

J

ournal

OF SCIENCE AND MATHEMATICS
EDUCATION IN SOUTHEAST ASIA

JUNE 2003

Volume 26 Number 1

ISSN 0126-7663



Southeast Asian
Ministers of Education
Organisation



Regional Centre
for Education in Science
and Mathematics

COMPLIMENTARY
SEAMEO - REF

JOURNAL

OF SCIENCE AND MATHEMATICS
EDUCATION IN SOUTHEAST ASIA

VOLUME: XXVI/NO. 1 JUNE 2003 ISSN 0126-7663

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SEAMEO Regional Centre for Education in Science and Mathematics
Penang, Malaysia

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The Journal of Science and Mathematics Education in Southeast Asia, the official journal of SEAMEO Regional Centre for Education in Science and Mathematics (RECSAM), is published twice a year in June and December.

It is a medium for the exchange of ideas and the dissemination of information in science, mathematics and technology education for a wide range of readership. Submission of quality papers relevant to the teaching and learning of science, mathematics and technology education at the primary and secondary levels and the area of teacher-education is invited.

Readers wishing to submit manuscripts for publication should refer to the notes on the inside back cover.

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Published and Printed by the SEAMEO Regional Centre for Education in Science and Mathematics (RECSAM), Gelugor, Penang, Malaysia.

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MAKING SENSE OF RADIAN MEASURE: EXPERIENCES WITH TECHNICAL STUDENTS IN BRUNEI DARUSSALAM

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Students sometimes wonder why they need to learn certain mathematics topics in schools. Frequently, fragmented information offered to students is of little use or application and was often delivered poorly to them. Topics or areas that do not make sense cause them to lose interest and lead to poor performance. This in turn leads them to develop a certain phobia to mathematics and to develop the habit of studying by memorising formulae and facts just to pass examinations. The Radian measure is one of the topics that comes under this category for students in Brunei, who often have no idea why they need to learn about angles measured in radians (after becoming familiar with measuring angles in degrees). This paper examines some of the instructional practices used in the process of teaching the topic of radian measure to the students of a Technical College in Brunei Darussalam. The study set out to investigate what the students already knew about radians, where their problems lay in understanding, and what might motivate them in learning mathematics in general. The instructional approach is examined and then the end-results are evaluated. Qualitative and quantitative data were collected through pre-tests and post-tests, interviews, observations and survey questionnaires.

INTRODUCTION

Students enrol in technical colleges in Brunei, particularly at the Maktab Teknik Sultan Saiful Rijal, in order to pursue the Diploma and Pre-Diploma courses such as Aircraft Engineering, Electrical and Electronics Engineering, Mechanical Engineering, Construction, Estate Management, Science,

can be generated by providing students with their desired classroom environments – that is, by making them aware of the history behind what they are learning through the teaching of mathematics in a more applicative and integrated form, by solving more authentic real-world problems, by introducing collaborative and cooperative learning and also by effective instructional strategies.

Research suggests that we need to change mathematics education because students are not learning the math they need or are expected to know (MathStar, 2003). Students' knowledge and skills are apparently learned without much depth of conceptual understanding. This problem becomes evident when we study performance on related items that require students to extend these skills, reason about them, or explain why they work. We have used the same basic methods to teach mathematics in almost all classrooms around the world. In traditional mathematics teaching, the emphasis is on teaching procedures; little attention is given to helping students develop conceptual ideas, or to connecting the procedures they are learning with the concepts that show why they work. It is interesting to see that most vocational trainers are not aware of new ideas about the use of mathematics. Most of them use the mathematical routines, merely algebraic algorithms, which they learned themselves a long time ago (Van der Kooij, 2001).

