

Dilemma of Mathematics

M. Azram, *Member, IACSIT*

Abstract—The pursuit of knowledge and the use of reason, based on sense and observation is a key ingredient for research. Mathematics is a creation of human mind concerned chiefly with ideas, processes and reasoning. In this paper, we will try to give a new comprehensive definition of mathematics to understand “what is mathematics”. We will discuss the controversial nature and position of mathematics and its scientific status. We will highlight the position of mathematics in different civilizations. We will highlight the mythical issues about Mathematics. We will also discuss the current state of mathematics i.e. mathematics in crises, especially pure mathematics and will put forward the remedial suggestions. We have gathered together some of these impressions; these are all tentative, nothing final about them, but these are here nonetheless.

Index Terms—Mathematics, mythical issues, scientific status, LOGS, crises of mathematics.

I. INTRODUCTION

Acquiring knowledge is a basic right for all citizens of a state. Advancing in the fields of medicine, mathematics, Education, physics, astronomy, geography, architecture, literature, science and technology, trade and business, banking system and defensive capability etc. are essential and important ingredient for the future development.

Mathematics is a way of thinking, a way of organizing a logical proof. As a way of reasoning, it gives an insight into the power of human mind. Over the last few years issues connected with learning and teaching mathematics have become a matter of the highest importance for everyone involved in education, training and research [1], [2]. According to Galileo Galilei, Mathematics is the language with which God has written the universe [3]. According to him, the universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word [4]. Mathematics is the entrance and pivotal ingredient of the Science. According to the famous Philosopher Kant, A Science is exact only in so far as it employs Mathematics [5]. So, all scientific education which does not commence with Mathematics is said to be defective at its foundation. Neglect of mathematics works injury to all knowledge. Mathematics is a creation of human mind concerned chiefly with ideas, processes and reasoning. It is much more than Arithmetic, more than Algebra more than Geometry. Also it is much more than Trigonometry,

Statistics, and Calculus. The study of mathematics can satisfy a wide range of interests and abilities. It develops the imagination. It trains in clear and logical thought. It is a challenge, with varieties of difficult ideas and unsolved problems, because it deals with the questions arising from complicated structures. Yet it also has a continuing drive to simplification, to finding the right concepts and methods to make difficult things easy, to explaining why a situation must be as it is. In so doing, it develops a range of language and insights, which may then be applied to make a crucial contribution to our understanding and appreciation of the world, and our ability to find and make our way in it. Many non-mathematicians have some sense of the importance of Applied Mathematics and Statistics, but they may view Pure Mathematics as something with little use. Consequently, we feel it is important to overcome these crises.

II. THE NATURE AND NEW DEFINITION OF MATHEMATICS

Mathematics is considered as immaterial in itself but has very strong links with the physical world. Mathematics differs from physics because Physics is mostly related with material substance and movement. It also differs from metaphysics because metaphysics does not have any direct relation with physical entities. This situation raised the question “what is mathematics”. Defining mathematics is simply like asking a blind person to describe the elephant. Obviously, the answer will be inconsistent, because he has never seen the real shape of an elephant. He will be just imagining, how elephant looks like. There are many definitions of mathematics in the literature on mathematics. According to the great Bertrand Russell “Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true [6]. Charles Steinmetz said “Mathematics is the most exact science, and its conclusions are capable of absolute proof, but this is so only because mathematics does not attempt to draw absolute conclusion. All mathematical truths are relative, conditional”. Benjamin Pierce said, “Mathematics is the science, which draws necessary conclusions”. According to Webster’s International Encyclopedia [7] “Mathematics is a field of thoughts concerned with relationship involving concepts of quality, space and symbolism”. In the next few lines we suggest a moderate answer to the same question, “what is mathematics”? We consider mathematics as “LOGS”, virtually means knowledge [8]. Whereas “L” in LOGS stands for language that is mathematics is a language, which is indispensable for expressing scientific ideas.

To strengthen my claim I re-write the saying of Galileo Galilei, “Mathematics is the language with which God has written the universe” [3]. According to him, the universe cannot be read until we have learnt the language and become

familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word [4].

