Australian Journal of Basic and Applied Sciences, 5(3): 51-59, 2011
ISSN 1991-8178

Islamic Mathematical Sciences

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Abstract: In Islam Mathematics is seen as an important tool for life. The Qur'an considers the rules of mathematics to be a pathway to discover God's will. This Paper discusses some of the contributions made in mathematics by some Muslim scholars. These remarkable inputs are due to the fact that mathematics is one of the renowned physical sciences which started right from antiquity. The inputs worth discussing in this paper include those of Al–Khawarizmi, Al–Kashi, Al–Khayyam and Al–Khazin. All have invented some vital concepts used in modern day mathematics.

Key words:

INTRODUCTION

It is being argued that Muslims adopted mathematics from the Indians and not from Greeks or the Chinese, those for India argue that an unnamed man came from India to Baghdad in 773 and taught the mathematical analysis of stars movements and composed an abridged version of a work pertaining to this matter, Al-fazari translated his work into Arabic. However, Muslim contributions to mathematics emerged in a full-fledged manner only in early ninth century. The earliest Muslim scholar to have written about arithmetic was al-Khawarimi, an active scholar in Baghdad during Ma'mun's era. The Arabic version of al-Khawarimi's arithmetic work is not available and believed to have lost but the later translation is available. Two versions of numerals spread in the Muslim world one in the East and one in the West, the eastern Muslims used the today's Arabic numeral while in the West the present day English of 1, 2, 3, was developed which is also Arabic and Indian.

The earliest known work on arithmetic was however, written by Ahmed Ibrahim al-uqlidisi in Damascus in 952 known as *al-Hisab al-Hindi or Hisab al-ghubar*. The significant aspect of al-uqlidisi's work is the author's use of decimal fractions an innovation that was until recently attributed to al-Kashi. The extraction of cube roots was first described by Kushay labiam, it was however Umar al-Khaym who systematized these problems. In arithmetics Muslims were concerned with the theory of numbers, ilm al-a'dad, as this filed was closely related to the study of magic squares and amicable numbers.

A work entitled *Kitab al-Mukhtasar fi hisab al-Jabra wa ' ilm al-muqabala* remains the most well known work in Islaimic al-gebra. The terms *gabr* and *muqabala* defer slightly from one writer to another, the first means the transposition of terms in order to make them all positive, by *muqaba* is meant the reduction of similar terms.

The differentiation between the two terms is to indicate the expressions used and reflect the origin of al-gebra in commerce and in dealing with complex questions of inheritance for instance term *mal* (property or wealth) was originally used for the unknown quantity in linear equations but later come to mean the square as opposed to the root. On the same note the word *shay* (thing) was applied to the quantity sought. It was Abu Kamal al-Shuja' (d.930) who made valuable contributions to the theory of *al-gebra* by turning it into powerful instrument for geometrical research.

Geometry sciences or '*Ilm al-Handasa*, developed during the creativity period (19-15 centuries); this period the translated works from India and Greek were corrected and annotated a phenomenon resulted Muslim writers make notable contributions to geometry. The remarkable work of jawhari (9th Century scholar) Nayrizi (10th Century scholars) and Nasr ad-Din at-Tusi (13th Century scholar) are only part of Islamic geometry. More than any other branch of mathematics geometry impinges of various sciences and technologies. Buildings, civil engineering, mechanical engineering, and other constructions, required the knowledge of geometry in one way or the other. Geometry was also applied to land surveying and to hydraulic works.

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In fact the term *handasah* was originally used to denote only geometry a more accurate usage than the modern use, as naming engineering *handasah* has became a source of ambiguity and reflect an attitude that engineering is totality an “applied science” as engineering is not simply an application of mathematics and science, for instance skills required for design and engineering structures are not necessarily mathematical. Muslims also applied geometry and geodetic measurement and to land surveying for fiscal purposes.

Trigonometry, an extension of geometry, is one of the branches of mathematics that Muslims made great contributions, as it constitutes close link with astronomy owing to the establishment of calendars and genomics. The theory and practice of Sundials, this constriction was widespread throughout the Muslim world. A work contains the first Arabic tables of tangents, adapted by al-Majriti al-Qordubi (d.1007), is attributed to Muhammad Musa but we are not certain whether or not this was Musa’s work. Habash Hasib al-marwazi who worked in Baghdad also developed tangent and cotangent with select and covenant used in genomics for ratios of right angle triangles. The tables by al-Habshi is considerable contribution to trigonometrically function, Habshi expressed the relationship of the right association of the sun, the declination and the inclination of the ecliptic. Al-Batani a famous as-Samarai (d.929) demonstrated the trigonometrically relationship, known already to al-Habshi, Abu Yousuf al-Baghdadi (d.1009) also contributed to these formulas.

