Mathematics Lessons Learned from Across the World

Prekindergarten–Grade 8

edited by
Johnny W. Lott
Carolyn J. Lott

NCTM
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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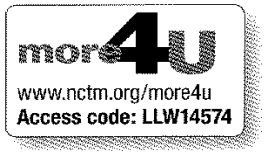

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Table of Contents

Introduction v

1. A Witch's Different Soups: Mathematical Problem Solving, Number Concepts 1
   Laurence Delacour—Sweden

2. Problem Solving with Basic Addition Facts 5
   Cheng-Yong Poon, Kee-Jiar Yeo, and Noor Azlan Ahmad Zanzali—Malaysia

3. Basic Operations Activities 11
   Nasim Asghari—Iran

4. Representation of Numbers by Place Value 17
   Olimpia Rosa Castro Mora—Peru

5. Investigating Area 23
   Christine Choquet—France

6. What Are We Talking About? 27
   Estela Vallejo Vargas—Peru

7. Number Pyramids as a Mathematically Rich Learning Environment for All Students 33
   Silke Ruwisch and Torsten Fritzlar—Germany

8. Mr. Splash Provides Problems 43
   Sharyn Livy, Tracey Muir, and Sandra Herbert—Australia

9. Who Will Win the Animal Race? Te tauomaoma o ngā kararehe 49
   Robin Averill and Dayle Anderson—New Zealand

10. Making Pink-Rose Drink 57
    Madihah Khalid and Nor Azura Hj Abdullah—Brunei

11. What Can We Learn from Natural Disasters to Prevent Loss of Life in the Future? 65
    Soledad Estrella, Raimundo Olfos, and Sergio Morales—Chile

12. Three Age-Changing Problem-Solving Activities 71
Hak Ping Tam and Yu-Jen Lu—Taiwan

13. Figures with Equal Areas in Convex Quadrilaterals 77
Kiril Bankov and Iliana Tsvetkova—Bulgaria

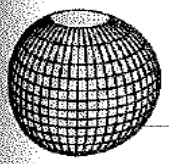
14. Areas of Circles and Irregular Shapes 85
Mohamed Sayed Ahmed Abou—Egypt

15. Taxicab Geometry: A Landscape of Investigation 89
Morten Blomhøj and Tinne Hoff Kjeldsen—Denmark

16. Folding Paper as a Learning Resource 97
Rosa María García Méndez and Mariana Sáiz—Mexico

17. The Signpost: Where Was This Photograph Taken? 103
Doug Clarke and Anne Roche—Australia

18. There is a Trouble in Paradise: Severe Water Shortage Problem in Cyprus 109
Nicholas Mousoulides and Constantinos Christou—Cyprus



Chapter 10

Making Pink-Rose Drink

Madihah Khalid
Nor Azura Hj Abdullah
Universiti Brunei Darussalam
Brunei

MATH CONTENT

Measurement of volume

Converting between measures

Multiply and divide compound quantities

Solve word problems

Use communication, connections, mental computation and estimation, visualization, and problem solving

MATERIALS NEEDED

Measuring cylinder

Water

Soda can filled with water

Red water (water mixed with red food coloring)

White water (water mixed with white food coloring)

Container that will hold about 2 L

Activity sheets for students

Poster paper or chart paper to record thinking and answers

Setting the Scene

Country of Context

Brunei has adopted English as the primary language of instruction. Thus, many students have to adapt to a new language while learning mathematics. Teachers of many classes in Brunei tend to be highly teacher centered, possibly as a result of students learning in a second language (Khalid 2008). However, more activities such as the one presented here allow flexibility in using groups for instruction as well as allowing more open-ended problems for class use. This activity is suitable for primary students ages eleven or older. The new Brunei Primary Mathematics Curriculum (CDD 2006) specifically mentions communication as one of the processes that need to be developed with the teaching of mathematical content.

Classroom Context

This sixty-minute activity allows for whole class instruction, group work in groups of four, and presentation time.

Teacher Notes

Task A

The first task is centered on the teacher talking to students about how to mix a drink for the class. Students then demonstrate how to measure and mix one version of the drink.

Task B

The teacher tells the entire class the following story:

Upin and Ipin are celebrating their birthday. They want to serve Opah's special pink-rose drink. So, Upin and Ipin asked for the recipe to make the drink. Opah listed the following items needed to make a jug of the drink:

- 1000 mL water
- 250 mL rose cordial syrup (or similar other flavored syrup available locally)
- 300 mL milk
- 260 mL soda

Upin and Ipin went to the grocery store and found the following:

- 1 bottle of mineral water is sold in a 1.5-L bottle
- 1 bottle of rose cordial is sold only in 1-liter bottles
- 1 1-L bottle of milk
- 1 can of soda contains 330 mL

The teacher asks the students to work in groups to help decide how much has to be bought to make five jugs of the drink. Additionally students are asked to record their thinking and answer on either poster paper or chart paper for sharing.