“O” stands for operation that is mathematics is a computational skill/tool to answer logical, quantitative questions. “G” stands for game that is mathematics is a game played within a domain with certain rules and regulations that is postulates/axioms determine the domain and related rules and regulations and provide foundations for further improvement. In line with my claim, Richard J. Trudeau said, Pure mathematics is the world's best game. It is more absorbing than chess, more of a gamble than poker, and lasts longer than Monopoly. It's free. It can be played anywhere-Archimedes did it in a bathtub. It is dramatic, challenging, endless, and full of surprises. David Hilbert said, Mathematics is a game played according to certain simple rules with meaningless marks on paper [9].

Lastly “S” stands for science that is mathematics is a science, which is in a creative process for precision, truth through logical reasoning.

In line with this claim, E. Kasner and J. Newman, said “Mathematics is the science which uses easy words for hard ideas” [10]. Benjamin Peirce called mathematics “the science that draws necessary conclusions”. Carl Friedrich Gauss quoted as “Mathematics is the queen of Sciences”. G. Dunnington *et al.* quoted as “Mathematics is the queen of sciences and arithmetic the queen of mathematics” [11]

III. SCIENTIFIC STATUS OF MATHEMATICS

The nature of mathematics as science or not is a controversial issue. According to scientific criteria for something to be science is that its nature must be empirical. Mathematics in itself is not empirical but rather immaterial. It will never meet the fundamental condition of modern scientific criteria; being empirical. Consequently, many scholars do not consider mathematics as science but they consider mathematics as instrument of science. Now, it is natural to raise this query that why a non-empirical knowledge cannot be considered as a science. The obvious answer by the scientist will be that a non-empirical object is not real. But on the other hand many scholars consider it as science and classify it as mathematical science. Some scholar argued as “science is reasoning; reasoning is mathematics; and, therefore, science is mathematics. These are people considering mathematics as “Mother of all Sciences”. Despite all, the scientific nature of mathematics is still a controversial issue.

However the Islamic scholars [12], as per their Islamic traditions consider mathematics as science, because Muslim scholars believe in the ontological reality of not only physical objects (**al-mahsusat**) but also non-physical objects (**al-ma’qulat**), such as mathematical objects. Therefore, Muslim scholars do not have reasons to deny the legitimate scientific status of the mathematics. Hence, mathematics is a science or not is a controversial issue and is still a problem within scientist community.

Now the problem is that if mathematics is not a science then what is its status in scientific world? One version of

mathematics is that it is instrument for science, because we used it to understand and study physical entities. In astronomy, mathematics has been used by Muslim scientists as scientific method together with demonstrative methods¹². Mathematics was also very often applied to help scientific research in geology and geography etc. Mathematics was used by Muslim scientists as indispensable instrument for understanding physical matters¹², same is the case with modern physicists. Mathematics has also been considered as methodological instrument for understanding philosophy, without which philosophy, according to Abu Yusuf Yaqub Ibn Ishaq Al-Khindi (801–873 CE), will never be understood properly. Even he To me the following are the main reasons for crises in mathematics in general and in pure mathematics in particular; wrote a book entitled Philosophy cannot be understood without Mathematics to show us how important mathematics is for understanding philosophy. He also says that scientific method can be demonstrated clearly only in mathematics. So according to Muslim scholars, mathematics is a legitimate theoretical science, with its clear theoretical status. Its instrumental role in philosophy can be understood by the fact that mathematics is situated between physics and metaphysics.

IV. POSITION OF MATHEMATICS IN THE ISLAMIC WORLD

Muslim scientists considered mathematics as a rational science. Further they classified rational science as theoretical science and practical science. They placed mathematics theoretical science. Theoretical sciences, in turn, were sub-divided as: physics, mathematics and metaphysics, while practical sciences into ethics, economics and politics. Obviously mathematics has a middle position between physics and metaphysics. Although it is immaterial, but it still have very strong link to the physical world. Metaphysics on the other hand, has no direct relation to the physical world. There are four main branches of mathematics that is **quadrivium**; which consist of arithmetic, geometry, astronomy and music. Second one is **Arithmetic** which deals with numbers; **Geometry** with spatial dimension and geometric forms: **Astronomy** with the observation, measuring and mapping of the stars, planets and spheres, and **Music** with melody, tones, and harmony. **Algebra** is a sub-branch of arithmetic.