The most important finding for the designers of the new programme developed for vocational education (engineering) for students in the age group 16-20 in Netherlands was the fact that mathematics as it is taught in general secondary education, is far away from what is needed for most fields of technical practice (Van der Kooij, 2001). In Brueni's case, the abstract, procedural approach they experienced in secondary schools was not compatible with the concrete nature of technical studies. Furthermore, when practitioners use mathematical tools to solve a problem, they always stay close to the context of the problem. Mathematicians are very well trained in handling value-free numbers as if they are entities in themselves, but most users of mathematics are using numbers as values for (physical) entities. Most of the time, they use the context of the problem as a kind of anchor for their calculations. The context of the problem that has to be solved is used as a guide for the way in which they choose a solving strategy. In general, these strategies are not easily translated into an algorithm

that can be used to solve similar problems in different contexts (Van der Kooij, 2001).

Vocational/technical educators have been promoting hands-on learning experience and knowledge transfer and this tradition is now expanded through the advent of constructivism. Constructivist pedagogy is reflected in current school-to-work initiatives that require students to be active learners who draw upon perceptual, cognitive and affective learning dimension as they test academic theories via tangible real-world applications (Brown, 1998).

BACKGROUND OF STUDY

The study described in this paper is part of a larger study with the title 'Enhancing Mathematical Achievement of Technical Education Students in Brunei Darussalam Using a Teaching and Learning Package.' In this larger study, students and teachers were surveyed on the actual and preferred teaching and learning environment via questionnaires and interviews. This information, coupled with information from the literature review was used to design a teaching and learning package that is students-centred and constructivist in nature. The package was implemented and then evaluated. The package consists of:

- An introduction to the course – including a background history of the topic, the requirements for this course, the syllabus and the timeline.
- Recommendation for teaching the course – includes curriculum, teaching, teaching style and assessment.
- Recommendation for students – includes a work contract.
- Lesson plans and a lesson plan checklist.
- Activity sheets.
- Worksheets for class work and homework.
- Assessment sheets – includes project work and tests.

- Implementation of the teaching material; and finally the
- Evaluation of the whole process.

The following research questions guided the study:

- What are the problems that students face in understanding radian measure?
- What is the students' actual and preferred classroom environment?
- What is the best way for a teacher to address this problem?
- What were the positive aspects with the way the class was conducted?
- How effective is intervention process?
- What are the implications for teaching and learning?

METHODOLOGY

A mixed qualitative and quantitative methodology approach was adopted for the research.

The qualitative components used in the study consisted of interviews, classroom observation and content analysis, whilst the quantitative components included a diagnostic test (pre-test), an achievement test (post-test) and survey questionnaires.

Designing the teaching/learning material

The instructional design and sequencing of the lessons was based on the current mathematics curriculum for the Bruneian National Diploma Year 1 and was introduced in November 1999. The curriculum committee of technical education department implemented it in early 2000. Among other things, it

- encourages the development of thinking skills through an intuitive rather than rigorous instructional approach.
- emphasises on the integration with other subject disciplines and learner directed application of skills.
- ensures that learning and assessment throughout will be work related.

(Programme Guide, 1999)

Frudenthal (1987) as quoted by (Romberg, 2001) argued that instruction should begin with activities that contribute to mathematization and that, when students learn mathematics divorced from experiential reality, it is quickly forgotten, or that they are unable to apply it. Students make sense of a situation by seeing and extracting the mathematics imbedded in activities and solving problems that they can relate to. This will motivate them to read and think about mathematics on their own.

From the classroom environment survey that was conducted earlier in the study, it was concluded that the students would prefer a more enhanced Student Cohesiveness, Teacher Support, Involvement, Innovation, Cooperation, Task Orientation, Individualisation and Relevance in their mathematics classes. Taking these results into account and considering the current instructional trend in technical education from the literature review, the features of the instructional approach that were emphasized during the implementation were:

- **Real life Problem Solving**
Each lesson emphasized the application of mathematics to real world situations that were relevant to the students' course of study and everyday life.
- **Balanced Instruction**
Besides emphasizing a on conceptual and contextual approach, some of the proven traditional approach were still used. *Practice* is stressed here because the authors believe that procedural proficiency is important. It sometimes led to a better conceptual understanding and problem solving because all of the three skills is interconnected. Paper and pencil skills are practical in certain situations, are not necessarily hard to acquire and are widely expected. The drawback is only when it is overemphasized with insufficient attention given to the conceptual basis for the procedures (Isaacs, 2002).
- **Appropriate Use of Technology**
The lesson included many activities where learning is enhanced through the use of a scientific calculator. No graphic calculators have

been used as yet in Bruneian classrooms. Computers were also used when looking for reference material and completing the project given. Links to interesting mathematics sites were given every now and then and students were encouraged to visit them.

