asymmetric behavior. In order to accommodate the asymmetric impacts of positive and negative shocks, Glosten et al. (1992) proposed the asymmetric GARCH or GJR specification for the conditional variance, which for \( r=s=1 \), is given by:

\[
h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 + \beta_1 h_{t-1} (Asymmetric \ Conditional \ Variance \ Model)\]

(Where \( I_{t-1} \) is an indicator function to distinguish between positive and negative shocks on conditional volatility)

In order to capture the dynamics of time-varying conditional correlation \( \Gamma_t \) Engle (2002) and Tse and Tsui (2002) proposed the following DCC model:

\[
\Gamma_t = (1 - \theta_1 - \theta_2) \Gamma_{t-1} + \theta_1 I_{t-1} + \theta_2 I_{t-1} (DCC \ model)\]

In which \( \theta_1 \) and \( \theta_2 \) are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlations on current DCC.

DCC is a popular estimation procedure which is reasonably flexible in modeling individual volatilities and can be applied to portfolios with a large number of assets (Pesaran and Pesaran, 2007). To capture the fat-tailed nature of the distribution of asset returns, it is more appropriate if the DCC model is used with a multivariate t-distribution, especially for risk analysis where the tail properties of return distributions are of most concern. Engle (2002) suggested that the log-likelihood function of the DCC model can be maximized by using a two step procedures. This procedure, however, will no longer be applicable to such a t-DCC specification and a simultaneous approach to the estimation of the parameters of the model which includes the degrees of freedom parameter of the multivariate t distribution would be needed (Pesaran and Pesaran, 2007).

The standardized returns used by Engle (2002) are as follows:

\[
z_{it} = \frac{r_{it}}{\sigma_{t-1}(\lambda)}\]

For estimation of cross-asset correlations, Engle proposes a two-step procedure:

(i) Individual GARCH (1,1) models are fitted to the ‘m’ asset returns separately, and then,

(ii) The coefficient of the conditional correlations, \( \phi \), is estimated by Maximum Likelihood Estimator (MLE) (assuming that asset returns are conditionally Gaussian).

This procedure has two main drawbacks:

(i) The Gaussianity assumption does not hold for daily returns and its use can under-estimate the portfolio risk

(ii) The two-stage approach is likely to be inefficient (although consistent) even under Gaussianity.

Pesaran, therefore, proposes an alternative formulation of conditional correlations \( \rho_{i,t-1}[\phi] \) that makes use of realized volatilities. Pesaran estimates correlations based on devolatilized returns that are nearly Gaussian.
$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{realized}} = \frac{r_{it}}{\sigma_{it}(p)}$$

For daily returns a value of $p=20$ tends to render $\tilde{r}_{it}$ nearly Gaussian.

The t-DCC estimation procedure was applied to a portfolio composed of six currency futures, four 10 year government bonds and five equity index futures over the period 02 January 1995 to 31 December 2006 by Pesaran and Pesaran (2007). They found that the results strongly reject the normal-DCC model in favor of a t-DCC specification. There has been a general trend towards a lower level of volatility in all markets, with currency futures leading the way.

Kearney et al. (2005) examined the correlation dynamics also by using daily data from 1993 to 2002 on the five largest Euro-zone stock market indices. They found that the presence of a structural break in market index correlations occurred at the beginning the process of monetary integration in the Euro-zone.

2.3 Tests of Mean-Reversion:

In the empirical applications we shall consider the mean reverting as well as the non-mean reverting specifications, and experiment with the two specifications of the conditional correlations that are based on standardized and devolatized returns.

The decomposition of $H_t$ allows separate specification of the conditional volatilities and conditional cross asset returns correlations. For example, one can utilize the GARCH (1,1) model for the variance $\sigma^2_{it-1}$, namely

$$V(r_{it}|Q_{t-1}) = \sigma^2_{it-1} = \tilde{\sigma}^2_i(1 - \lambda_1 - \lambda_2) + \lambda_1 \sigma^2_{i,t-2} + \lambda_2 r^2_{it-1}$$

Where, $\tilde{\sigma}^2_i$ is the unconditional variance of the $i$th stock return.