V. MYTHICAL ISSUES

The following are some of the common mythical issues about mathematics [13], [14].

- Mathematics is hard.
- Men are better in Mathematics than women.
- Mathematics requires logic, not intuition.
- Mathematics is not creative.
- You must always know how you got the answer.
- In Mathematics you must get always right answer.
- It is bad to count on your fingers.
- Mathematician does problems quickly, in their head.
- Mathematics required a good memory.
- Some people have a “Mathematics Mind” and some don't.

- Calculators can do all the mathematics for me.
- You'll never use this stuff again.

VI. MATHEMATICS IN CRISES

To me the following are the main reasons for crises in mathematics in general and in pure mathematics in particular;

- Marketability

From 2001 to 2007 the number of students enrolled in mathematics major in Australian universities declined by 15%. The number of students taking advanced mathematics at high school dropped by 27% between 1995 and 2007.

- Insufficient funds for research and development. Algebra, algebraic topology, and general/geometric topology will vanish. Algebraic K-theory and general/geometric topology (VU Amsterdam April 27, 2011)
- Lack of inspiration by problems of real world.
- The position of mathematics, as that of any science, is at bottom determined by the development and the position of the forces of production, of technology and economy.
- Poor and weak teacher ignorant of importance and relevance of mathematics. 26% of full time mathematics teacher in UK have no proper education in Mathematics (Education Studies in Mathematics, 56 pp 343-356, 2004 Kluwer Academic Publisher)
- In early 60's, due to so considered difficult mathematics courses and lack of trained manpower for each one, mathematical ideologues in a number of countries began to carelessly destroy established schemes of phased teaching of mathematics and entered the age of biology, which cannot replace mathematicians and theoretical physicists.
- Non-mathematics departments such as Engineering, Economics and Business, ICT etc. prefer that their own staff member should teach mathematics rather than asking the relevant mathematics department to delegate mathematicians to teach. There are always positive, negative arguments but consequently, many mathematics departments are in crises.
- Unfortunately mathematics is usually applied by engineers and/or industrial scientists, and not by mathematicians. In other words, mathematicians even applied mathematicians do not apply mathematics; the engineers or scientists do it. Their invented tools are utilized and enjoyed by others. In consequence, mathematicians are losing inspiration by problems of the real world, living in their own imaginary world.
- Mathematicians especially people in pure mathematics ignore the applied science, industries, financial world etc.. Consequently no research fund, no sponsored students for supervision, no consultancy from the real World and some time at a risk of losing job.
- This is a fact that mathematics describes only some aspects of the real world. With the advancement of technology, machines such as computers have become mathematical machines to compete with mathematicians in numerical and symbolic calculations. End result is obvious.
- Each mathematics department should establish a "mathematical clinic", inviting people to come for

mathematical solutions and this is how mathematics can be connected to external world.

- Most of mathematicians are spending their time trying to prove properties of objects that do not correspond to anything in the real world, while many phenomena of the real world remain unexplained, they are poorly modeled, or their features are not explored. Mathematician should link themselves with activities aimed toward our survival and wellbeing such as manufacture, agriculture, medicine etc.

VII. PURE MATHEMATICS

Emmy Noether an influential German mathematician has written a letter to the Editor of The New York Times, published May 5, 1935, quoting that "Pure mathematics is, in its way, the poetry of logical ideas" but;

- Many non-mathematicians have some sense of the importance of Applied Mathematics and Statistics, but they may view Pure Mathematics as something with little use.
- The easiest way to think of it is that **pure mathematics** is mathematics done for its own sake, while **applied mathematics** is mathematics with a practical use. But in fact, it's not that simple, because even the most abstract mathematics can have unexpected applications. For example, the branch of mathematics known as "number theory" was once considered one of the most "useless", but now plays a vital part in computer encryption systems. If you've ever bought something online, you can thank number theorists for letting you do it safely.
- Pure mathematics is not necessarily applied mathematics, more difficult and less marketable.
- The dilemma of pure mathematics is to present its results to a broader scientific and general public.

