Indian Journal of Pure & Applied Physics
Vol. 50, December 2012, pp. 887-891

Low-energy structure of isotopes $^{152\text{-}156}$Sm

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Received 28 March 2012; revised 20 June 2012; accepted 3 October 2012

The $^{152\text{-}156}$Sm isotopes are classified as deformed nuclei. The phenomenological model is presented to describe the complete low-energy structure of $^{152\text{-}156}$Sm isotopes by taking into account the Coriolis mixing between states. The parameters fitted to the model are calculated. The energy spectra of positive-parity states which are found to be in good agreement with the experimental data are presented. It is found that the non-adiabaticity of rotational energy bands occurred at high spin due to the Coriolis effect. Few new states are predicted.

Keywords: Nuclei, Energy spectra, Band, Low-lying state, Coriolis effect

1 Introduction

Analogous to liquid drop idea, collective model is proposed by Bohr and Mottelson[1]. The collective motion is interpreted as vibration and rotation of nuclear surface. The instantaneous coordinate of a point on the nuclear surface may be described quite generally by an expansion in spherical harmonics with time-dependent coefficient. In axially deformed nuclei, the low-energy quadrupole mode with multipolarity λ=2 is dominant. We recall that the parity π is given by $(-1)^{\lambda}$ in even-even nuclei.

The low-lying, collectively magnetic dipole excitations so-called low-lying "scissor-mode" in deformed nuclei are discovered in the last years[2]. The opposite oscillations between neutrons and protons generate isovector magnetic dipole resonance. Taking account into the Coriolis mixing of the isovector collective M1 states with low-lying states will lead for the non-adiabaticity of electromagnetic properties to occur. The role of M1 and E2 excitations will be discussed in the future.

In the present paper, the complete low-energy structure of deformed nuclei by considering the rotational bands Coriolis states mixing[3,4] is described. Since $^{152,154\text{-}156}$Sm nuclei are classified as deformed nuclei, these nuclei are the best candidates to study the collective properties of low-lying states. They are quite well studied experimentally[5-8]. By (*t, p*) reaction on even Sm isotopes, the excitation spectrum was established[9] below 2-3 MeV.

2 The Model

The basic states of the Hamiltonian include ground (gr), $\beta_n-(0^+_{\beta_1},0^+_{\beta_2})$, γ-vibrational and $K^{\pi}=1^+_{\nu}$ rotational bands. As n is the number of included β-vibrational states, so ν is the number of 1^+ collective states.

The nuclear Hamiltonian is written in the two-partition form:

$$H=H_{rot}(I^2)+H^{\sigma}_{K,K'}(I) \quad \dots (1)$$

$H_{rot}(I^2)$ is the rotational part and

$$H^{\sigma}_{K,K'}(I)=-\omega_K\delta_{K,K'}-\omega_{rot}(I)(j_x)_{K,K'}\chi(I,K)\delta_{K,K'\pm1} \quad \dots (2)$$

$(j_x)_{K,K'}$ is the matrix element describing the Coriolis coupling of rotational bands and $\omega_{rot}(I)$ is the angular frequency of core rotation yielded from:

$$\omega_{rot}(I)=\frac{dE_{cor}(I)}{dI}$$

ω_K is the head energy of respective K^+ bands which is the lowest energy level with I = 0 and

$$\chi(I,0)=1,\ x(I,1)=\left[1-\frac{2}{I(I+1)}\right]^{\frac{1}{2}}$$

wave function of the nuclear Hamiltonian:

$$\psi^I_{\nu}=\sum_K\varphi^I_{\nu,K}\left|IMK\right\rangle \quad \dots (3)$$

where $\varphi^I_{\nu,K}$ represents the Coriolis mixing coefficient of basis states and

$$\left|IMK\right\rangle = \sqrt{\frac{2I+1}{16\pi^2}}\left\{\sqrt{2}\psi^I_{gr,K}D^I_{M,0} + \sum \frac{\psi^I_{K^{'},K}}{\sqrt{1+\delta_{K^{'},0}}}\right.$$

$$\left.\times\left[D^I_{M,K^{'}}(\theta)b^+_{K^{'}} + (-1)^{I+K^{'}}D^I_{M,-K^{'}}(\theta)b^+_{-K^{'}}\right]\right\} \quad \ldots (4)$$

$\psi^I_{K^{'},K}$ are the amplitudes of basis states mixing from the $(4+\nu)$ bands includes the ground $\left|0\right\rangle$ states bands and the single-phonon $b^+_{\lambda=2}\left|0\right\rangle = b^+_K\left|0\right\rangle$ with all the mentioned rotational bands before.

