

# **MATHEMATICAL THINKING IN BRUNEI CURRICULUM: IMPLEMENTATION ISSUES AND CHALLENGES**

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*Mathematical thinking has been identified as one of the important processes that should be developed through the teaching of mathematics content in the new Brunei mathematics curriculum. This new curriculum was introduced starting from early 2006. The emphasis on mathematical thinking in the new curriculum makes teachers in the elementary level anxious and eager to learn more about how it should be incorporated in their teaching. Many issues and challenges that arise as a result of these would be discussed in this paper. This paper will also look at how mathematical thinking is defined in the curriculum, discuss the key window for considering mathematical thinking and also discuss how mathematical thinking can be developed through lessons.*

## **INTRODUCTION**

Mathematical thinking is the mathematical mode of thought that we use to solve any problem in our daily life including problems in classrooms. It can be defined as applying mathematical techniques, concepts and processes, either explicitly or implicitly, in finding the solution of problems. Students are expected to be able to use the fundamentals of mathematics to solve real-life problems including simple ones such as finding the number of ways three children can stand in a line. However, in many instances, Bruneian school children do not apply mathematical thinking (Clements, 2002). Many are not interested in mathematics, have mathematics phobia, could not relate mathematics to the real world and some suffer from the syndrome of learning mathematics for the sake of passing examinations. There are many who practice rote memorisation and would forget much of what they have learnt the following year.

Experts in many countries including Brunei now realised the importance of incorporating mathematical thinking into their curriculum. Children are encouraged to use thinking skills and problem solving strategies during mathematics lessons and not just learn mathematical skills and concepts. It is feared that if mathematical thinking is not emphasised, our children would end up learning mathematics by rote memorisation, without understanding and without the ability to think intelligently.

## **HOW MATHEMATICAL THINKING IS DEFINED IN BRUNEI CURRICULUM.**

The new Brunei Mathematics Syllabus (CDD, 2006a; 2006b) both for upper and lower Primary schools, were introduced and implemented in early 2006 starting with Primary 1 (pupils around 6 years old) and Primary 4 (pupils around 10 years old)

simultaneously. The main reason for change has more to do with reform effort, because teaching of mathematics in Brunei should be up to date with current research findings and technology.

Brunei Darussalam feels that having a well educated workforce with the ability to think and analyse, using varied reasoning and problem-solving skills in an integrated manner, as necessary for national development. Therefore the mathematical foundation at school level must adequately provide them with these basic skills. The aims of teaching/learning of mathematics as were stated in the curriculum document are to:

- provide children with skills and understanding which will enable them to cope confidently with the mathematics of everyday life
- stimulate children's interest in mathematics and encourage good learning habits, the ability to solve mathematical problems and think clearly and logically
- develop students ability to interpret and communicate mathematical ideas
- help children to appreciate how mathematics can help interpret the world in which they live, in line with the national aspirations of MIB, and
- provide a solid foundation for those who may wish to continue studies in mathematics

Among the specific objectives of the curriculum are to provide learners with the opportunity to:

- develop understanding of numbers; systems of measurement; basic geometrical relations
- tackle non-routine problems systematically
- apply what has been learned to solve real problems
- develop positive attitude and confidence
- conduct simple mathematical investigations
- develop the ability to calculate both mentally and on paper; estimate; communicate mathematical ideas clearly
- appreciate the aesthetic nature of mathematics

(CDD, 2006b, p. 3)

The aims stated above are interrelated. If children are provided with the skills and understanding that will enable them to cope with mathematics of everyday life, then it will stimulate children's interest in mathematics and encourage good learning habit and thus help children to appreciate mathematics. Concurrently, the ability to solve mathematical problems and to think clearly and logically will follow. The objectives are more specific. Children are to be provided with opportunities to develop positive attitude and confidence as well as understanding in a number of topics, develop the ability to calculate both mentally and on paper; estimate; communicate mathematical ideas clearly and hence be able to tackle and solve real problems systematically as well as conduct mathematical investigations. From these objectives it can be assumed

that mathematical thinking is embedded through out the teaching of mathematical content to give children the opportunity to appreciate the aesthetic nature of mathematics.

