Mathematics Lessons Learned from Across the World

Grades 7–12

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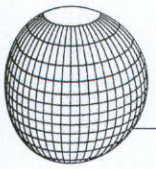

Lesson 4

Everyday Mathematics: Profit and Loss, Cost Price, and Selling Price

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MATH CONTENT

Collecting data to maximize profit

Direct proportion

Solution of simultaneous equations or inequalities

Calculating total profit and total expenditure

Reasoning and argumentation

MATERIALS NEEDED

An activity page for each student

Poster paper and markers for each group of students

Spreadsheet software, if available, for each student or group of students

Setting the Scene

Country of Context

In Brunei, students exhibit the ability to apply their mathematical knowledge to problem solving in the real world. Further, students demonstrate their abilities to develop mathematical reasoning and argumentation. The Brunei mathematics curriculum encourages activities

that are differentiated in nature, allowing less able students to solve the problem in their own ways while challenging more able students to work in more sophisticated ways. Activities should be interesting enough to promote positive attitudes toward mathematics and should be usable as performance assessments, since Brunei promotes school-based assessments. Open problems with more than one solution are encouraged in schoolwork.

Classroom Context

This lesson was designed for a one-hour class period for students ages fourteen to sixteen in Brunei. It is adapted from Rayner (2005, pp. 360–61) and is for use by students either in small groups or working alone.

Teachers are encouraged to prepare a rubric to assess students' finished products and to monitor their progress on the activity. Students are expected to communicate, discuss, and argue among themselves while working on the activity. They may also use an electronic spreadsheet to construct tables and plot graphs.

Teacher Notes

The suggested method of instruction for the activity is detailed in table 4.1.

Table 4.1. Method of instruction for task A

Duration	Teacher activity	Student activity
5 min.	Introduction • Form groups of four or five students. • Give a brief introduction of the task and objectives of the activity. • Distribute the activity page with the problem to be solved to all students. • Give students ample time to read the activity page.	• Students should read and understand the problem before deciding on the next step.
28 min.	Activity • Ask each group of students to discuss the problem among themselves for 5 minutes before attempting to solve the problem. • Monitor students' progress. • Distribute posters and markers for students to use in displaying their work.	• Students should organize and write their work on the activity sheet. • Students should discuss how to solve the problem. • As students determine solutions, they should prepare a poster demonstrating ideas, methods of solution, and complete solution. • At the end of the activity, students should submit their written work on the activity sheet and the poster.

Table 4.1. *Continued*

25 min.	**Group presentation and discussion** • Give each group 5 minutes to present its poster. • Invite one group to present its complete solution, if time permits. • Ask relevant questions if students do not provide enough evidence in their presentation.	• Students must participate in the discussion and must answer any question posed by the teacher or other students. • Students should speak clearly and audibly.
2 min.	**Conclusion** • Summarize the lesson by highlighting the different methods of solving the problem.	• In their journals, students write what they learned and explain their choice of the best method for solution.

Possible solutions are shown in table 4.2.

Table 4.2. Possible solutions of task A

Type A Amazonia	**No. of TVs**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
	Total cost price ($)	900	1050	1200	1350	1500	1650	1800	1950	2100
	Total profit made ($)	480	560	640	720	800	880	960	1040	1120
Type B Bulovania	**No. of TVs**	**14**	**13**	**12**	**11**	**10**	**9**	**8**	**7**	**6**
	Total cost price ($)	4200	3900	3600	3300	3000	2700	2400	2100	1800
	Total profit made ($)	1400	1300	1200	1100	1000	900	800	700	600
Money needed to buy TVs of type A and type B ($)		5100	4950	4800	4650	4500	4350	4200	4050	3900
Extra money on hand ($)		−600	−450	−300	−150	0	150	300	450	600
Profit made on all TVs ($)		1880	1860	1840	1820	1800	1780	1760	1740	1720
If profit were to be added to extra money on hand (or if profits were to be deducted after paying debt) ($)		1200	1410	1540	1670	1800	1930	2060	2190	2320

Table 4.2. *Continued*

Total money collected (once all TVs are sold) ($)	6380	6360	6340	6320	6300	6280	6260	6240	6220
	(Original money + money owed) Collected back + total profits − money owed				Original money + total profit	Money spent collected back + extra money on hand + profit			

Using the information from table 4.2, students may give the following answers:

- 10 type A and 10 type B TVs provide the maximum profit without any money owed.
- 6 type A and 14 type B TVs may be the optimal solution if interest-free loans can be obtained from family.
- 14 type A and 6 type B TVs may be the optimal solution if maximum money on hand plus profit after all televisions are sold is desired.

1. Form two simultaneous equations where A is the number of type A televisions and B is the number of type B televisions:

$$A + B = 20$$
$$150A + 300B = 4500, \text{ or } A + 2B = 30$$

   Solving these two simultaneous equations yields 10 type A and 10 type B televisions to be sold.
2. Using graphs of the following constraint inequalities, we obtain a quadrilateral area for the possible number of type A and type B televisions to be sold.

$$A \geq 6$$
$$B \geq 6$$
$$A + B \leq 20$$
$$150A + 300B \leq 4500, \text{ or } A + 2B \leq 30$$

The objective function $C = 80A + 100B$ to be maximized for profit might be considered as $C = 4A + 5B$, where C is a constant but the profit ratio of $^{80}/_{100}$ could be thought of as $^{4}/_{5}$. This objective function was drawn to move the maximum point of the quadrilateral formed as shown in figure 4.1.

