



**TECHNICAL PROGRAM**  
of the International Workshop  
**Rogue waves, dissipative solitons, plasmonics, supercontinuum,**  
**and special fibres**

ICFO – The Institute of Photonic Sciences,  
Castelldefels (Barcelona),  
July 25-26, 2014

**Organizers:**

Prof. Nail Akhmediev (Australian National University)  
Dr. Yaroslav Kartashov (ICFO – The Institute of Photonic Sciences)

## Solitons in quadratic nonlinear media with $PT$ -symmetric potentials

Fatkhulla Abdullaev

*Kulliyah of Sciences, International Islamic University of Malaysia, Bandar Indera Mahkota, 25200 Kuantan Malaysia*

**Abstract:** The existence and stability of solitons in the quadratic nonlinear media with spatially localized parity-time- ( $PT$ )-symmetric modulation of the linear refractive index are studied. Families of stable one- and two-hump solitons are found. The exact bright solitonic solutions for special forms of the  $PT$ -symmetric potentials are found. The examples of dynamics and excitations of solitons obtained by numerical simulations are also given.

**Summary:** Recently a lot of attention has been devoted to investigations of nonlinear localized modes in the Kerr nonlinear media with  $PT$ -symmetric defects. It is interesting to address the possibility of the existence of defect modes and their stability in another class of widely used optical media, which is the  $\mathcal{X}^{(2)}$  materials. In the present paper, we study the existence of solitons in the media with quadratic nonlinearity and localized  $PT$ -symmetric potentials.

We consider the system of equations for the dimensionless fields of the first and second harmonics  $u_1, u_2$

$$iu_{1\xi} + d_1 u_{1\xi\xi} + [V_1(\xi) + iW_1(\xi)]u_1 + 2u_1^* u_2 = 0,$$

$$iu_{2\xi} + d_2 u_{2\xi\xi} + qu_2 + [V_2(\xi) + iW_2(\xi)]u_2 + u_1^2 = 0,$$

where  $\xi, \zeta$  are dimensionless transverse and longitudinal coordinates,  $V_i(\xi), W_i(\xi)$  are real and imaginary parts of the modulations of the refractive indices for both harmonics,  $q$  is the mismatch parameter.

First we consider the case when the gain-loss modulations are only in the first harmonic equation. Experimentally such model can describe a medium with active dopants, typically having rather narrow spectral resonances, i.e. affecting only a limited range of frequencies. We investigate analytically and numerically families of solitons for the particular choice of the localized  $PT$ -symmetric potential in the form

$$V_{1,2}(\xi) = V_{01,02} \sec h^2(\xi), W_1(\xi) = W_{01} \sec h(\xi) \tanh(\xi).$$

We found families of one- and two-hump solitons. The properties of nonlinear modes bifurcating from a linear limit of small fundamental harmonic fields are investigated. It is shown that the fundamental branch, bifurcating from the linear mode of the fundamental harmonic is limited in power. The power maximum decreases with the strength of the imaginary part of the refractive index. The modes bifurcating from the linear mode of the second harmonic can exist even above the  $PT$ -symmetry-breaking threshold. We show that the fundamental branch bifurcating from the linear limit can undergo a secondary bifurcation colliding with a branch of two-hump soliton solutions [1].

For the specially selected forms of the  $PT$ -symmetric defects we found the exact bright solitonic solutions. The solutions are of the form

$$u_1, u_2 \propto \sec h^2(\xi); u_1 \propto \sec h(\xi), u_2 \propto \sec h^2(\xi); u_1 \propto \sec h(\xi) \tanh(\xi), u_2 \propto \sec h^2(\xi)$$

It is demonstrated that solitonic solutions can exist for a gain-loss modulations without  $PT$ -symmetry [2].

### References

- 1.F.Moreira, F.Kh. Abdullaev, V.V. Konotop, and A. Yulin, Phys.Rev. A 86, 053815 (2012).
- 2.F.Kh.Abdullaev and B.A. Umarov, J.Phys. Conf.Series, in press (2014).