2. The Nature of Mathematics:

Mathematics is one of the foundation knowledge of utmost importance in understanding the behavior of the universe and its contents, as well as the rationale behind every event or phenomena that takes place. The significance of this science is comparable to that of the basic necessities of life that are vital for survival in modern day living.

Mathematics is often understood as a study that deals with quantities, magnitudes, and the relations between numbers and symbols. In science, the study of a particular substance or an event, which usually include the study of its structure, order, and other properties evolved from elemental practices of counting, measuring, and description of the substance or event, which are all related to mathematics. Thus, mathematics can be thought of as a science that deals with logical reasoning and quantitative calculation. Over the years, its development has involved an increasing degree of idealization and abstraction of its subject.

Mathematics embraces three main subject matters including *arithmetic*, *geometry* and *analytical mathematics*. Each of these divisions is then divided into either pure or abstract, wherein abstract mathematics considers magnitude or quantity abstractly, without relation to matter. On the other hand, pure mathematics treats magnitude as subsisting in material bodies, and is consequently interwoven with physical considerations.

Arithmetic is the most useful of all sciences, and probably, no other branch of human knowledge that is more widely spread among the masses as arithmetics. Arithmetic consists of mathematics that is related to the use of integers, rational numbers, real numbers, or complex numbers under the operations of addition, subtraction, multiplication, and division. It is the foundation of all mathematics, pure or applied.

Geometry consists of mathematics that explains the use of properties, measurement, and relationships of points, lines, angles, surfaces, and solids. Under geometry, there are trigonometry and conic sections. Trigonometry is the branch of mathematics that deals with the relationships between the sides and the angles of triangles as well as the calculations based on them. The study of conic sections involves the study of curves that result from the intersection (section) of a (right circular) cone (conic) and a plane.

Analytical mathematics involves the separation of an intellectual or material into its constituent parts for individual study. It is also the study of such constituent parts and their interrelationships in making up a whole. Under analytical mathematics, there are algebra, analytical geometry, and calculus. Algebra is the branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or members of a specified set. It has been described as a way of doing arithmetic when unknown quantities are involved. In fact, every time one works with an equation or formula that includes an unknown quantity, the process being used is algebra. The letters used, represent quantities that have general relationships that hold for all members of the set. A pair of binary operator is usually defined on the set.

Although algebra uses the same basic operations with numbers as arithmetic such as addition, subtraction, multiplication, and division, the power of algebra is that it can solve problems that could not be figured out by arithmetic alone (such as a rate, time, or distance word problem). Analytic geometry, on the other hand, involves the use of algebra to study geometric properties thus, it operates on symbols which are defined in a coordinate system. Calculus is one of the branch of mathematics that deals with limits and the differentiation and integration of functions of one or more variables. This branch of mathematics provides serves as an important tool in engineering studies. An important tool in our daily life, the study mathematics continues to grow and new applications of it to our life are being discovered every day.

3. Al–khawarizmi's Contribution to Mathematics:

The involvement and the contributions of Al–Khawarizmi in the development of mathematics, can be described as one of the greatest. Some of the branches of mathematics that matured greatly under the deliberation of Al–Khawarizmi include the study of algebra and the discovery of the number zero, both of which, are of great significance in mathematics today.

Algebra may be divided into classical algebra (equation solving or 'find the unknown number' problems) and abstract algebra (the study of groups, rings, and fields), which is also called 'modern algebra'. The development of algebra can be traced or outlined from the history of Egyptian algebra, Babylonian algebra, Greek geometric algebra, Diophantine algebra, Hindu algebra, Islamic algebra, European algebra (since 1500) and lastly, modern algebra. Since algebra flourished from arithmetic, recognition of new numbers, such as irrationals, zero, negative numbers and complex numbers is an important part of its history. Algebraic notation progressed through various stages including the rhetorical (verbal) stage, the syncopated stage (use of abbreviated words), and the symbolic stage with which we are all familiar.

