Performance Analysis of OFDM-PSK System for Optimization of Fiber Optic Dispersion

Muhammad Ibn Ibrahimy
Department of Electrical and Computer Engineering, Faculty of Engineering
International Islamic University Malaysia (IIUM), Kuala Lumpur, Malaysia
E-mail: ibrahimy@iium.edu.my
Tel: 603-61964504; Fax: 603-61964488

Fowzia Akhter
Department of Electrical and Computer Engineering, Faculty of Engineering
International Islamic University Malaysia (IIUM)
Kuala Lumpur, Malaysia

Hasin Reza Siddiquei
Department of Electrical and Computer Engineering, Faculty of Engineering
International Islamic University Malaysia (IIUM)
Kuala Lumpur, Malaysia

Abstract

The Performance of Orthogonal Frequency Division Multiplexing (OFDM) combined with Phase Shift Keying (PSK) has been analyzed in presence of fiber optic dispersion. The influence of dispersion on the OFDM spectrum is investigated with fiber lengths, bit rates and source linewidths. The inter-channel interference occurs due to dispersion-induced broadening of OFDM spectrum. The power penalty suffered by the system is evaluated at bit error rate, \( \text{BER}=10^{-9} \) for a single mode fiber operating at 1.55\( \mu \text{m} \). The results obtained from the OFDM-PSK have been compared with OFDM-QAM system. The obtained results demonstrate that the influence of dispersion is lower on OFDM-PSK spectrum in comparison with OFDM-QAM spectrum having same bandwidth.

Keywords: OFDM-PSK, Dispersion, BER performance

1. Introduction

In modern communication technology such as in optical fiber communication system, efficient utilization of communication resources (bandwidth and power) is the key challenge to meet the high bit rate demand in future application. There are several approaches to reduce system bandwidth as well as transmission power in digital communication techniques [1]. Among them, advanced digital modulations and multiplexing techniques are widely used to reduce system bandwidth significantly. It is well known that M-ary modulation techniques have the property of reducing bandwidth by the order of \( N \), where \( N \) is the number of bits used to form a symbol [1].

In M-ary modulations, a large number of symbols that is, \( M=2^N \) symbols are modulated with the M carriers which are closely separated either by M-phases or M-frequencies or M-magnitudes. The
transmission power as well as system performance mainly depend on the modulation types. It has been shown in previous study [1] that the probability of inter symbol interference is more in case of M-ary phase shift keying (PSK) compare to that of M-ary frequency shift keying (FSK). Although M-ary FSK gives better performance, bandwidth requirement is the higher than M-ary PSK.

It is well established that orthogonal frequency division multiplexing (OFDM) provides us more compact spectrum. Therefore with the combination of OFDM multiplexing and PSK modulation, an efficient communication system can be obtained.

Optical fiber is a well known waveguide capable of propagating light wave in the range of near infrared [2]. The available bandwidth in optical carrier is in the order of $10^3$ times than that of microwave link. Therefore, if OFDM-PSK system is combined with optical fiber, the speed of data communication can be enhanced dramatically.

There are numerous papers on the dispersion compensation of optical fiber communication system by using OFDM. However, there is no paper what has combined OFDM & PSK techniques to compensate dispersion. Arthur James Lowery and Jean Armstrong [3] proposed that a combination of Orthogonal Frequency Division Multiplexing and Optical Single Sideband Modulation (OSSB) can be used to adaptively compensate for chromatic dispersion in ultra-long-haul 10 Gbps Standard Single-Mode Fiber (S-SMF) links. But the main problem is required power is very high. Arthur James Lowery, Liang Du and Jean Armstrong [4] proposed that OFDM with optical single sideband modulation can adaptively compensate for dispersion in 4000-km 32×10Gbps WDM SMF links with 40% spectral efficiency having no reverse feedback path. But the limitation of this system is the number of channels could be increased to the bandwidth of the optical amplifiers. Brendon J.C. Schmidt, Arthur James Lowery and Jean Armstrong [5] proposed that Optical OFDM using a simple direct-detection receiver can post-compensate for dispersion in 320km of SMF28e fiber at 20 Gbps. But in this case the system performance is limited mainly by optical amplifier noise and the frequency response of the arbitrary waveform generator (AWG). Daniel J. F. Barros and Joseph M. Kahn [6] proposed that OFDM can compensate for linear distortions, such as GVD and PMD. But the main problem of this system is a large fraction of the transmitted energy is wasted on the cyclic prefix samples, leading to a power penalty.