$\lambda_1$ and $\lambda_2$ are stock specific volatility parameters (individual stock return volatilities). Under the restriction $\lambda_1 + \lambda_2 = 1$, the unconditional variance disappears in the above equation and we have the Integrated GARCH (IGARCH) model, which tells us that conditional variance is non-stationary, and then the shock to variance is permanent.

A more general mean reverting specification is given by

$$q_{it,t-1} = \tilde{p}_{ij}(1 - \lambda_1 - \lambda_2) + \lambda_1 q_{ij,t-2} + \lambda_2 \tilde{r}_{i,t-1} \tilde{r}_{j,t-1}$$

where, $\tilde{p}_{ij}$ is the unconditional correlation between $r_{it}$ and $r_{jt}$ and $\lambda_1 + \lambda_2 < 1$. One would expect $\lambda_1 + \lambda_2$ to be close to 1 in order to be non-mean reverting (does not come back to the mean or equilibrium). The non-mean reverting case can be obtained when $\lambda_1 + \lambda_2 = 1$. Therefore, in order to test the existence of non-mean reversion, we need to put a restriction of $\lambda_1 + \lambda_2 = 1$.

RESULTS AND DISCUSSIONS

In this section, three types of empirical tests are conducted, namely, comparison of Gaussian DCC model and t-DCC model, plotting the Estimated Conditional Volatilities &Correlations and finally testing for linear restrictions. The comparison of Gaussian DCC model and t-DCC model serves as a preliminary step to determine which model is relatively more significant.

Since we are primarily interested in volatility modeling, we set $\mu_{t-1} = 0$, and estimate the DCC models on Shariah compliant indices daily returns over the period 26 October 2007 to 9 March 2011. We did not encounter the case of non-convergence, and furthermore obtained the ML estimates of the Gaussian DCC and t-DCC model on stock indices daily returns.

3.1 ML estimates of the Gaussian DCC and t-DCC model on stock indices daily returns:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-Ratio[Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>lambda1 RCNSHA</td>
<td>.47540</td>
<td>.069414</td>
<td>6.8487[.000]</td>
</tr>
<tr>
<td>lambda1 RMYEMAS</td>
<td>.82526</td>
<td>.039230</td>
<td>21.0365[.000]</td>
</tr>
<tr>
<td>lambda1 RINSHA</td>
<td>.87029</td>
<td>.021271</td>
<td>40.9141[.000]</td>
</tr>
<tr>
<td>lambda1 RDJSHA</td>
<td>.91688</td>
<td>.009680</td>
<td>92.9146[.000]</td>
</tr>
<tr>
<td>lambda1 RUSSHA</td>
<td>.90369</td>
<td>.008785</td>
<td>91.4809[.000]</td>
</tr>
<tr>
<td>lambda2 RCNSHA</td>
<td>.23049</td>
<td>.029038</td>
<td>7.9374[.000]</td>
</tr>
<tr>
<td>lambda2 RMYEMAS</td>
<td>.15477</td>
<td>.017901</td>
<td>8.9435[.000]</td>
</tr>
<tr>
<td>lambda2 RINSHA</td>
<td>.11546</td>
<td>.017901</td>
<td>6.4497[.000]</td>
</tr>
<tr>
<td>lambda2 RDJSHA</td>
<td>.076119</td>
<td>.0085110</td>
<td>8.9435[.000]</td>
</tr>
<tr>
<td>lambda2 RUSSHA</td>
<td>.090974</td>
<td>.0089946</td>
<td>10.1143[.000]</td>
</tr>
<tr>
<td>delta1</td>
<td>.95709</td>
<td>.0085867</td>
<td>108.0633[.000]</td>
</tr>
<tr>
<td>delta2</td>
<td>.022644</td>
<td>.0028470</td>
<td>7.9547[.000]</td>
</tr>
</tbody>
</table>