Algebra is the branch of analytical mathematics that investigates about quantities using letters to symbolize them. The *Mathematics Dictionary* defines algebra as a generalization of arithmetics – for example, the arithmetic fact that $2 + 2 + 2 = 3 \times 2$ or $4 + 4 + 4 = 3 \times 4$ and etc are all special cases of the general algebraic statement that $x + x + x = 3x$, where x is any number. In line with that, Ibn Khaldun, a famous Muslim scholar, defined algebra as a 'subdivision' of arithmetic. It is a craft in which it is possible to discover the unknown from the known data if there exists a relationship between them.

The earliest evidence of the use of algebra was contained in the Rhind papyrus, one of the oldest known mathematical manuscripts. Much later, a Greek mathematician by the name of Diophantus solved problems using what would now be called algebra, and even worked out a symbolism of his own. Some call him the "father of algebra", since his work inspired Al–Khawarizmi, who many also call the "father of algebra". Although Al–Khawarizmi wrote many books on different fields (such as geography), one of his works dealt with the development of solutions to mathematical problems in which there was an unknown quantity – that is, the study of algebra.

In 820 AD, Al–Khwarizmi wrote his classical work on algebra, the *Al–Jabr wa–al–Mugabala* (the science of reduction and cancellation). In 1140, the book was translated into Latin and the title became *Liber algebrae et almucabal*. The Arabic word for reduction, *Al–Jabr*, became what we know today as Algebra. Therefore, it can be said that algebra got its European name through the title of Al–Khawarizmi's math book. The title of his book, *Al–Jabr wa–al–Mugabala*, which was quite long, would often be shortened by translators to the word *Al–Jabr*.

Mathematicians in the West first learned about the new techniques of algebra from Al–Khawarizmi's book. They began describing any aspect of mathematics of solving for unknown quantities as *Al–Jabr.* By the 16th century it appeared in the English language as "algiebar". Eventually, *Al–Jabr* was spelled out using the characters of the Latin alphabet, ending up with, 'algebra'. By following these translations, it can be said that algebra was studied seriously in Europe during the 1500s, and by the end of the 17^{th} century, it was a fairly complicated mathematical tool.

Referring to the Al–Khawarizmi's own words, the purpose of his book, the *Al–Jabr wa–al–Mugabala*, was to teach:

"what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds are concerned."

This might not sound like the contents of an algebra text. In fact, only the first part of the book is a discussion of what we would today recognize as algebra. However, it is important to realize that the book was intended to be highly practical, and that algebra was introduced to solve real life problems that were part of everyday life in the Islamic empire at that time.

Early in the book, Al–Khawarizmi describes the natural numbers in terms that are almost funny to us who are so familiar with the system, but it is important to understand the new depth of abstraction and understanding here – Al–Khawarizmi said:

"When I consider what people generally want in calculating, I found that it always is a number. I also observed that every number is composed of units, and that any number may be divided into units. Moreover, I found that every number which may be expressed from one to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled just as before the units were: thus arise twenty, thirty, etc. until a hundred: then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand; ... so forth to the utmost limit of numeration."

Having introduced the natural numbers, Al–Khawarizmi also discussed on solutions of equations, which was the main topic of the first section of his book. His equations were either linear or quadratic and are composed of units, roots and squares. For example, to Al–Khawarizmi, a unit was a number, a root was x, and a square was x^2. However, unlike the familiar algebraic notations that we use today, Al–Khawarizmi's mathematics was purely documented in words with no symbols being used. Al–Khawarizmi used both algebraic methods of solution and the geometric method of completing the square.

He first reduces an equation to one of the six standard forms, using the operations of addition and subtraction, and then shows how to solve these standard types of equations. The reduction is carried out using the two operations of *Al–Jabr* and *Al–Mugabala*. The word *Al–Jabr* from the title of his work, means transposing a quantity from one side of an equation to another, whereas the word *Mugabala* signifies the simplification of the resulting expressions. Symbolically, *Al–Jabr* means restoring the balance of an equation by transposing terms. According to David Eugene Smith:

"In the 16th century it is found in English as algebra and almachabel, and in various other forms but was finally shortened to *algebra.* The words mean restoration and opposition, and one of the clearest explanations of their use is given by Beha Eddin (1600 AD) in his *Kholasat Al–Hisab* (essence of arithmetic): The member which is affected by a minus sign will be increased and the same added to the member, this being algebra; the homogeneous and equal terms will then be cancelled this being *Al– Muqabala*."