By solving the Schrodinger equation:

$$H^{\sigma}_{K,n}\psi^I_{K^{'},n} = \varepsilon^{\sigma}_n\psi^I_{K,n} \quad \ldots (5)$$

We obtained wave function and energy of states with positive parity.

Total energy of states is determined by following:

$$E^{\sigma}_n(I) = E_{rot}(I) + \varepsilon^{\sigma}_n(I) \quad \ldots (6)$$

Energy of rotational core $E_{rot}(I)$ can be determined by different methods. In the present paper, we used Harris parameterization of the angular momentum and energy[10].

$$E_{rot}(I) = \frac{1}{2}\Im_0\omega^2_{rot}(I) + \frac{3}{4}\Im_1\omega^4_{rot}(I) \quad \ldots (7)$$

$$\sqrt{I(I+1)} = \Im_0\omega_{rot}(I) + \Im_1\omega^3_{rot}(I) \quad \ldots (8)$$

where $\Im_0$ and $\Im_1$ are the adjustable inertial parameters of rotational core. A method of defining the even-even deformed nuclei inertial parameters using the experimental data up to $I \le 8\hbar$ for ground band is suggested in recent paper[11,12] and presented in Table 1. The linear dependence of moment of inertia for states $J_{\text{eff}}(I)$ on the square of angular frequency of rotation $\omega_{\text{eff}}(I)$ are shown in Fig. 1.

The rotational frequency of the core, $\omega_{rot}(I)$ is calculated by solving the cubic equation, which the real root is as follows:

$$\omega_{rot}(I) = \left\{\frac{\tilde{I}}{2\Im_1} + \left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}}$$

$$+\left\{\frac{\tilde{I}}{2\Im_1} - \left(\left(\frac{\tilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right)^{\frac{1}{2}}\right\}^{\frac{1}{3}} \quad \ldots (9)$$

Table 1 — Inertial parameters of rotational core used in the calculations

Nucleus	$\Im_0(\text{MeV}^{-1})$	$\Im_1(\text{MeV}^{-3})$
^{152}Sm	24.74	256.57
^{154}Sm	36.07	178.88
^{156}Sm	39.22	98.36

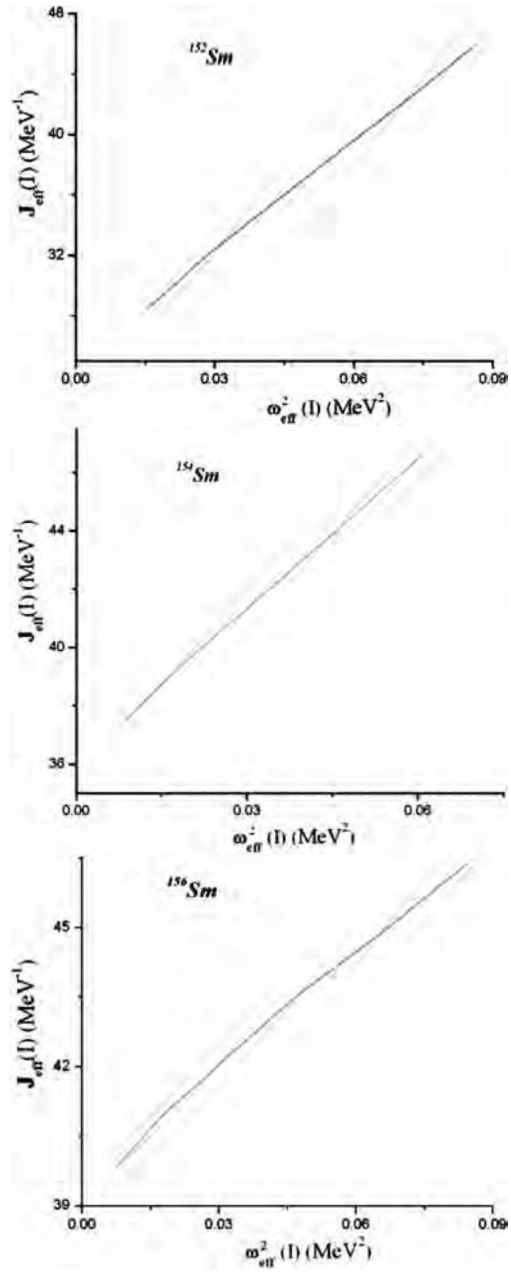

Fig. 1 — Linear dependence of $J_{\text{eff}}(I)$ on $\omega^2_{\text{eff}}(I)$

where $\tilde{I}=\sqrt{I(I+1)}$. Eq. (9) gives value of $\omega_{rot}(I)$ at the given spin I.