- **Cooperative Learning**

Students were divided into groups of four. A lot of the activities given were to be completed within the group. Besides this, lessons included time for whole group instruction as well as individual learning.

- **Varieties of Activities**

To avoid the students from getting bored, A variety of activities were created. Having interest and motivation in studies was considered very important.

- **Non-Traditional Assessment. (Authentic Assessment)**

In line with a resolve to vary the assessment methods, non-traditional assessment was used. Besides solving of real life problems in class, these kinds of problems were also set as assignments and at the end of this topic, students were to develop a concept map of what they had learnt.

RESULTS

Pre-test

The pre-test (see Appendix 4) consisted of ten objective questions in section A and ten subjective, open-ended questions in Section B. Three questions from section A and four from section B were on the topic of radian measure. This instrument also acted as a diagnostic test besides being used as a tool to compare students' understanding later.

SECTION A

Table 1

*Percentage of students answering Q1 – Q3 in section**A. Correct answer for Q1 is D, Q2 is B and Q3 is B*

		A	B	C	D	E
Q1	RTE	33	0	8	59	0
(sc)	ELE	29	0	0	71	0
Q2	RTE	67	0	25	0	0
(C)	ELE	71	14	0	7	0
Q3	RTE	0	8	33	17	25
(P)	ELE	0	0	93	0	0

SECTION B

Table 2

Percentage of students answering Q1 – Q4 in section B

Q1	RTE	0% - radian & gradian	75% - radian only	25% - no answer
	ELE	17% - radian & gradian	67% - radian only	17% - no answer
Q2	RTE	17% - correct	0% - correct	83% - could not do
	ELE	0% - nearly correct	8% - got the formula but use it for the wrong one	92% - could not do
Q3	RTE	8% - correct	25% - could do something	67% - could not do
	ELE	0% - correct	8% - could do something	92% - could not do
Q4	RTE	0% - correct	8% - could do 1 step	92% - could not do
	ELE	8% - correct	8% - 1 step and 25%, 2 steps	58% - could not do

From the pre-test results above, it can be concluded that not many students know or remember that there are other units for measuring angle besides the 'degree' they are familiar with. In the RTE class, 83% of the students

and in the ELE class, 67% of the students did not include angle in radian as the answer to the question. Only 17% in the ELE class and 25% in the ELE class gave both degree and radian angle measurement. Sixty seven percent from both classes were able give the right answer as to how the angle in radian is measured, and the same percentage could manipulate numbers to give the right answer to the area of sector shown in the diagram.

For section B, 75% of the students in the RTE class, and 67% of the students in the ELE class, knew only radians as the other alternative unit of measurement. Only 17% in the ELE class could give two other unit of angle measurement. Only 17% from RTE class and none from ELE class could convert angles from degree to radian and vice versa. Although they could define the angle in radians as in Q2, Section A, they fail to see that by transposing the numbers they could solve Q3 in Section B. Only 8% from RTE class could solved the problem. Question 4 of section B required a deeper understanding and problem-solving skill. Only 8% from the ELE class could obtain the correct solution.

From interviews and the results above, it is clear that students are not very familiar with other units of angle measurement. Many thought that degrees are a good enough measurement for angles and could not see the benefit of learning about radians.

As for finding the arc length and area of sector from section A, 67% of the students from both classes were able to solve the problem. However, in Section B it could be seen that 83% in RTE and more than 92% in ELE were unable to convert angles in degrees to radians and vice versa. The score was equally poor in questions 3 and 4 where it involved word problems and problem solving. It could be concluded that students' have problems in problem solving and word problems.

The average score on the pre-test was 25% and the average was approximately the same in both classes.