In other words, A*l–Jabr* is the process of removing the negative terms from an equation whereas *Al–Muqabala* is the process of reducing positive terms of the same power when they occur on both sides of an equation. That is, given $x^2 + 5x + 4 = 4 - 2x + 5x^3$, *Al– Jabr* gives $x^2 + 7x + 4 = 4 + 5x^3$, *Al– Muqabala* gives $x^2 + 7x = 5x^3$. After the reduction process, he solves the equation. For example, to solve the equation $x^2 + 10x = 39$ he writes:

"A square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square which combined with ten of its roots will give a sum total of 39? The manner of solving this type of equation is to take one–half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this which is 8, subtract from it half the roots, 5 leaving 3. The number three therefore represents one root of this square, which itself, of course is 9. Nine therefore gives the square."

Al–Khawarizmi continued his study of algebra on how the laws of arithmetic extend to his algebraic objects. For example he discussed on how the multiplication operation can be carried out on expressions such as $(a + bx)(c + dx)$. The next part of Al–Khawarizmi's algebra consists of applications and worked examples. He also investigated on the rules for finding the area of figures such as the circle, and also finding the volume of solids such as the sphere, cone, and pyramid. The final part of the book deals with the complicated Islamic rules for inheritance, but requires little from the earlier algebra beyond solving linear equations. Al–Khawarizmi's algebra can therefore be regarded as the foundation and cornerstone of the science of algebra.

In 1857, a Latin translation of a Muslim arithmetic text was discovered at the Library of the University of Cambridge. Entitled *Algoritmi de numero Indorum,* the work opens with the words: 'Spoken has Algoritmi. Let us give deserved praise to God, our leader and defender.' It is believed that this was a copy of Al–Khawarizmi's arithmetic text, which was translated into Latin in the twelfth century by an English scholar. This translated version of Al–Khawarizmi's text found its way to Italy, Spain, and England. Its name, through various modifications, became Alchwarizmi, Al–Karismi, Algoritmi, Algorismi, which named of the new art, Algorithm. Thus, A–Khawarizmi had also left his name to the history of mathematics in the form of Algorism, the old word for arithmetic.

An algorithm is a set of instructions that indicates a method for accomplishing a task. For example, an algorithm can be developed for tying a shoe, making cookies or determine the area of a circle. The individual steps are written that no judgment is ever required to successfully carry them out. The length of time required to complete an algorithm is directly dependent on a number of step involved. The more steps, the longer it takes to complete. Consequently, algorithm are classified as fast or slow depending on the speed at which allow a task to be completed. Typically, fast algorithms are usable while slow algorithms are unusable. In line with these descriptions of the word algorithm, Al–Khawarizmi had discussed on equations of first and second order, and found solutions of equations of the third and even fourth degree algorithm can be written to solve any conceivable problem. It is noteworthy that the word algorithm is derived from A–Khawarizmi's Latin form of his name, Algorismus.

One impressive achievement of Al–Khawarizmi in the area of mathematics is the introduction of 'Arabic numerals' in the ninth century. His arithmetics, which were synthesized Greek and Hindu knowledge, also contained his own contribution of fundamental importance to mathematics and science. 'Arabic' numerals, which Al–Khawarizmi refers to as Indian numerals, are the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These symbols were immensely more convenient to use than the Roman system (i, ii, iii, iv, v, vi, vii, viii, ix, etc.). Al–Khawarizmi's work on *Algoritmi de numero Indorum* (Concerning the Hindu Art of Reckoning) was based presumably on an Arabic translation of Brahmagupta in which, he gave a full account of the Hindu numerals. He was the first to expound the system with its digits 0,1,2,3, … ,9 and the decimal place value which was a fairly recent arrival from India. Number symbols that are similar to those used today are actually a very ancient accomplishment in India. The Hindu numerals, like most new mathematics were not welcomed by all. In 1299, there was a law in the commercial center of Florence forbidding their use – to this day this law is respected. This can be seen from the fact that checks are still written in longhand.

Al–Khawarizmi explained the use of the number zero, a numeral of fundamental importance developed by the Arabs. The number 307, therefore, means that there are 3 hundreds, 0 tens, and 7 ones. Although the value would be written as three hundred seven, the number could not be written in symbols without zero.