Explaining the Coriolis interaction, given the value of the perturbed (experimental) energies, $E^1_{\exp}$ and $E^2_{\exp}$, it is possible to calculate the interaction matrix element $\omega_{rot} j_x$ from the pure energies E^1_{theor} and E^2_{theor}, such that :

$$\begin{pmatrix} E^1_{theor} & \omega_{rot} j_x \\ \omega_{rot} j_x & E^2_{theor} \end{pmatrix}\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = E^{1,2}_{\exp}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad \dots (10)$$

3 Results

The parameters fitted with the model are presented in Table 2. The lowest energy for ground-state and β_n bands was taken from experimental energies, since they are not affected by the Coriolis forces at spin I=0: $\omega_{gr}=E^{\exp t}_{gr}(0)$ and $\omega_{\beta_n}=E^{\exp t}_{\beta_n}(0)$.

The headband energies for the collective I^+ states in ^{152,154,156}Sm nuclei are assumed to be ω_1=3 MeV because the $K^{\pi}=1^+$ bands have not been observed experimentally for these nuclei, respectively[13]. Coriolis rotational states mixing matrix elements $(j_x)_{K,K'}$ and γ- and head energies ω_γ were determined by using the least square fitting method of the diagonalize matrix.

$$\begin{pmatrix} \omega_K-\varepsilon & \omega_{rot} j_x \\ \omega_{rot} j_x & \omega_{1^+}-\varepsilon \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \omega_K\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

Currently, the experimental energy spectrum for the β_2-band in the ^{156}Sm nucleus is not available. No calculation is done for this band.

In Table 2, the value of interaction matrix elements, $(j_x)_{K,K'}$ and proximity of headband energy, ω_K for certain band determined the strength of states mixing of that band with other bands[11] Larger value of matrix elements leads to strong states mixing. For ^{152}Sm nucleus, the matrix element $(j_x)_{\beta_2,1}$ of the β_2 - and 1$^+$ bands is larger than other matrix elements. Unlike ^{152}Sm nucleus, the matrix elements $(j_x)_{\gamma,1}$ of the γ^- and 1$^+$ bands in ^{154}Sm and ^{156}Sm are larger than other matrix elements.

Tables 3-5 present the calculated Coriolis mixing coefficients, $\phi^I_{\nu,K}$ which represents mixture components of other bands in certain band. Structure of ^{152,154,156}Sm can be understood by these calculated values.The theoretical energy spectra of positive-parity states in ^{152,154,156}Sm are shown in Figs 2-4,

Table 2 — Parameters used in the calculations

Nucleus	ω_{β_1}	ω_{β_2}	ω_1	ω_γ	$(j_x)_{gr,1}$	$(j_x)_{\beta_1,1}$	$(j_x)_{\beta_2,1}$	$(j_x)_{\gamma,1}$
^{152}Sm	0.685	1.083	3.0	1.0	0.742	0.821	0.864	0.855
^{154}Sm	1.099	1.203	3.0	1.380	0.345	0.403	0.408	0.417
^{156}Sm	1.068	-	3.0	1.365	0.749	0.872	-	0.903

Table 3 — Structure of ^{152}Sm states

I	gr	0_{β_1}	0_{β_2}	I^+	γ	gr	0_{β_1}	0_{β_2}	I^+	γ
	Ground-state band					β_1				
2	-0.9997	-0.0025	-0.0016	-0.0227	-0.0014	0.0032	-0.9994	-0.0064	-0.0326	-0.0065
4	-0.9993	-0.0065	-0.0043	-0.037	-0.0044	0.0086	-0.9982	-0.0169	-0.0536	-0.0199
6	-0.9987	-0.0109	-0.0073	-0.0483	-0.0076	0.0148	-0.9964	-0.0283	-0.0704	-0.0343
8	-0.9981	-0.0153	-0.0103	-0.0576	-0.0108	0.0211	-0.9942	-0.0397	-0.0845	-0.0485
10	-0.9975	-0.0197	-0.0132	-0.0657	-0.014	0.0275	-0.9917	-0.0508	-0.0968	-0.0621
12	-0.9968	-0.0239	-0.016	-0.0729	-0.017	0.0338	-0.9888	-0.0615	-0.1078	-0.0752
	γ					β_2				
2	0.0022	0.0078	-0.0302	-0.0326	-0.9990	-0.0025	-0.0075	0.9987	0.0395	-0.0316
3	-	-	-	0.0473	0.9989	-	-	-	-	-
4	-0.0069	-0.0248	0.0888	0.0630	0.9937	-0.006	-0.0182	0.9938	0.0585	-0.093
5	-	-	-	0.0698	0.9976	-	-	-	-	-
6	-0.0122	-0.0442	0.1452	0.0853	0.9846	-0.009	-0.0277	0.9855	0.0686	-0.1526
7	-	-	-	0.0866	0.9962	-	-	-	-	-
8	0.0175	0.0643	-0.1942	-0.1028	-0.9733	0.0115	0.0355	-0.9753	-0.0736	0.2049
9	-	-	-	0.1000	0.9950	-	-	-	-	-
10	-0.0226	-0.0843	0.2352	0.1168	0.9610	-0.0134	-0.0416	0.9645	0.0759	-0.2492
11	-	-	-	0.1112	0.9938	-	-	-	-	-
12	-0.0274	-0.1038	0.2691	0.1282	0.9485	-0.0149	-0.0464	0.9538	0.0766	-0.2865