Implementing the material

The instructional approach for implementation follows the guidelines of the Core-Plus Mathematics Project (CPMP), Department of Mathematics and Statistics, Western Michigan University. Lessons are organized around cycles of instructional activities intended primarily for small-group work

in the classroom and for individual work outside of the classroom. The five-phase cycle of classroom activities, (launch, explore, summarize and share (which I call *checkpoint* in my lesson plan), apply and practice) were designed to actively engage students in investigating and making sense of problem situations, in constructing important mathematical concepts and methods, in generalizing and proving mathematical relationships, and in communicating their thinking and the results of their efforts. Many classroom activities are designed to be completed by students working together collaboratively in heterogeneous groupings –in pairs or in groups of three or four.

Lesson One: (Refer to Appendix 1)

The lesson was launched with a question, 'What does an angle of $p/4$ mean?' After a brief discussion, a review of angles in the activity sheet was carried out. The discussion involving the question of 'Why is 1 degree $1/360^{\text{th}}$ of a circle? What other unit of angle measurement do you know?' was particularly interesting. This created an opportunity to explain that it might be because the Babylonians who invented the degree, had a base 60 numerical system. It was an ancient creation and in fact anybody could create one's own unit of measurement. Those units still in use are the radian – 400 in a circle, and the radian – 2π in a circle. It is convenient to define an angle based on the circumference of a circle of radius 1 because measuring an arc length is equivalent to measuring the angle. The activity sheet continues to look at the relationship between degree and radian and students are to discover for themselves how to convert degree to radian and vice versa. Later, they were given the application worksheet to work on.

Lesson Two: (Refer to Appendix 2)

This lesson concerns the arc length and area of sectors. After a brief review of the previous lesson, the second lesson was launched with two questions:

- Q1: You are given a choice of running two tracks today. One track is a quarter of a circle whose radius is 20 m and the other is one third of a circle whose radius is 15 m. You are feeling tired today and would like to choose the shortest possible route. Which one would you choose?

Q2: A whole pizza with radius 8 cm is cut into six pieces. A piece of that pizza costs \$2.00. Another whole pizza with radius 10 cm is cut into eight pieces and a piece of that pizza costs \$2.50. Assuming that the ingredients are the same, which one is more value for money?

Following this, students started on the activity sheet where they were to discover for themselves how to obtain the arc lengths and the area of sectors. At the end of the activity sheet, they were required to summarise their discoveries and the return to the two initial questions.

Lesson Three: (Refer to Appendix 3)

In this lesson, students were required to apply arc length and sector area formula to solve a range of work-related problems. An application worksheet of four problems was given out and each group was asked to present the solution to one problem to the whole class. A particularly interesting problem to the students is the one where Eratosthenes of 240 B.C measured the radius of the earth. This problem makes the students see how useful radian measure is. Students were encouraged to do more problems on their own to hone their skill and understanding.

At the end of the lesson, the students were asked to prepare a concept map of what they have learnt in this topic (radian measure).

Comments

From observation, it could be seen that most students enjoy the activities and the authentic problems. But one or two in-service students (mature students) who were used to the traditional teaching method were not happy. They were reluctant to join the discussion in the beginning but towards the end, their attitude improved. This observation was confirmed from interviews with some of the students. One said that they enjoyed the class and thought they could now relate mathematics to everyday life. Another says that she had never enjoyed a mathematics class before this one. The negative comments came from the mature student who said that at first she could not follow the approach and thought that the activity sheets were a waste of time because she could not see the purpose of it. However she said that she enjoyed working on the authentic problems.

The teacher of RTE who observed the learning process thought that the different style and the activities should be useful. The teacher of ELE said that he had always tried to relate his lessons to real-world, authentic problems and thought that he could use some of the problems that were used in class.

Result: (From the post-test)

The post-test (see Appendix 5) was given four weeks after they had finished learning about radian measure. The post-test contains the same number of questions and is of the same format and are parallel to the pre-test. The result is as shown in Table 3 and Table 4.