A key part of the example is also the concept called "positional notation," or place value. According to the previous example, the 3 in 307 is in the third place from the right, meaning it signifies group, or units, of the hundred. The second place is for group of ten, and the first is for groups of ones. The Hindu system was based on the value ten, and each change in the position of number meant at increase by ten. Thus, the fourth position was for thousand and so on. Similarly, he had also developed the decimal system – therefore, the overall system of numerals, 'algorithm' or 'algorism' is named after him. In addition to introducing the Indian system of numerals (now generally known as Arabic numerals), he developed at length several arithmetical procedures, including operations on fractions. It was through his work that the system of numerals was first introduced to Arabs and later to Europe, through its translations in European languages. Al–Khawarizmi's algebra and his introduction of the Hindu numerical system to the Arab world made him a highly significant link between time and cultures.

4. Al–kashi's Contribution to Mathematics:

A mathematician as well as an astronomer, Al–Kashi dedicated a larger portion of his years as scholar to the former. Al–Kashi, whom is well–known for his accurate calculation of the constant π (pi), had also interests in other areas such as the study of polygons, the study of error control in computations and many others.

Al–Kashi's extraordinary ability in optimizing mathematical problems as well as in producing calculations of controlled error elevated him above the rest, making him one of the first modern mathematicians. Al–Kashi used geometry as a tool for his calculations unlike others whom use it for constructions. He carried out his computations by applying techniques in which he controlled the maximum error through repeated checks at various stages of the computation. Al–Kashi invented a wide application of this iterative algorithm. Some of the applications of his iterative algorithm include calculations related to arches, vaults, and domes (qubba). Al–Kashi had also calculated surface area of stalactite vaults (muqarnas) – he had established approximate values for such surfaces.

In his *Key of Arithmetic*, one of Al–Kashi greatest works, he had discussed a large number of rules for determining the areas of plane figures such as triangles, quadrangles, and other regular polygons of the circle as well as other more complicated figures such as the volume and lateral surfaces of truncated pyramids and cones of the sphere as well as other regular polyhedral. Striving to achieve high precision in his calculations, Al–Kashi considered inscribed and circumscribed regular polygons with 3 x 22 = 805 306 368 sides unlike Archimedes who restricted himself to polygons with 3 x 25 = 96 sides. He reduced the calculation of k = 3 x 22 to extracting the square root twenty eight times in succession. Al–Kashi selected this value of k because of the minute difference that resulted between the perimeters of the polygons that inscribe into and circumscribe around a circle whose diameter D was 600 000 times the diameter of the Earth, which is lesser than the breadth of a horsehair. In Al–Kashi's estimates, D was the diameter opinion of the sphere of the fixed stars, so that natural scientists would never encounter large circle. He accomplished his calculations in sexagesimal fractions whose use made the extraction of roots easier as compared with the use of decimal fractions.

In another work of Al–Kashi, the *Al–Risala Al–Muhitiyya (Treatise on the Circumference)*, he determined the value of π with an accuracy that surpassed by far not only all previous attempts, but even the achievements of many later European scholars. Al–Kashi calculated the π in the same way as Archimedes did in his Measurement of the circle. After determining the perimeter of the inscribed polygon with 3×2^{28} sides, Al–Kashi calculated the perimeter for the related circumscribed polygon and assumed that the length of the circumference of the circle inscribed into and circumscribed around by the respective polygons, was equal to the arithmetical means of these perimeters. He discovered the value of the constant pi to be π = 3° 8 29‴ 44 ‴. Converting the value of the constant to the decimal system, he arrived at π = 3. 3.141593. His work on determining the value of π remained until 200 years. Al–Kashi also determined the value of 1° with the same degree of accuracy to that of π. He represented it as a root of a cubic equation and solved it by means of his own rapidly converging iterative method. Following these discoveries, Al–Kashi came up with the method of approximate integration to measure the volumes of truncated cones and hollow cupolas. There were no mathematicians after him who could challenge his work.

Al–Kashi also played a significant role in the architectural and civil engineering field. In medieval Italy, it was a common practice to pay artisans according to the surface area they had completed building. In the seventeenth–century, Safavid Iran architects were paid a percentage on each building based on the cubit measure of the height and thickness of the walls. This is asserted by O' Conner and Robertson when they quoted:

"The Persians determine the price for masons on the basis of the height and thickness of walls, which they measure by the cubit, like cloth. The King imposes no tax on the sale of buildings, but the Master Architect, that is Chief of Masons, takes two percent of inheritance allotments and sales. This officer also has a right to five percent on all edifices commissioned by the King. These are appraised when they are completed and the Master Architect, who has directed the construction, receives as his right and salary as much as five percent of the construction cost of each edifice."