REFERENCES

- [1] E. M. Dickey, "Challenges of Mathematics Teaching Today: How Can School Leaders Help?" *NASSP Bulletin*, pp. 1-10, 1997.
- [2] P. Ernest, B. Greer, and B. Sriraman, *Critical Issues in Mathematics Education*, Montana Mathematic Enthusiast, University of Montana, USA, 2009.
- [3] E. A. Burt, *The Metaphysical Foundations of Modern Science*, Dover Publications Inc., 2003.
- [4] M. L. Lial, C. D. Miller, and E. J. Horns, "Beginning algebra," *Harpercollins College Div.*, pp:21992,
- [5] A. Schultze, *Teaching of Mathematics in Secondary Schools*, The Macmillan Co., 1931.
- [6] R. Bertrand, *Mysticism and Logic and other Essays*, George Allen & Unwin Ltd., 1917.
- [7] D. H. Micheal, *The New Webster's International Encyclopedia*, D.S.M.A.X., USA, 1991.
- [8] M. Azram, "Metrazibility Via Separation Axioms," *J. Sc. & Tech.*, ISSN 0250-5339, Uni. Pesh, vol. 24, no. 1-2, pp. 59-63, 2000.
- [9] N. Rose, *Mathematical Maxims and Minims*, Raleigh NC: Rome Press Inc., 1988.
- [10] E. Kasner and J. Newman, *Mathematics and the Imagination*, New York: Simon and Schuster, 1940.
- [11] D. G. Waldo, "Carl Friedrich Gauss, Titan of Science," *The Mathematical Association of America*, 2004.
- [12] M. Kartenegro. (2008). Mathematics in the Islamic Scientific. [Online]. Available: <http://i-epistemology.net/component/content/article/76-mulyadhi-kartenegro/608-mathematics-in-the-islamic-scientific-tradition.html>
- [13] C. Mukisa. (2011). Maths Myths That Are Killing Your Child's Achievement. *The Maths Insider AmazonStore*. [Online]. Available: <http://www.mathsinsider.com/math-myths/>

[14] Student Learning Assistance Center (SLAC) Texas State University-San Marcos. Mind Over Math. [Online]. pp. 30-43. Available: <http://www.txstate.edu/slac/math/skills/12Myths.pdf>



Mohammad Azram received his B.S. and M.Sc. (with distinction) in mathematics from the University of Peshawar, Pakistan, in 1974 and 1976 respectively. He started his teaching career as a lecturer in the Department of Mathematics, University of Peshawar in 1977. He received his M.S. and Ph.D in mathematics from the University of Idaho (USA) in 1985 and 1989 respectively. His Ph.D. thesis (graph theoretic version of Reidemeister moves) was supervised by the then a leading mathematician Prof. Charles O. Christenson. His research interest is in the area of Low-dimensional Cell-complexes/Knot Theory, Point-set Topology,

Mathematical Physics and Topological Algebras. As a teacher, he served the University of Peshawar in the capacity of lecturer, assistant prof., assoc prof. & prof. As a leader/administrator, he served the University of Peshawar as a director (Ph.D./M.Phil programs), the head (Dept. of Computer Science) and member (Academic Council, the Senate, affiliation committee & Peshawar University Teacher's Association). He has joined the International Islamic University of Malaysia in 1999. Apart from many other academic and administrative responsibilities, he has served IIUM as a professor of Mathematics, the deputy dean (Academics Affairs, Centre for Postgraduate Studies) and the head (Dept of Sc, Faculty of Engg). During his stay at IIUM, he won Two Gold and Two Bronze Medals in research exhibitions. He has also been awarded the best teacher award for the session 2008/09.