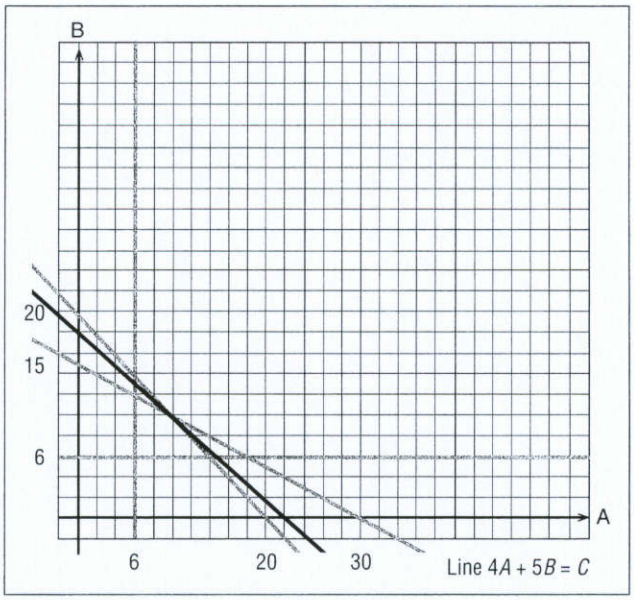

Fig. 4.1. Graphs of constraints in problem

Extensions

The following problem may be offered to students as an extension:

> Students in a school photography club decided to sell T-shirts for fundraising. They plan to sell long-sleeve T-shirts for $30 each, at a $10 profit, and short-sleeve T-shirts for $23 each, at an $8 profit. They were given $500 to order the T-shirts. After discussion, they decided that they should order at least 10 of each type of T-shirt.
>
> 1. Assuming that they can sell all the T-shirts that they purchase, what are the minimum and maximum numbers of T-shirts that they should order? Explain your responses.
> 2. What are the minimum and maximum profits that they can make? Explain your responses.

Research Notes

According to the description offered by the Brunei Ministry of Education (MOE) for year 8 student work for everyday mathematics (MOE 2008a), students are expected to be able to explain the concepts of profit and loss and cost and selling price of an object and be able to solve problems on finding the cost, selling price, profit, or loss. Student work should include solving simultaneous equations, and students should be able to explain how to solve word problems by identifying and translating given information into mathematical statements and then solving any equations obtained.

In Brunei, year 9 student work for graphical representation of inequalities (MOE 2008b) includes writing the inequalities that define a region (usually unshaded) where the equations

of the boundary lines are given or not given, graphing them, and explaining how to obtain the region defined by a system of linear inequalities. Students are also expected to determine the maximum or minimum of $ax + by$ for a defined region by evaluating the expression at the vertices of the polygon formed by the region.

In the United States, the Common Core State Standards for Mathematics (CCSSM) for high school state that students should be able to "create equations and inequalities in one variable and use them to solve problems"; "create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales"; and "represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context" (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010, p. 65).

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Activity Page

Money-Making Decision

How many of each?

As the owner of an electronics store, you have enough room to stock no more than 20 televisions. Two types of televisions are available, as shown in figure 1. You can stock either Amazonia (type A) or Bulovania (type B) televisions, which cost $150 and $300, respectively. You make an $80 profit on each Amazonia television and $100 profit on each Bulovania television sold. For advertising purposes and sales, you must stock at least 6 of each type of television. You may spend only $4500 in purchasing the televisions. How many of each type of television should you purchase so that you may maximize your profit?

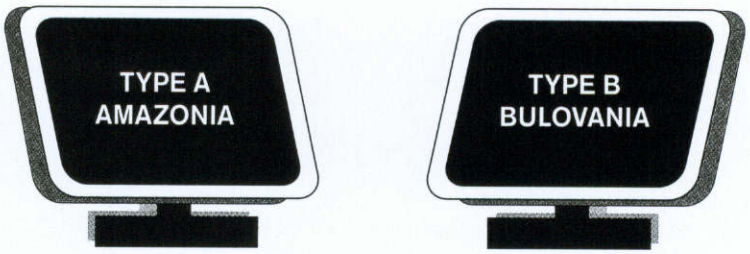

Fig. 1: Two types of televisions

If you buy 6 Amazonia televisions and the rest are Bulovania televisions, how much do you have to pay? What is your expected profit?

If you buy 6 Bulovania televisions and the rest are Amazonia televisions, how much do you have to pay? What is your expected profit?

How many of each type of television should you purchase to obtain the maximum profit? Explain your responses.

If you could buy any number of televisions that you want (with no restriction to 6 of each type), how many of each type would you buy for maximum profit? Explain your responses.