The same custom had existed in the Arab world. It was also useful to know, more or less, the amount of material that is needed like gold for gilding, bricks for construction or paint, and such things. Payment made according to the cubit measure was common in Ottoman architectural practice where a team of architects and surveyors had to make cost estimates of projected buildings and supply preliminary drawings for various options. It is believed that the formulas and method of calculation used in determining these wages as well as the cost of the building materials involved were of Al–Kashi's.

In addition to facilitating estimates of wages and building materials before construction, Al–Kashi's formulas may also have been used in appraising the price of a building after its completion. His sophisticated formulas were like the simple formulas found in the arithmetic books that were useful for everyday life. This was al–Kashi's objective for writing his *Key of Arithmetic.*

Besides these, Al–Kashi also worked on solutions of system of equations where he had developed methods to find the n^{th} root of a number – a method that we recognize today as the Horner's method. It is believed that this method had also appeared in Chinese mathematics in 1303 in the *Ssu–yüan–yü–chien* (Precious Mirror of the Four Elements).

Due to the highly significant contributions of Al–Kashi's work to the history of the development of mathematics, some of the European's mathematicians have discussed some of his works whereas some have even utilized his work in their own. For instance, Dold–Samplonius had discussed several aspects of Al–Kashi's *Key to Arithmetic.*

Dold–Samplonius had discussed on the measurement of the muqarnas, which refers to types of decorations used to hide the edges and joints in building such as mosques and palaces. The decoration resembles as a stalactite and consists of three–dimensional polygons, some with plan surfaces, and some with curve surfaces. He had also discussed on the qubba, the dome used as funerary monument for a famous people. Some claim that Al–Kashi's use of the decimal fractions had enabled him to attain considerable fame. The generally held view that Stevin had been the first to introduce decimal fractions was shown to be false in 1948 when P. Luckey showed that in the *Key to Arithmetic*, Al–Kashi gives a clear description of decimal fraction just as Steven does. However the claim that Al–Kashi was the inventor of decimal fractions as was done by many mathematicians following the work of Luckey is somewhat doubtful and far from the truth since the idea had been present in the work of several mathematicians of Al–Karaji's school. Luckey had also commented on the algorithm developed by Al–Kashi in calculating the n^{th} roots of a number, a special case of the algorithm given many centuries later by Ruffini and Horner.

Although Al–Kashi's contributions were not as many as other Muslim scholars like Al–Khawarizmi and Al–Khayami, he is still regarded as one the greatest mathematicians in the Muslim society.

5. Khayyam's Contribution of Mathematics:

The work of Umar Khayyam is probably one of the works of Muslim scholars that are well–recognized in the West. According Seyyed Hossein Nasr:

"The name Umar Khayyam has become very familiar in the English–speaking world through the beautiful, although sometimes free, translation of his *Ruba'iyat* or *Quartrains* by Fitzgerald. In his own time, however, Khayyam was known as a metaphysician and scientist rather than as a poet, and he is remembered in Persia today most of all for his mathematical works … "

The contributions of Umar Khayyam in the field of mathematics are enormous – his focus areas spanning from algebra to geometry and many more. He had also researched and commented on the works of many Greek mathematicians, Euclid in particular, and Archimedes, to name a few.

In geometry, Umar was greatly interested in the theory of parallels. Umar tried to bridge the gap between the Parallel Postulate with the fourth postulate, which asserts that all right angles are equal. Umar accepted the first 28 propositions of Euclid's *Elements* but he sought to replace propositions 29 with eight propositions of his own which, in his opinion, should be added to the book of the *Elements.* Proposition 29 was precisely the proposition in which Euclid began the theory of parallel lines based on a doubtful fifth postulate. Umar took this as a point of departure for his theory of parallels, a principle that he attributed to Aristotle, namely that:

"Two convergent straight lines intersect and it is impossible that two convergent straight lines should diverge in the direction of convergence."

Scholars have noted that nothing similar to Umar's principle could be found in any of the known writings of Aristotle. To establish the validity of the hypothesis of the right angle, Umar employed the method of 'reductio ad absurdurn' (method of contradiction), which is one of the most important logical tools in mathematics.