Maximized Log-Likelihood = 12627.2
The upper panel of the above results presents the maximum likelihood estimates of \( \sigma_i \) and \( \beta_i \) (Volatility parameters) for the five stock indices returns, and \( \gamma_i \) and \( \delta_i \) (Meanreverting parameters, \( i = 1, 2, 3, 4, 5 \)). We can observe that all volatility parameters are highly significant, with the estimates of \( \sigma_i \), \( i = 1, 2, 3, 4, 5 \) very close to unity implying a gradual volatility decay. The lower panel of the table reports the estimated unconditional volatilities and correlations of the vector of stock indices.

The unconditional volatilities and correlations of returns are given at the lower panel of table 1. China appears to have the highest volatility and Malaysia’s EMAS appears to have the lowest volatility. In terms of correlations, the lowest appears to be between the Malaysia’s EMAS index and the US index implying the potential benefit of portfolio diversification for the investors.

Furthermore, we conducted the ML estimates of the t-DCC model to serve as a preliminary step to determine which model is relatively more significant.

### Table 2: ML estimates of the t-DCC model on stock indices daily returns

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>T-Ratio [Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 ) RCNSHA</td>
<td>.84010</td>
<td>.026338</td>
<td>31.8975 [.000]</td>
</tr>
<tr>
<td>( \lambda_1 ) RMYEMAS</td>
<td>.90802</td>
<td>.021001</td>
<td>43.2375 [.000]</td>
</tr>
<tr>
<td>( \lambda_1 ) RINSHA</td>
<td>.89634</td>
<td>.019974</td>
<td>44.8747 [.000]</td>
</tr>
<tr>
<td>( \lambda_1 ) RDJSHA</td>
<td>.91207</td>
<td>.012618</td>
<td>72.2835 [.000]</td>
</tr>
<tr>
<td>( \lambda_1 ) RUSSHA</td>
<td>.88517</td>
<td>.014303</td>
<td>61.8886 [.000]</td>
</tr>
<tr>
<td>( \lambda_2 ) RCNSHA</td>
<td>.14508</td>
<td>.022720</td>
<td>6.3859 [.000]</td>
</tr>
<tr>
<td>( \lambda_2 ) RMYEMAS</td>
<td>.083170</td>
<td>.018135</td>
<td>4.5861 [.000]</td>
</tr>
<tr>
<td>( \lambda_2 ) RINSHA</td>
<td>.090932</td>
<td>.016659</td>
<td>5.4583 [.000]</td>
</tr>
<tr>
<td>( \lambda_2 ) RDJSHA</td>
<td>.078771</td>
<td>.010632</td>
<td>7.4087 [.000]</td>
</tr>
<tr>
<td>( \lambda_2 ) RUSSHA</td>
<td>.10579</td>
<td>.012661</td>
<td>8.3558 [.000]</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>.9142</td>
<td>.031069</td>
<td>29.4249 [.000]</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>.027091</td>
<td>.0053934</td>
<td>5.0230 [.000]</td>
</tr>
<tr>
<td>df</td>
<td>6.0523</td>
<td>.48443</td>
<td>12.4936 [.000]</td>
</tr>
</tbody>
</table>

Maximized Log-Likelihood = 13073.2

From the above ML estimates of the t-DCC model on stock indices daily returns, we could see that all return volatility estimates are statistically significant and near to unity implying a gradual decay in volatility under t-DCC model as well. The maximized Log-Likelihood value (13073.2) is significantly larger than that obtained under the normality assumption (12627.2). The estimated degrees of freedom for the t-normal distribution is 6.05 which is below 30. This suggests that the t-distribution is more appropriate in capturing the fat-tailed nature of the distribution of stock returns.
### Table 3: Ranks of the unconditional volatilities of the five indices (from lowest to highest)