### The conceptual framework

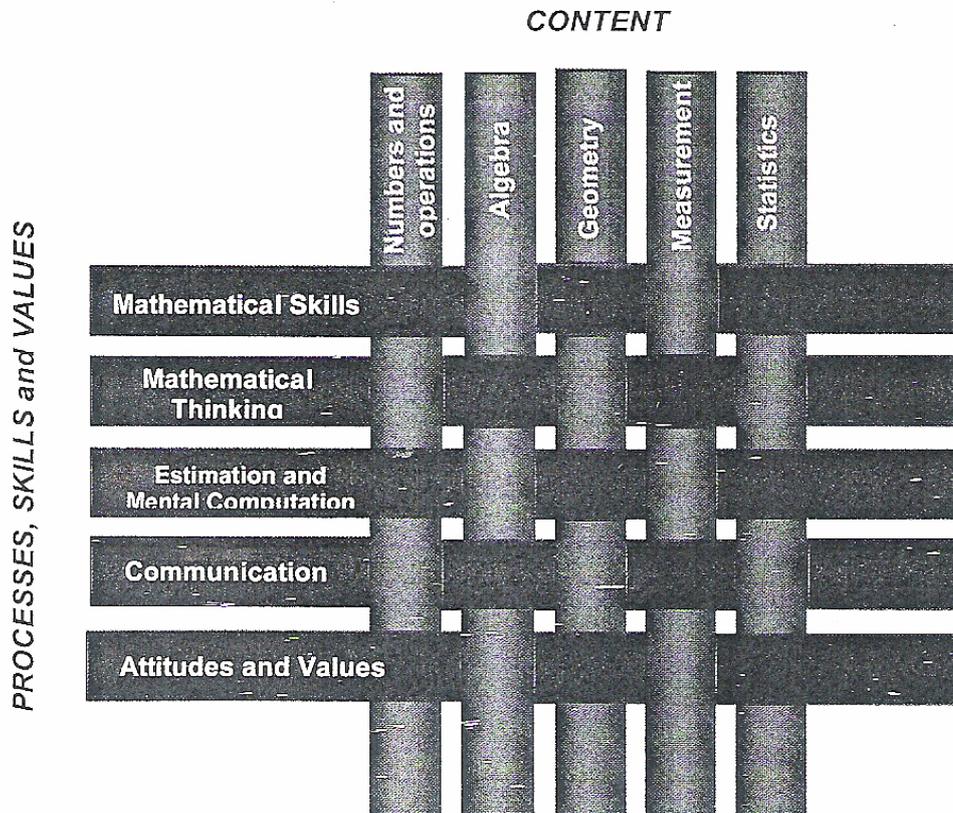


Figure 1: The Conceptual Framework

Figure 1 illustrates how content and process is presented in the curriculum document. They are supposed to relate and interweave with each other in a way that mathematical processes, skills and values are developed through the teaching of mathematical content, and mathematical content complements and supplements the development of mathematical processes, skills and values. All of these are supposed to happen during the teaching and learning of mathematics.

In the syllabus, mathematical thinking and problem solving are mentioned together. Teachers must encourage children to use thinking skills and problem solving strategies during mathematics lessons (CDD, 2006b, p. 7). Among the sub-processes of the mathematical thinking and problem solving processes that are listed in the syllabus are: guessing and checking, drawing diagrams, making lists, looking for patterns, working backwards, classifying, identifying attributes, sequencing, generalising, verifying, visualising, substituting, re-arranging, putting observation into words, making predictions as well as simplifying the problem and solving part of problems.

## **Instructional Approaches**

The curriculum recommends the use of a variety of representations to facilitate the development of the content knowledge and processes. Active learning is encouraged and the use of different representations is to be implemented according to the age and stages of the pupils. In the early years, concrete materials are supposed to help children develop basic mathematical concepts. As children move on, diagrams, real-world examples, verbal representations, ICT and symbolic representation will help children to proceed from the concrete to more abstract ways of thinking. The rationales for choosing the six different representations are:

*Concrete:* Since children at the elementary level are at the concrete operational stage according to Piaget, concrete experience is important in providing the foundation for the development of the abstract concepts. In fact, concrete mode also helps older pupils in secondary level of their schooling.

*Real-life:* Real life application is important to make children see the relevance of mathematics and will help them make sense of whatever mathematics they are learning.

*Diagram:* Use of diagrams, charts, graphs and pictures could clarify concepts more clearly to students and help build connection between concrete and symbolic models.

*Verbal:* Children need to be able to convey thoughts and interpret statements made by others in the process of communicating and learning mathematics. This mode also refers to thinking about, clarifying meanings and constructing simple arguments.

*ICT:* Incorporating ICT in lessons will enrich students learning experience by making lessons more interesting and more understandable because of the interactive nature of ICT.

*Symbolic:* In order for children to appreciate this mode, it should be linked with other modes since children usually find it difficult and meaningless although this is the popular mode in traditional teaching.

Understanding each representation and connecting them with one another requires mathematic thinking process and mathematical thinking should be incorporated when any of these representations is used. These representations can also clearly express the connotative meaning of concepts (Katagiri, 2006). Any of the representation can be used to represent the four fundamental processes which mathematical thinking very often proceeds – specialising, generalising, conjecturing and convincing, mentioned by Mason, Burton and Stacey (1982)

## **Assessment and Evaluation**

It is recommended in the new curriculum that assessment is conducted on a continuous basis through out the school year and assessment tasks should include “problem solving, mathematical thinking and creative work” (CDD, 2006, p. 6).