Umar's formulation and application of the principle to serve as the starting point for his theory of parallel showed that Umar was fully aware of his predecessors who had successively committed the logical mistake of 'petitio principi' in their proofs. That he did not fall into the same error was indeed, one of Umar's achievements. Some of Umar's conclusions drawn from the hypothesis of acute or obtuse angles are essentially the same as the basic theorems of the non–Euclidean theory of parallel lines of Bolyai–Lobachevski and Riemann, respectively. Umar's theory of parallels largely influenced the work of later Muslim scholars, particularly Nasir al–Din, who in turn influenced the development of the theory of parallels in Europe in the seventeenth and eighteenth centuries to a considerable degree.

Another interest of Umar was in the theory of ratio and proportions. Umar replaced Euclid's definition by applying his first principle and defined equal ratios by what might be described as something like a limit process: the ratios are equal when they can be expressed by the ratio of integer numbers with as great a degree of accuracy as required. Umar's proof lay in establishing the equivalence of the definitions of equality and inequalities in both commensurable and incommensurable ratios.

He based his demonstrations on an important theorem on the existence of the fourth proportional d with the three given magnitudes a, b, and c. He tried to prove it by using the principle of the infinite divisibility of magnitude, which unfortunately was insufficient for his purpose. However, his work marked the first attempt at a general demonstration of the theorem, since the Greeks had not treated it in a general way. Umar's work on ratio and proportion was taken up later by Muslim mathematicians, particularly Nasir al–Din and his followers. European mathematicians of the fifteenth to the seventeenth centuries took up similar studies but it is difficult to assess the influence of Umar and his followers in the East upon the later mathematicians in the West.

Umar's contribution in algebra is probably one of the most significant ones in the history of development of mathematics. Umar's greatest original contribution to algebra was his systematic attempt to solve all types of cubic equations by the intersections of conic section after having classified them completely. According to Sarton, this achievement due to Umar was one of the highest peaks – perhaps the very highest – of medieval mathematics. This work is found in his algebra, which he completed in Samarqand and dedicated to his patron, the chief judge of that city, Abu Tahir in 1070. The study of cubic equations began, first with Archimedes' problem of the section by a plane of a given sphere into two segments of which the volumes are in a given ratio. About a century later, Muslim mathematicians found a geometrical solution to isolated cubic equation by using conic sections.

However, Umar's construction of a geometrical theory of cubic equations may account for the most successful accomplishment ever undertaken. In giving some background of the mathematics before Umar Khayyam's time, Kasir stated that:

"It is true that the Greeks were aware of the existence of equations of the third degree, for we know that they attempted to solve isolated cases of such equations by means of geometry. Their solution however, was incomplete, although the results which they left, as we expressed by algebraic symbols would take the form of the cubic equation. But there is no trace of algebra in any of the geometrical work of the Greeks."

Umar noted in this treatise that he had found an approximated solution with an error of less than one percent, and he repetitively remarked that it was impossible to solve this equation by Euclidean means (with compass and straight edge) since it required the use of conic sections. This is the first statement in the mathematical literature so far extant, that equations of the third degree cannot be generally solved by Euclidean means. In 1637 Descartes presented the same particular supposition, which was later proved by F. Wantzel in 1837.

Umar's conception of algebra is indicated by his definition as the science that aims at the determination of numerical and geometrical unknowns. He went on further when he mentioned that if the numerical solution is not supplemented by geometrical construction or vice versa, then "the art of algebra could not be verified. Umar was the first to make a systematic and comprehensive classification of all types of equation in which no term of degree higher than three occurs. His task was to solve every type of the equations by supplementing the numerical solution of the equation by a geometric construction. Unfortunately, when he began to consider equations of the third degree he was forced to abandon this procedure. However, he did not despair in his failure to apply the algebraic method of the solution of the cubic equation and in fact, he challenged the algebraists to come and overcome the difficulty. There is overwhelming evidence that Umar tried to find an algebraic method for these roots when he wrote:

"As for a demonstration of these types, if the object of the problem is an absolute number, neither we nor any of the algebraists have succeeded, except, in the case of the first three–degree, namely number, thing, and square, but maybe those after us will."