Although optical fiber provides us enormous bandwidth, the system performance severely degrades due to dispersion. It is therefore very much important to investigate how an OFDM-PSK system is influenced in presence of dispersion. The main objective of this paper is to propose OFDM-PSK system for adaptive dispersion compensation of long haul optical fiber system. It is expected that this system will overcome all the above mentioned problems. The system performance will be evaluated at bit error rate $10^{-9}$ for a single mode fiber operating at 1.55µm.

The rest of the paper is organized as follows: Sec. 2 discusses the proposed system model, Sec. 3 represents the theoretical analysis of the proposed system, and Sec. 4 explains the results obtained from this system, respectively and followed by conclusions in Sec. 5

2. System Model

In our proposed system, PSK symbols are modulated by the carriers having frequency orthogonality. As the orthogonal carriers provide us more compact spectrum, dispersion effect will be less in OFDM-PSK system. Consequently, the proposed system allows us to transmit huge amount of bits than the conventional optical fiber communication. This system consists of mainly transmitter, transmitting medium and receiver.

Figure 1 shows the structure of a transmitter. In the transmitter, at first serial data stream is then converted into parallel data. Then the digital data is converted into analog data by using digital to analog converter (DAC). At the output of DAC, we get the even data and odd data which are modulated by using phase shift keying technique i.e. PSK technique. Then these are added together and as a result we get the PSK signal. After that, each channel signal is modulated by sub-carriers which
are orthogonal to each other i.e. the frequency spacing between carriers is $\frac{1}{T}$, where $T$, be the symbol period. The signals from different channels are then multiplexed and at the output of the MUX we get the OFDM-PSK signal.

**Figure 1:** Transmitter for generation of OFDM-PSK signal

![Transmitter for generation of OFDM-PSK signal](image)

Figure 2 shows the transmitting medium. The OFDM-PSK signal is first modulated and then passed to the optical fiber.

**Figure 2:** Transmitting medium for OFDM-PSK signal

![Transmitting medium for OFDM-PSK signal](image)

Figure 3 shows the structure of a receiver. The receiver picks up the OFDM-PSK signal, which is then demultiplexed. To regain the each channel signal filters are used. Here we’ve used cosine filter. The cut off frequency to regain each signal is determined from the OFDM-PSK signal. When the signals from each channel are separated then there remain some noises such as shot noise, thermal noise etc. To remove these noise each channel signal is filtered by a low pass filter. And at last we get a noise free output signal. This returns parallel streams, each of which is converted to a binary stream using an appropriate symbol detector. These streams are then re-combined into a serial stream, which is an estimation of the original binary stream at the transmitter.
3. Theoretical Analysis

OFDM-PSK is first generated and then transmitted through the fiber. At the receiver side, the detected signal is analyzed to observe effect of dispersion on it.

3.1. Generation of OFDM-PSK Signal

The baseband signals are first modulated by using M-ary PSK modulation technique. A detailed description of M-ary PSK can be found in [1] where the expression for PSK symbol is represented as:

\[ v_{PSK}(t) = au_1(t) + au_2(t) \]  

(1)

Where we have that \( u_1(t) = \sqrt{\frac{2}{T_s}} \cos \omega_o t \), and that \( u_2(t) = \sqrt{\frac{2}{T_s}} \sin \omega_o t \) and \( a = \sqrt{0.1E_s} \). We can write Eq. (1) as:

\[ v_{PSK}(t) = \sqrt{0.2\left(\frac{E_s}{T_s}\right)} \cos \omega_o t + \sqrt{0.2\left(\frac{E_s}{T_s}\right)} \sin \omega_o t \]  

(2)

And since \( \frac{E_s}{n} = P \) we have for OFDM-PSK signal:

\[ f_{n_{OFDM-PSK}}(t) = \sum_{n=0}^{M} \left(\sqrt{0.2P_s} \cos n \omega_o t + \sqrt{0.2P_s} \sin n \omega_o t \right) \]  

(3)

PSK signal for 4 bit symbol \( D_{k+3}D_{k+2}D_{k+1}D_k \) from which 2 bit represent by \( A_k(t) \) modulates the balance modulator whose input carrier is the even function \( \sqrt{P_s} \cos \omega_o t \) and \( A_0(t) \) modulates the modulator with odd-function carrier.