Teachers are encouraged to keep a portfolio for each child's work and this portfolio is to contain all assessed work plus samples of other work.

However, in Brunei, the most common assessment style are tests and examinations such as school based examination (mid and end of year examination) which still calls for rote memorisation and emphasised procedural skills. The importance paid to the centralised examination (PSR and PMB) makes teachers teach towards a test that do not really emphasised understanding.

### **KEY WINDOWS FOR CONSIDERING MATHEMATICAL THINKING**

According to Clements (2002), the traditional skill-drill approach is not working in Brunei and students struggle to make sense of the lessons. Teaching mathematics through problem solving is one way to make students make sense of the lessons and it encourages students to think critically and apply problem solving approaches. The focus should be on teaching mathematical topics through problem-solving contexts and enquiry-oriented environments where teachers can help students construct a deep understanding of mathematical ideas and processes by engaging them in creating, conjecturing, exploring, testing, and verifying. Pupils can also apply thinking and understanding to solve content related problem. Besides that, reflective thinking – which is also considered as another important aspect of mathematical thinking can also be encouraged through journal (log) writing after each lesson.

Cockcroft (1982) recommended problem solving as a means of developing mathematical thinking for daily living, saying that problem-solving ability lies 'at the heart of mathematics' (p.73) because it is the means by which mathematics can be applied to a variety of unfamiliar situations. Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill which can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer.

### **HOW TO DEVELOP MATHEMATICAL THINKING THROUGH THE LESSON**

The way mathematics is taught in Brunei is still very traditional (Lim (2000); Khoo (2001); Clements (2002)). A number of commentators have identified problems with the traditional way of teaching mathematics. There are weaknesses in pedagogy and assessments that emphasise rote learning procedures (mastery of mathematical techniques) where conceptual understanding and development of mathematics thinking is not a priority. The traditional method has produced students who fail to understand the connectedness of mathematics, unable to solve non-standard problems and lacks reasoning power especially providing arguments. Over time students come to adopt a passive role, and think of mathematics as a dead body of knowledge which they have to memorize, rather than as a set of higher-order thinking tools which will increase their abilities to deal with a complex world (Schoenfeld 1992).

In the Primary IV, V and VI mathematics attainment targets (CDD, 2006b, p. 42, 77 and 106 respectively), mathematical thinking and problem solving were listed at the end of other topics. The repercussion of this is that teachers would think that they would only have to deal with mathematical thinking and problem-solving last, that is after they have completed other topics and not to weave in mathematical thinking and problem solving through out the syllabus as discussed earlier. Teachers should be aware of this and should not have the tendency to only do problem solving at the end of the year, after every other syllabus was completed.

Mathematical thinking should be taught in every lesson. Lessons taught should include problems or questions by teachers (which can be routine or non-routine/open-ended or close-ended) that calls for thinking strategies to answer and to solve. Games and activities that emphasise mathematical thinking should also be included in the lessons so that lessons are fun and children are interested to learn. A few good examples are shown below (CDD, 2006b, P. 75 and p. 41). The first two is for recognizing patterns and relationships (induction) given addition and multiplication pattern:

Example 1: Fill in the blanks

Top row	1	2	3		5	6
Bottom row		8		16	20	

Example 2: Study the following number grid

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Describe the pattern you see in row, columns and diagonals of the addition facts tables.

Draw any 2x2 square on this table. Use addition to find patterns

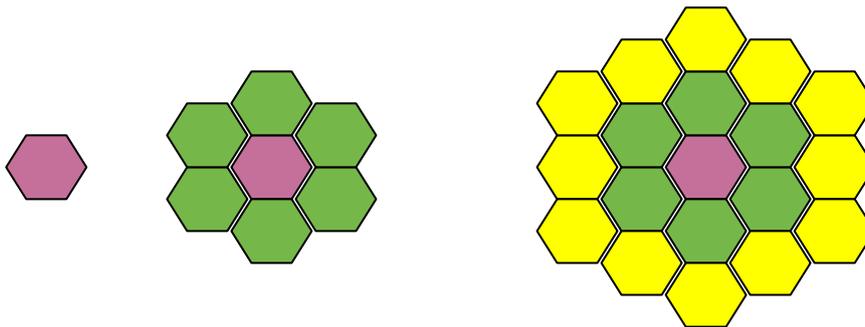
Example 3 and 4 would test pupils' induction and deduction ability which are two important aspects of mathematical thinking.

Example 3: Study the multiplication pattern for 3 and 7. They are the same because the digits in the ones place for both of these multiples are the same but in the reverse order.

3	6	9	12	15	18	21	24	27	30	33
7	14	21	28	35	42	49	56	63	70	77

Investigate similar patterns for other pair of multiples.