SECTION A

Table 3

*Percentage of students answering Q1 – Q3 in section A
Correct answer for Q1 is D, Q2 is A and Q3 is C*

		A	B	C	D	E
Q1	RTE	33	0	8	59	0
(sc)	ELE	29	0	0	71	0
Q2	RTE	67	0	25	0	0
(C)	ELE	71	14	0	7	0
Q3	RTE	0	8	33	17	25
(P)	ELE	0	0	93	0	0

SECTION B

Table 4
Percentage of students answering Q1 – Q4 in section

Q1	RTE	25% - correct	7% - did something	58% - no answer
	ELE	36% - correct	21% - did something	43% - no answer
Q2	RTE	42% - correct	33% - nearly got it	25% - got it wrong
	ELE	93% - correct	0% - did something	7% - no answer
Q3	RTE	17% - correct	0% - did something	83% - could not do
	ELE	79% - correct	7% - did something	14% could not do
Q4	RTE	8% - correct	0% - did something	92% - could not do
	ELE	14% - correct	36% - did something	50% - could not do

While the two tests contained similar items, the post test was slightly more difficult than the pre test. This is because, having just learnt about this topic, they were expected to do better in the post test if the two tests were of similar degree of difficulty. The post test required some thinking to test for understanding rather than rote learning. From Section A, there was a marked improvement in the percentage of students who obtained the correct answer. As for Section B, it could be seen marked improvement in all of the questions took place. There was an improvement in procedural skill as were measured in question 2, and the ELE class did very well in this item. There was also improvement in problem solving skills as measured in question 3 and 4.

The average score from the post-test was 50%. Although this is a two-fold increase, the increase was most probably due to better effort from ELE class. In fact the RTE class showed no improvement over the pre-test. From observation, students in RTE class were not very serious in answering the test questions. It might have been because their class was at two in the afternoon and a couple of students were late for class.

DISCUSSION

Overall, students enjoyed the teaching/learning process even though it had limited success. Much could still be done to revise the material to suit the students' contextual knowledge. Outcomes also demonstrated that the younger students who had been exposed to some investigation and hands-

on activities at school were more ready for the changes in the teaching and learning approach. The more mature students appear to need more time to familiarize themselves with the new approach.

Comparing the results of the pre-test and the post-test, it can be said that there was a marked improvement in the results although the improvement came mostly from the ELE class.

From the concept map results, it was found that the differences between the two classes are not that distinct. Both classes produced good concept maps. A small difference is that the RTE class could list down much of what they have learnt but could not make the connection from one to another, whereas the students in ELE class could.

It can be concluded that technical students could succeed in mathematics if the teaching approach was more active, student centred, relevant and contextual in nature. Since the student do not do well with the traditional approach, and since their course is more applied and concrete in nature, teaching of mathematics could be improved via the approach described in this paper.

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APPENDIX 1

Lesson 1

Approximate time: 1 hour

Title:

Radian measure. (Introduction).

Converting angles in degrees to radian and vice versa.

Aim:

- 1) To introduce angle.
- 2) To introduce other angle measurement i.e. gradian and radian.
- 3) To establish the relationship between degree and radian by first using the:
 - a) Calculator
 - b) Defining the angle $\theta = s/r$ (in radian) and $180^\circ = \pi$ radian.
- 4) To convert angles given in degree to radian and vice versa. (Worksheet accompanying)
- 5) To practice calculator skill especially converting angles in degree, minutes seconds to degree and vice versa.

Key skills:

- 1) problem solving 2) calculator skills
- 3) class interaction & co-operation.

Approach:

- 1) Launch: Start by posing a question whether they know any other unit of angle measurement. What does it mean when an angle is written as $\pi/4$? Explain that angles can be measured in a few different units, the popular ones being degree, followed by radian and also gradian.
- 2) Explore: Give out the activity sheet and investigation 1 sheet. Work with them along the way to get how to convert degree to radian and vice versa. Give some examples.

Discuss in detail: Activity sheet question: 'Why in ancient time, 1 degree is $1/360$ th of a circle?'