For orthogonal sub carrier, the carrier spacing between each channel is \( \frac{1}{T} \cdot v_{n_{OFDM-PSK}}(t) \) in the time domain signal.
Here, 16-PSK symbols have been used. Each symbol consists of four bit having the bit period of \( T = 5 \times 10^{-10} \) sec. Thus symbol period is \( T_S = 4 \times T = 2 \times 10^{-9} \) sec. To transmit the OFDM-PSK signal, 16 different orthogonal carriers have been used. The carrier frequency for 1st channel is 1GHz. The sampling period of the carrier is \( 7.874 \times 10^{-12} \) sec, where the no of sample is 128. The carrier frequency of the 16\(^{th}\) channel is 16GHz and time period for 16\(^{th}\) carrier \( 16 \times 10^{-9} \) sec. The time domain signal for the 16th channel is shown in Figure 4.

**Figure 4:** Time domain signal for 16\(^{th}\) channel having carrier frequency 16 GHz

After First Fourier transform of the time domain signal the frequency domain signal can be obtained [7]:

\[
v(f) = F \left[ v_{OFDM - PSK}^{OFDM - PSK}(t) \right]
\]  

(4)

**Figure 5:** Spectrum of 16\(^{th}\) channel having bandwidth 2 GHz

Frequency domain spectrum for 16\(^{th}\) channel is shown in Figure 5. Here the bandwidth of the channel signal is 2 GHz. The power level of the main lobe is so high but for the side lobe it is very low. After the multiplexing of frequency domain signal from each channel OFDM-PSK signal can be developed. The OFDM-PSK spectrum is shown in Figure 6.
It is found from Figure 6 that, in case of single sideband, individual channel can not be identified due to overlapping of channels. To identify the individual channel, double sideband is used in our calculation. The effect of dispersion on OFDM-PSK spectrum is included in the next section.

3.2. Transmission of OFDM-PSK Signal through Optical Fiber

The dispersion-induced distortion on OFDM-PSK spectrum can be analyzed by means of fiber transfer function. The transfer function of single mode fiber is given by [8]:

\[
H(f) = \left(1 + \frac{if}{f_2}\right)^{-\frac{1}{2}} \exp\left[-\frac{\left(\frac{f}{f_1}\right)^2}{2\left(1 + \frac{if}{f_2}\right)}\right]
\]

(5)

Where the parameters \( f_1 \) and \( f_2 \) are given by:

\[
f_1 = (2\pi\beta_2 L \sigma_\lambda)^{-1} = \left(2\pi |D| L \sigma_\lambda\right)^{-1}
\]

(6)

\[
f_2 = (2\pi\beta_3 L \sigma_\sigma)^{-1} = \left[2\pi (S + 2|D|/\lambda)L \sigma_\lambda^2\right]^{-1}
\]

(7)

Where, \( \sigma_\lambda \) is the source linewidth and \( D \) and \( S \) are the 2\(^{nd}\) order and 3\(^{rd}\) order dispersion parameters respectively.

The impulse response of the fiber transfer function is, \( h(t) = F^\dagger[H(f)] \).

The frequency-dependent response of single mode fiber is shown in Figure 7 with fiber length 50Km. The figure plotted for the dispersion coefficient of 20ps/km.nm with a laser operating at 1.5\( \mu \)m having a spectral width of \( \sigma_\lambda = 1 \)nm.
Figure 7: Optical fiber transfer function spectrum with fiber length, L=50 Km and laser linewidth, $\sigma_\lambda=1$nm, $D=20 \text{ ps/nm.km}$, $S=0$

![Optical fiber transfer function spectrum](image)

After passing the OFDM-PSK signal through the optical fiber of 1000km length, the OFDM-PSK spectrum obtained at the fiber output is shown in Figure 8.

Figure 8: Optical fiber output for 16 channel OFDM-PSK spectrum with fiber length, L=1000 Km and laser linewidth, $\sigma_\lambda=1$nm

![Optical fiber output](image)

When the length of the fiber increases the cut off frequency of the fiber decreases and if the signal bandwidth is greater than the fiber cut off frequency, total signal can’t pass through the fiber. As a result, at the receiver we get the distorted signal and it’s very difficult to detect the original signal. Dispersion-induced change in magnitude of OFDM-PSK signal is calculated in the next section.

3.3. Effect of Dispersion on OFDM-PSK Spectrum

The influence of dispersion on the normalized power spectral density of OFDM-PSK spectrum can be obtained by [9]:

$$I_{av} = \frac{1}{f} \left[ \int_0^f P(f)df \right]$$

(8)
Where $P(f)$, is the normalized power spectral density.