Example 4: Deduce simple rules from given patterns



## THE ISSUES AND CHALLENGES

There are certain issues and challenges that teachers in Brunei faced, in order to incorporate mathematical thinking and problem solving in their teaching. The issues involve standardized tests, teachers, school administrators, parents and pupils. The first is the presence of external examinations that encourage teacher-centered classroom practices as well as the pre-occupation with preparing students for examination (Majeed, Aldridge & Fraser, 2001). As long as there is standardized test at the end of the elementary level, teachers would continue to provide drill and practice to get good results. The stakeholders such as parents, school authorities, teachers, ministry of education officials and even pupils themselves are happy as long as they score high marks in standardized examinations. They are not bothered by conceptual understanding or the ability to solve non-routine problems.

From the studies done in Brunei by Lim (2000) and Clements (2002), it was found that Bruneian children has a low level of conceptual understanding of mathematical ideas involved and they rely more on procedural approaches or rote memorization. Clements (2002) also commented that the drill and practice routines has generated a situation where most of the students had come to regard mathematical “understanding” being the same as being able to answer examinations questions correctly.

Traditional testing methods in mathematics have often provided limited measures of student learning, and equally importantly, have proved to be of limited value for guiding student learning. The methods are often inconsistent with the increasing emphasis being placed on the ability of students to think analytically, to understand

and communicate, or to connect different aspects of knowledge in mathematics (e.g., Ridgway, 1988). One consequence of this type of teaching and assessment is that students learn in school that problems mostly have neat, unique solutions, and that methods to solve problems will be provided to them.

The second challenge is connected to the teachers. Besides being burdened to teach a heavy syllabus (before the introduction of this new one), they are not properly trained and are in the dark as how to teach mathematical thinking to the students. They need professional development and practical sessions such as “lesson study” to adequately understand what is expected from them. They have been used to cramming students with drill and practice because the emphasis was different with the old syllabus and teachers performance are measured by how well their students performed in public examinations.

Mathematics in the classroom should model the elements of active engagements in the practice of mathematical thinking if students are to come to understand and use mathematics and to learn to think mathematically. Learning mathematics is about learning to work in the ways that mathematicians work, and is about acquiring the thinking skills that mathematicians use. These skills are important for scientists as well as mathematicians. Pólya (1957) argued that mathematics resembles the physical sciences in its dependence on conjecture, insight, and discovery. He argued that for students to understand mathematics, their experience with mathematics must be consistent with the way mathematics is done by mathematicians.

The teacher is responsible for creating an intellectual environment in the classroom where serious engagement in mathematical thinking is the norm. Effective teaching requires deciding what aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask students having varied levels of expertise, and how to support students without taking over the process of thinking for them. Therefore, teachers needed the professional training very badly and the best way of providing this would be through lesson study. Lesson study has been proven to be a successful way of professional development in many countries especially Japan because it is conducted in a real environment.

The third challenge is changing the expectation of school administrators and ministry officials. Some school administrators are only interested in good results in order to boost their school image. Some teachers that I have met mentioned that many teachers are requested to complete teaching of the whole syllabus in July so that the rest of the month until end of October can be used for revision which activities are usually practicing the past-year papers again and again. There was no emphasis on mathematical thinking during these times at all.

Parents eagerness for their children to get good results made them send their children for extra tuition. Again outside tuition are not more than drill and practice. They are more concerned with their children getting good results because with good results, their children would be selected to attend premier (elite) schools. Lastly, the children

themselves are still too young to understand the consequences of their learning. They will listen to their parents or teachers because it is expected for them to obey the wishes of their elders. Furthermore, not being exposed to other kinds of teaching like teaching through problem-solving, makes them think that the best way to learn is by rote memorization.

## **DISCUSSION AND CONCLUSION**

Brunei still has a long way to go in order to make the implementation of mathematical thinking a success in their curriculum. However, the change to a new curriculum is a good start. More effort is needed to train teachers and to make teachers aware of the importance of mathematic thinking to make it a success and a good avenue is to implement lesson study for teachers nation wide. Another step is to deemphasize the importance of the standardised test at the end of the elementary year. The third is to change the nature of test from testing pupils on questions requiring procedural skills alone to questions that require thinking skills and problem-solving. Changing assessment is crucial in the introduction of any new curriculum.

A preliminary study was conducted to assess nine Primary 5 children from three semi-urban schools recently (Madiah, 2007). These students were asked to answer (explain via interview) five questions that require mathematical thinking, to test whether they can check results and correct mistakes, make plausible estimates of quantities that are unknown, model and define new concept, judge statements and create proves and organize unsorted data and draw conclusions (FLAG, 2006). It was found that children are not used to thinking about these problems, could not explain the problems or answers clearly, were afraid of giving wrong answer and give up easily. I am positive that they will perform better once they get used to thinking mathematically.

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