The Babylonians invented this based on their Babylonian base 60 numerical system. Hours and minutes are similarly divided into 60's (of course, there are minutes of time and minutes of angle - there are 60 minutes in a degree, and, similarly, there are seconds of time and seconds of degree - there are 60 seconds in a minute, 60 seconds in a degree).

The reason they were asked this question was to make them aware that it is a creation of ancient time and in fact we can invent another unit of measurement that we can call a 'madihah' in which there are 1234 in a circle. Besides degree, the other unit for angle measurement that is still in use is the gradian - of which there are 400 in a circle (German engineering unit? British military use?) Another one that is of very important use is the radian). The reason why radian is defined to be 2π is because the circumference of a circle of radius 1 is 2π . So in a circle of radius 1, one radian subtends an arc length of exactly 1. This makes measuring arc length equivalent to measuring angle. The activity sheet tries to build the relationship between degree and radian and how they can easily convert angle from degree to radian and vice versa.

- 3) Checkpoint: How do you change angles in degree to radian and vice versa?
- 4) Apply: Once a definite way is found, emphasise this new-found skill with exercises. (Worksheet 1).
- 5) Homework: Give unfinished exercise or additional problems if needed as homework.

APPENDIX 2

Lesson 2

Approximate time: 1 hour

Title:

Radian measure: Length of arc and area of sector.

Aim:

- 6) To revise the definition of a radian
- 7) To define the arc and sector
- 8) To deduce and use the arc length formula

- 9) To deduce and use the area of sector formula.
- 10) To solve real world problems in radian measure.

Key skills:

- 2) Problem solving
- 3) Calculator skills
- 4) Class interaction & co-operation.

Approach:

- 6) Launch:

Q1: You are given a choice of running two tracks today. One track is a quarter of a circle whose radius is 20 m and the other is one third of a circle whose radius is 15 m. You are feeling tired today and would like to choose the shortest possible route. Which one would you choose?

Q2: A whole pizza with radius 8 cm is cut into six pieces. A piece of that pizza costs \$2.00. Another whole pizza with radius 10 cm is cut into eight pieces and a piece of that pizza costs \$2.50. Assuming that the ingredients are the same, which one is more value for money?

- 7) Explore: Give out the activity sheet and investigation 2 sheet. Work with them along the way to get to the arc length formula and the area of sector formula. Give some examples.
- 8) Checkpoint: How do you find the length of arc? How do you find the area of sector?
- 9) Apply: Once a definite way is found, emphasise this new-found skill with exercises. (worksheet 1)
- 10) Homework: Give unfinished exercise or additional problems if needed as homework.

APPENDIX 3

Lesson 3 .

Approximate time: 1 hour

Title:

Radian measure: Apply arc length and sector area formula to solve a range of work related problems.

Aim:

- 11) To apply arc length and sector area formula to solve a range of work related problems.
- 12) To summarize the topic on radian measure in a concept map.

Key skills:

- 5) problem solving
- 6) calculator skills
- 7) class interaction & co-operation.

Approach:

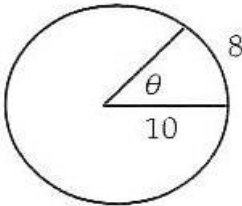
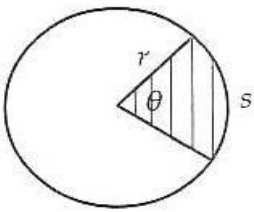
- 11) Give examples of real life problems and solve. The Eratosthenes problem.
- 12) Give a sheet of exercises and divide the problems among the groups.
- 13) Let the students discuss on how to solve the problems among themselves with the teacher facilitating the class.
- 14) Let each group present their solution to the class.
- 15) Practice: Let the students do other problems on the sheet individually.
- 16) Homework: Give unfinished exercise or additional problems if needed as homework.

Assessment:

As an assessment for this whole topic students are encouraged to come up with a concept map that will link everything that they have learned in this topic.