Answers may include the following:

- 4 bottles of mineral water
- 2 bottles of rose cordial
- 2 cartons of milk
- 4 cans of soda

A suggested rubric for assessing the posters and sharing the group work is given in table 10.1.

Table 10.1.
Suggested rubric for assessing posters

Level	Descriptors
4 Distinguished	• Shows complete understanding of a problem's mathematical concepts and procedures • Performs algorithms completely and correctly • Always uses appropriate mathematical notation (e.g., labels answers) • Identifies all relevant information and shows complete understanding • Selects a creative and efficient strategy for solving the problem • Shows clear evidence of a complete and systemic solution process • Gives an elaborate and effective explanation of the solution process • Succinctly explains "solution" to others • Uses precise mathematical language
3 Proficient	• Shows near complete understanding of a problem's mathematical concepts and procedures • Performs algorithms completely: computations may contain minor errors • Often uses appropriate mathematical notation • dentifies most relevant information and shows a general understanding • Selects an appropriate strategy for solving the problem • Shows some evidence of a systemic solution process • Gives an effective explanation of the solution process • Uses appropriate mathematical language
2 Apprentice	• Shows some understanding of a problem's mathematical concepts and procedures • Performs algorithms that may contain major computational errors • Seldom uses appropriate mathematical notation • Identifies some relevant information and shows limited understanding • Selects a strategy but unable to effectively arrive at a solution • Shows little evidence of a systemic solution process • Gives some explanation of the solution process but may be vague or difficult to interpret • Uses some mathematical language appropriately

Table 10.1. (continued)

Level	Descriptors
1 Novice	• Shows limited or no understanding of a problem's mathematical concepts or procedures • Performs algorithms that may contain major computational errors • Misuses or omits appropriate mathematical notation • Identifies little or no relevant information • Selects an inappropriate strategy for solving the problem • Shows no evidence of a systemic solution process • Gives minimal explanation of the solution process; may fail to explain or omits significant parts of the problem • Does not use mathematical language or uses it inappropriately

Extensions

This problem may be adapted with recipes from anywhere in the world. A different problem could be created by considering buying in local markets where more exact amounts can be purchased. Then a question of how much to buy may include fractional quantities not allowed when preexisting containers must be purchased. To do this, the teacher must be very aware of local buying and selling practices.

Research Notes

Communication is a mathematical practice that seems to be necessary in today's classrooms. Besides playing a role as an instrument for thinking mathematically, language is important for verbal representations, together with other representations such as diagrams, concrete materials, and symbols (Khalid 2008). In Brunei, some students are unwilling to speak in class as recognized by Martin (1996) when he said that apart from choral chanting, there is very little verbal output by students in Brunei classrooms. He added that when there is verbal output it is very often in the form of simple verbal recall statements. Salleh (2005) suggests that pupils in the Bruneian culture are not used to giving their views in a classroom setting. This activity is one of many now being tried to change the culture.

In the *Common Core State Standards for Mathematics*, the grade 4 (age 10) standards say that students should "use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit" (National Governors Association Center for Best Practices [NGA Center] and Council of Chief State School Officers [CCSSO] 2010, p. 31).

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Activity Sheet

Task A

Read the recipe for making the pink-rose drink; decide how to make any measurement conversions needed, and be prepared to demonstrate to the class how to make the drink in the empty jug.

Pink-Rose Drink Recipe

Ingredients

- 1000 mL water
- 250 mL red water
- 300 mL white water
- 260 mL soda

Supplies provided by teacher:

- 1.5 L bottle of water
- 1 L red water
- 1 L white water
- 1 can of soda water
- 1 empty jug

Task B

Upin and Ipin are celebrating their birthday. They want to serve Opah's special pink-rose drink. So, Upin and Ipin asked for the recipe to make the drink. Opah listed the following items needed to make a jug of the drink:

- 1000 mL water
- 250 mL rose cordial syrup (or similar other flavored syrup available locally)
- 300 mL milk
- 260 mL soda

Activity Sheet

Upin and Ipin went to the grocery store and found the following:

- 1 bottle of mineral water is sold in a 1.5-L bottle
- 1 bottle of rose cordial is sold only in 1-liter bottles
- 1 1-liter bottle of milk
- 1 can of soda contains 330 mL

Decide how much has to be bought to make 5 jugs of the drink.

Record your thinking and answers on either poster paper or chart paper for sharing.