With the above statement of Umar, the statements of Boyer (1985), Cajori (1931), Smith (1958), and Story (1918) who pointed out that Umar mistakenly believed that algebraic solutions to general cubic equations were impossible can be easily rejected. Umar merely stated that he did not succeed in solving it algebraically but he knew that it was possible. Umar's hope became a reality, for in the 16th century; the cubic equation was solved algebraically by the Italian mathematicians, Scipio del Ferro (1515), Cardano (1545) and Tartaglia (1535) even though the solution was very much obscure and uncertain, and gave rise to much dispute.

In his Algebra, Umar asserted that he could find higher powers of numbers by a law which he had discovered which did not depend on geometric figures. He wrote in his algebra:

"I have taught how to find the sides of the square–square, of the square–cube, of the cube–cube etc. to any extent, which no one had previously done."

Umar referred to a book he had written which dealt with this problem but unfortunately up until today the manuscript of this work has not been found. The next significant advance on this problem was made by Newton in the seventeenth century when he proved the binomial theorem for any rational number. If it is granted that this discovery of the Pascal Triangle was first made by Umar, then Umar has the prior claim to a result which up to recent years was usually credited to Michael Stifel (1544) but which already occurred in the work of Al–Kashi, a Persian mathematician during the first decades of the fifteenth century.

6. Al–khazin's Contribution to Mathematics:

As a mathematician Al–Khazin made a number of contributions to the study of mathematics particularly in the field of number theory.

Al–Khazin wrote a commentary on Ptolemy's Almagest. Only one fragment of this commentary has survived and a translation of it contains a discussion by al–Khazin of Ptolemy's argument that the universe is spherical. Ptolemy wrote:

"of different figures of equal perimeter, the one with more angles is greater in capacity, and therefore it is necessary that a circle is the greatest of surfaces (i.e. of all plane figures with a constant perimeter) and the sphere the greatest of solids."

Al–Khazin gives 19 propositions relating to this statement by Ptolemy. The most interesting results show that an equilateral triangle has a greater area than any isosceles or scalene triangle with the same perimeter. Other results among the 19 are based on propositions given by Archimedes in on the sphere and cylinder. The lack of proof to these theories is probably not due to Al–Khazin – it is probable that he had taken these theories from some unknown source.

The work Al–Khazin which is believed to have been motivated by the work of Al–Khujandi. Al–Khujandi claimed to have proved that $x^3 + y^3 = z^3$ is impossible for whole numbers x, y, z which of course is the n = 3 case of Fermat's Last Theorem. In a letter Al–Khazin wrote:

"I demonstrate earlier ... that what Abu Muhammad Al–Khujandi advanced – may God have mercy on him – in his demonstration that the sum of two cubic numbers is not a cube is defective and incorrect."

This seems to have motivated further correspondence on number theory between Al–Khazin and other Muslim mathematicians. His main was to show how, if we are given a number, to find a square number so that if the given number were added to it or subtracted from it the result would be square. In modern notation the problem is given a natural number a, find natural numbers x, y, z so that $x^2 + a = y^2$ and $x^2 - a = z^2$. Al–Khazin proves that the existence of x, y, z with these properties is equivalent to the existence of natural numbers u, v with $a = 2uv$, and $u^2 + v^2$ is a square (in fact $u^2 + v^2 = x^2$). The smallest example of a satisfying these conditions is 24 which Al–Khazin gives $5^2 + 24 = 7^2$, $5^2 - 24 = 1^2$.

He also gives a = 96 with $10^2 + 96 = 14^2$, $10^2 - 96 = 2^2$. Although, rather strangely, he seems to discount this case by other statements. Rashed suggests this may be because 96 = 2 x 48 = 2 x 6 x 8 and $6^2 + 8^2 = 10^2$ is not a primitive Pythagorean triple.

Al–Khazin had also worked on the theory of sinus for spherical triangles. He had written on this topic in his *Al–Masa'il Al–Adadiyya*, an article he wrote together with Ibn Al–Qiffi in *Matalib Juz'iyya mayl Al–Muyul Al–Juz'iyya wa Al–matali fi–kuraal–mustakima*. In another book of his, the *Kitab fi Al–Ab'ad Al–Ajram*, Al–Khazin reported investigations on the diameter of the star between 1st distances to the 6th. Even though the observation is done quite rough, no good explanation on what the diameter is all about, no calculation and theory is found, but further it was develop and modified, inspired the other thinker to give splendid astronomy discoveries.

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