<table>
<thead>
<tr>
<th>No.</th>
<th>Indices</th>
<th>Unconditional Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FTSE Bursa Malaysia EMAS Shariah Index</td>
<td>0.009775</td>
</tr>
<tr>
<td>2</td>
<td>Dow Jones Shariah Index</td>
<td>0.014757</td>
</tr>
<tr>
<td>3</td>
<td>FTSE Shariah USA Index</td>
<td>0.017113</td>
</tr>
<tr>
<td>4</td>
<td>FTSE Shariah India Index</td>
<td>0.023693</td>
</tr>
<tr>
<td>5</td>
<td>FTSE Shariah China Index</td>
<td>0.033635</td>
</tr>
</tbody>
</table>

The on-diagonals explain the volatility of indices. If the unconditional volatility is close to zero, it means that the particular index is less volatile. If the unconditional volatility is close to one, it means that the particular index is more volatile. Unconditional volatilities of the five indices returns are very low which range between 0.0098 and 0.034, which implies that returns on those five Shariah complaint stock indices are, overall, less volatile. Furthermore, we could see that FTSE Bursa Malaysia EMAS Shariah Index, Dow Jones Shariah Index and FTSE Shariah USA Index are relatively less volatile compared to FTSE Shariah India Index and FTSE Shariah China Index.

### Table 4: Unconditional correlations of five Shariah indices

<table>
<thead>
<tr>
<th>China Market</th>
<th>Malaysia Market</th>
<th>India Market</th>
<th>Dow Jones</th>
<th>USA Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJ</td>
<td>Malaysia</td>
<td>China</td>
<td>Kenya</td>
<td>Malaysia</td>
</tr>
<tr>
<td>Malaysia</td>
<td>India</td>
<td>India</td>
<td>India</td>
<td>Malaysia</td>
</tr>
<tr>
<td>India</td>
<td>China</td>
<td>DJ</td>
<td>USA</td>
<td>DJ</td>
</tr>
</tbody>
</table>

Note: Each column represents the ranking of unconditional correlation between the index specified in the table header and other indices.

To have a clear picture of the relative correlation among Shariah indices, we ranked the unconditional correlations as follows (from lowest to highest).

The above rankings indicate some interesting facts. Firstly, almost all indices have low correlations with the FTSE Shariah USA Index excepting Dow Jones Shariah index. Secondly, FTSE Bursa Malaysia EMAS Shariah Index shows that it has relatively low correlation with other indices. These imply that in order to fully benefit from portfolio diversification, portfolios should include USA and Malaysian stock markets. Last but not the least, FTSE Shariah India Index has relatively high correlation with other stock markets. An investor outside of India must be careful when he/she selects portfolios.

From an USA investor’s perspective, we notice that there is relatively low correlation between FTSE Shariah USA Index and other stock indices excepting Dow Jones Shariah index (0.895). It is a concern for the investors as any movement in the return of either of the two indices causes the other to move in the same direction.

### Plotting the Estimated Conditional Volatilities & Correlations for stock indices daily returns:

![Conditional Volatilities of Stock Indices Returns](image1)

**Fig. 1:** Conditional volatilities of stock indices returns over the period 31 December 2007 to 09 March 2011

It can be seen from the Figure 1 that, the conditional volatilities of all stock indices returns move more closely together over time. However, there is relatively high volatility in stock indices returns in year 2008 due to the US financial crisis. Overall the results show that returns on those five Shariah complaint stock indices are stable. This confirms that Shariah-compliant equities are less volatile than their conventional counterparts, both in times of crisis as well as in times of stability. One reason for this is because excessive financial leverage is prohibited.
Fig. 2: Conditional correlations of FTSE Shariah China Index returns with other indices over the period 31 December 2007 to 09 March 2011

From the Figure 2, we can see that conditional correlations of returns on FTSE Shariah China Index with other indices are not constant but are changing. We notice that FTSE Shariah China Index has relatively less correlation with FTSE Shariah USA Index and Dow Jones Shariah index while relatively high correlation with FTSE Bursa Malaysia EMAS and FTSE Shariah India Index Shariah Index. This also confirms our results which were presented earlier. These results offer opportunities to the investors to gain from their portfolio diversifications.