APPENDIX 4

SECTION A (Objective type questions)

- What is the value of θ if $\cos \theta = 0.5$, and θ is an acute angle?
 - 60°
 - 1.046 rad
 - neither 60° nor 1.046 rad
 - both 60° and 1.046 rad
 - none of the above.
- In a circle whose radius is 10 cm, a central angle θ intercepts an arc of 8 cm. What is the radian measure of that angle?
 - 80
 - 0.8
 - 1.6
 - 8
 - none of the above
- Given that $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ and s is the length of arc, r the radius, θ the angle that subtends the arc, A the area of the shaded sector, find the area of the shaded sector if $s = 20 \text{ cm}$ and $r = 10 \text{ cm}$.
 - 200 cm^2
 - 100 cm^2
 - 50 cm^2
 - 120 cm^2
 - none of the above

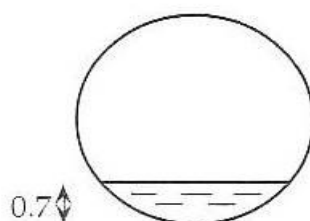
SECTION B (Subjective type questions)

- Do you know of any other angle measurement besides degrees? If so, list them.
- Convert:
 - $236^\circ 25' 35''$ to radians.
 - 1.345 rad to degrees, minutes and seconds.

3. A railroad track is to be laid out in the shape of a circle in which the radius is 100 yards. The track must have a central angle of 83° . Find the length of the track.
4. A cross section of a pipe is filled with water as shown in the diagram below. The diameter of the pipe is 3 cm and the height of water that fills the pipe is 0.7 cm.

Find: a) The wetted perimeter of the pipe.

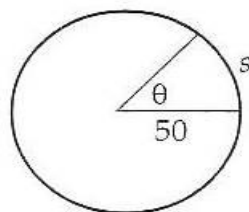
b) The area of the cross section of water shown.



APPENDIX 5

SECTION A (Objective type of questions)

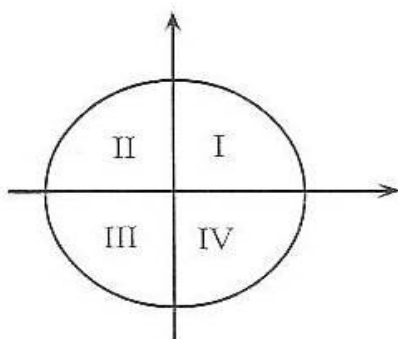
1. What is the value of \hat{E} if $\sin \hat{E} = 0.5$, and \hat{E} is an acute angle?
 - A. 30°
 - B. 0.5236 rad
 - C. neither 30° nor 0.5236 rad
 - D. both 30° and 0.5236 rad
 - E. none of the above.
2. In a circle of radius 50cm, a central angle θ intercepts an arc of s cm. If $\theta = 0.5$ radian, what is the length of s ?
 - A. 25 cm
 - B. 100 cm
 - C. 10 cm
 - D. 2.5 cm
 - E. none of the above



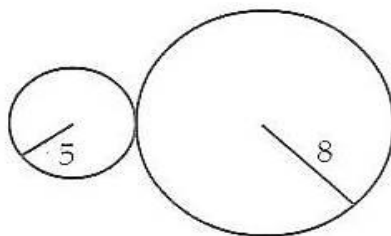
3. Given that $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ and s is the length of arc, r the radius, θ the angle that subtends the arc, A the area of the shaded sector, find the length of the Arc shown if $A = 200 \text{ cm}^2$ and $r = 10 \text{ cm}$.
- A. 80 cm B. 60 cm C. 40 cm
D. 20 cm E. none of the above

SECTION B (Subjective type of questions)

1. In which quadrant of the circle does 2.35 radians falls? Explain



2. Convert:
- a) $198^\circ 33' 40''$ to radians.
- b) 2.123 rad to degrees, minutes and seconds.
3. The rotation of the smaller gear (radius 5 in) forces the larger gear (radius 8 in) to rotate. If the smaller gear rotates through 50° , through how many degrees does the larger gear rotate?



4. A cross section of a pipe is filled with water as shown in the diagram below. The diameter of the pipe is 5 cm and the height of water that fills the pipe is 4 cm. Find:
- The wetted perimeter of the pipe.
 - The area of the cross section of water shown.