### 3.4. Receiver Input Signal Power and Noise Power

The signal power received after photo detection by a P-i-N photo detector can be given by [9]:

$$S = \left(R I_{av} P_{in}\right)^2$$

Where,
- $R$ = Responsivity of the photodiode
- $P_{in}$ = Input power

$P_{in}$ is given by:

$$P_{in} = \frac{\left(0.2 P_s A_{av}(t)\right)^2}{2}$$

Where,
- $P_s$ = Signal power.
- $A_{av}(t)$ = Average magnitude of symbols

$$A_{av}(t) = \frac{1}{M} \sum_{n=1}^{M} S_n(t)$$

Where,
- $S_n(t)$ = Individual signal magnitude

For PSK, let us consider to transmit a symbol for every 4-bits. There are then $2^4=16$ different possible symbols and we shall have to be able to generate 16 distinguishable signals.

The shot noise induced in P-i-N photo detector can be given by [2].

$$N_{shot} = 2q\sqrt{S B_e}$$

Where,
- $q$ = Electronic charge.
- $B_e$ = noise equivalent electrical bandwidth

Thermal noise current spectral density can be given by [2]:

$$I_{th} = 10^{-11} A / \sqrt{Hz}$$

So, thermal noise [2]:

$$N_{th} = (I_{th})^2 * B_e$$

Total noise is given by:

$$N_{total} = N_{shot} + N_{th}$$

### 3.5. Evaluation of Bit Error Rate (BER)

The probability of error for PSK system can be given by [9]:

$$P_e(m) = \frac{2(m-1)}{m} \text{erfc} \left( \frac{S}{2N_{total}} \right)$$

Where, $m = \sqrt{M}$ and $M = 2^k$. This result is valid for $k$ is even i.e. for $M = 16, 64, 256$ etc. For $k$ odd i.e. for $M = 8, 32, 128$, etc. $P_e$ can be obtained as:

$$P_e(M) \approx \frac{4 \sqrt{MN_{total}}}{\sqrt{6\pi S_{av}}} \cdot \exp \left( -\frac{3S_{av}}{2MN_{total}} \right)$$
The average power on each vector is $\frac{m^2 S}{12}$, where $m \geq 1$

Now the BER is given by,

$$BER = \frac{P_e}{\log_2 M}$$

(15)

3.6. Reception of OFDM-PSK signal

At the receiver, the OFDM-PSK signal is demultiplexed. For the separation of each channel signal cosine filter was used whose the transfer function is given by [10].

$$|H_e(f)| = \begin{cases} 
\frac{2T \cos \pi f}{T} & \text{for } |f| \leq \frac{1}{2T} \\
0 & \text{elsewhere}
\end{cases}$$

(16)

The corresponding impulse response $h_e(t)$, found by taking the inverse Fourier transform of $H_e(f)$ is,

$$h_e(t) = \sin c \left( \frac{1}{T} \right) + \sin c \left( \frac{t-T}{T} \right)$$

(17)

Since the frequency response of cosine filter is very sharp, it can be effectively used for demultiplexing purpose.

At the output of the filter the signal becomes:

$$P_{rf}(f) = P_e(f) * H_e(f)$$

(18)

All channel signals are demultiplexed at the receiver. An example of demultiplexing is presented in Figure 9.

**Figure 9:** Demultiplexing of 1st channel using cosine filter

Similarly, original signals can be extracted from different channels. In order to compare the effect of dispersion on OFDM-PSK system, the same calculation is performed on OFDM-QAM system with same bandwidth.
4. Results and Discussion
Following the theoretical analysis presented for the proposed system model in the previous section, the bit error rate has been evaluated at 64 Gb/s. The analysis is carried out for a single mode fiber operating at 1.55µm. The dispersion co-efficient of the fiber is considered as 20 $pS/nm.Km$. To evaluate the BER, signal to noise ratio (SNR) is evaluated for a P-i-N photo detector in presence of shot and thermal noises.

The base receiver sensitivity may be defined as the input power required to achieve a BER=$10^{-9}$ considering only the effect of receiver noise. The additional signal power with respect to base receiver sensitivity (BRS) may be termed as power penalty at BER=$10^{-9}$ due to the effect of various noise, dispersion and crosstalk power from adjacent channels.

The performance of OFDM-PSK system has been analyzed by the BER performance showed by the system. BER performance has been evaluated with fiber length, laser linewidth and Bit rate.

4.1. Evaluation of BER Performance with Fiber Length
The OFDM-PSK system has been analyzed for the variation of fiber length and it is found that as the fiber length increases the BER increases. As a result, the power penalty required to achieve base receiver sensitivity increases.