**Testing For Mean Reversion Of Volatility:**

In this section, we focus on the problem of testing the null hypothesis that the volatility is non-mean reverting.

We wish to test: $H_0: \lambda_{1t} + \lambda_{2t} = 1$

Under $H_0$ the process is non-mean reverting and the unconditional variance for this asset does not exist.

**Table 3:** Testing for mean reversion of volatility of Shariah compliant indices returns

<table>
<thead>
<tr>
<th>Indices</th>
<th>$t - \hat{\lambda}_1 - \hat{\lambda}_2$</th>
<th>Std. errors</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTSE Shariah China Index (RCNSHA)</td>
<td>.29412</td>
<td>.050684</td>
<td>5.8029</td>
</tr>
<tr>
<td>FTSE Malaysia EMAS Shariah Index (RMYEMAS)</td>
<td>.019970</td>
<td>.0082197</td>
<td>2.4296</td>
</tr>
<tr>
<td>FTSE Shariah India Index (RINSHA)</td>
<td>.014255</td>
<td>.0046089</td>
<td>3.0930</td>
</tr>
<tr>
<td>Dow Jones Shariah index (RDJSHA)</td>
<td>.0069975</td>
<td>.0020951</td>
<td>3.3400</td>
</tr>
<tr>
<td>FTSE Shariah US Index (RUSSHA)</td>
<td>.0053334</td>
<td>.0015548</td>
<td>3.4302</td>
</tr>
</tbody>
</table>

The above result shows statistically significant mean reverting volatility for all Shariah compliant indices. In terms of the speed of mean reversion, however, most of them are slow excepting that of China which is the fastest to get back to equilibrium.

**The final Concluding Remarks and Policy Implications:**

An humble contribution of this study is our first attempt at estimating the dynamic conditional correlations among the five Shariah-compliant stock indices through the application of a recently-developed dynamic multivariate GARCH approach with a view to helping both the domestic and international Shariah investors to diversify their portfolios by hedging against unforeseen risks.

Investors willing to take risks may invest in the Chinese stocks because of its relatively higher volatility and faster movement towards mean reversion. However, the risk-averse investors may invest in the Malaysian stocks because of its relatively lower volatility and slower mean-reversion process.

Correlations among the five indices are not constant but are dynamic and time-varying. Hence the investors should monitor these correlations and manage their investment portfolios accordingly.
Different financial markets offer different opportunities for portfolio diversification. For instance, the Chinese investors can gain the most by diversifying into the US stock market because of the low correlations between the Chinese and the US stock markets.

Finally, the timing of investment is also important. There are times when the Chinese and the US indices are negatively correlated and hence the investors may also gain by timing their portfolio diversification properly.

**Limitations of the study and Suggestions for future research:**

The Shariah Complaint Stock indices were not established long before, therefore, this paper failed to discuss model evaluation and forecasting of t-DCC model due to relatively a short period of data. The future researchers may try to focus on the model evaluation and forecasting of t-DCC model.

The choice of indices is somewhat arbitrary. Many other available indices could have been considered and might have produced additional or even different results.

The theoretical foundation and framework of this study also leave something to be desired. The underlying theory is of crucial importance. Otherwise, the studies such as this may be accused of being an exercise of number crunching or statistical data mining. However, developing a theory in such an area would be really challenging since the Islamic finance is now at its nascent stage of development. Nonetheless, efforts should be directed towards this end in the future.

**Disclaimer:**

The above works represent the humble effort and limited knowledge and experience of the authors. Errors, misrepresentations and flaws in argumentation and expression reflect the authors’ weaknesses. In the interest of the pursuit of the truth, the authors welcome any feedback, comments and inputs. Allah knows best.

**REFERENCES**