Figure 10: Effect on BER performance of OFDM-PSK system due to variation of fiber length operating at 1.55µm having dispersion coefficient, $D=20\ pS/nm.Km$ for bit rate 64 Gb/s

From Figure 10, it can be said that if the fiber length lies between 1~1000 Km, the effect on BER performance of OFDM-PSK system is not so severe but when the fiber length increases from 1000 Km, the effects becomes so severe.

4.2. Evaluation of BER Performance with Laser Linewidth
The OFDM-PSK system has also been analyzed for the variation of fiber laser linewidth. Figure 11 shows the effect of variation of laser linewidth. It is found from the analysis that as the laser linewidth increases the probability of BER increases. As a result the power penalty required to achieve the base receiver sensitivity increases. The effect of variation of laser linewidth on the BER performance of OFDM-PSK system is so severe.

Here for the laser linewidth, $\sigma_{\lambda}=2, 3$ and 4 nm the value of power penalty has been calculated at a bit rate 64 Gb/S.
From the above results, it can be concluded that, the BER performance of OFDM-PSK signal degrades with the increase in laser linewidth. But this effect is so distressing when the value of linewidth increases from 2\text{nm}.

**Figure 11:** Effect on BER performance of OFDM-PSK system due to variation of laser linewidth operating at 1.55\text{µm} having dispersion coefficient, D = 20 pS / nm.Km for bit rate 64 Gb/s

4.3. Evaluation of BER Performance with Bit Rate

With the increase in bit rate, the speed of data communication increases but the probability of error increases i.e. BER increases and as a result, the power penalty increases. The performance of OFDM-PSK system has been analyzed with the variation of bit rate at laser linewidth, $\sigma = 2\text{nm}$ and for the fiber length, L=1000 Km. Figure 12 shows the effect on power penalty due to variation of bit rate. As the bit rate increases, the power penalty increases.

**Figure 12:** Effect on BER performance of OFDM-PSK system due to variation of bit rate for laser linewidth, $\sigma = 2\text{nm}$ and for fiber length, L=1000 Km

At L=1000 Km and bit rate =2.0 GHz, the power penalty is 0.00. For the increase in bit rate from 2.0 GHz to 2.5 GHz for the same fiber length the increase in power penalty is 0.47 dB. When the
bit rate increases from 4.5 GHz to 5.0 GHz, power penalty increases from 1.01 dB to 1.1 dB i.e. increase in power penalty is 0.09 dB.

From the above result, we can see that the BER performance of OFDM-PSK system suffers due to variation of bit rate. When bit rate increases the power penalty required to receive base receiver sensitivity increases.

4.4. Comparison of BER Performance

To compare the performance of OFDM-PSK system with a conventional system, here OFDM-QAM system has been chosen. The bandwidth of this OFDM-QAM signal is same as that of OFDM-PSK signal [11]. This OFDM-PSK signal has been allowed to pass through the same fiber that was used for the transmission of OFDM-QAM signal.

Figure 13 shows the comparative value of power penalty required to achieve the base receiver sensitivity between OFDM-PSK system and OFDM-QAM system.

![Figure 13: BER Performance of OFDM-PSK System and OFDM-QAM System due to Variation of Fiber Length](image)

From the above result, it can be said that BER performance of OFDM-PSK signal is more improved than the OFDM-QAM signal for the variation of fiber length.

5. Conclusion

The performance of OFDM-PSK system has been investigated in presence of group velocity dispersion of a single mode fiber. The analysis is carried out in terms of BER and power penalty with fiber length, laser linewidth and bit rate. In this simulation, the power penalty caused by fiber dispersion is evaluated at BER of $10^{-9}$ for a single mode fiber having dispersion coefficient of 20 $ps/nm.km$ at 1.55 $\mu m$ wavelength. It is found that the influence of dispersion on OFDM-PSK spectrum is almost negligible. The effect of dispersion due to variation fiber length on OFDM-PSK system is not so severe than that of variation of laser linewidth. The proposed system model is also analyzed in terms of bit rate, the obtained results demonstrate that the increase in bit rate results more probability of error in bits and as a result power penalty increases more. To compare the performances of OFDM-PSK system with a conventional system, an OFDM-QAM system having the same bandwidth as that of OFDM-PSK signal has been sent through the same fiber. With the variation of fiber length, the BER performance of these two systems has been analyzed and obtained results have been compared. So the result obtained from this investigation proves that the OFDM-PSK system provides more improved performance on the variation of fiber length than an OFDM-QAM system. Hence at last it can be said that OFDM-QAM system has higher tolerance on the effect of fiber optic dispersion.
References