Table 4 — Structure of ^{154}Sm states

I	gr	0_{β_1}	0_{β_2}	I^+	γ	gr	0_{β_1}	0_{β_2}	I^+	γ
			Ground-state band					β_1		
2	1.0	0.0002	0.0002	0.0076	0.0001	0.0003	-0.9999	-0.0037	-0.0142	-0.0011
4	-0.9999	-0.0006	-0.0005	-0.0134	-0.0004	0.0009	-0.9996	-0.0114	-0.0250	-0.0041
6	0.9998	0.0011	0.0010	0.0184	0.0009	0.0017	-0.9991	-0.0215	-0.0347	-0.0080
8	0.9997	0.0016	0.0015	0.0228	0.0013	-0.0027	0.9984	0.0329	0.0435	0.0124
10	0.9996	0.0022	0.0021	0.0266	0.0018	-0.0037	0.9975	0.0449	0.0515	0.0172
12	0.9995	0.0029	0.0026	0.0301	0.0023	-0.0048	0.9964	0.0571	0.0589	0.0221
			β_2					γ		
2	0.0003	0.0039	-0.9999	-0.0151	-0.0019	-0.0002	-0.0013	-0.0021	0.0139	0.9999
3	-	-	-	-	-	-	-	-	0.0216	0.9998
4	-0.0009	-0.0121	0.9996	0.0262	0.0067	-0.0008	-0.0047	-0.0075	0.0280	0.9996
5	-	-	-	-	-	-	-	-	0.0344	0.9994
6	-0.0016	-0.0228	0.9990	0.0358	0.0129	-0.0016	-0.009	-0.0145	0.0390	0.9991
7	-	-	-	-	-	-	-	-	0.0451	0.9990
8	0.0025	0.035	-0.9982	-0.0439	-0.0197	-0.0024	-0.0138	-0.0223	0.0480	0.9985
9	-	-	-	-	-	-	-	-	0.0543	0.9985
10	0.0034	0.048	-0.9972	-0.0508	-0.0265	-0.0032	-0.0187	-0.0303	0.0554	0.9978
11	-	-	-	-	-	-	-	-	0.0623	0.9981
12	0.0042	0.0612	-0.9959	-0.0568	-0.0333	-0.004	-0.0236	-0.0383	0.0617	0.9971

Table 5 — Structure of ^{156}Sm states

I	gr	0_{β_1}	I^+	γ	gr	0_{β_1}	I^+	γ
		Ground-state band				β_1		
2	0.9999	0.0008	0.0155	0.0005	-0.0012	0.9996	0.0280	0.0043
4	0.9996	0.0025	0.0277	0.0019	-0.0039	0.9986	0.0503	0.0158
6	0.9992	0.005	0.0392	0.0039	-0.0079	0.9969	0.0715	0.032
8	0.9987	0.008	0.0499	0.0064	-0.0128	0.9944	0.0914	0.0513
10	0.9981	0.0113	0.0598	0.0091	0.0185	0.9911	0.1100	0.0723
12	0.9974	0.015	0.0689	0.0121	-0.0247	0.9871	0.1272	0.094
			γ					
2	-0.0009	-0.005	0.0277	0.9996				
3	-	-	0.0436	0.999				
4	0.0034	0.0187	-0.0564	-0.9982				
5	-	-	0.0708	0.9975				
6	0.0069	0.0377	-0.0795	-0.9961				
7	-	-	0.0947	0.9955				
8	0.0108	0.0604	-0.0983	-0.9933				
9	-	-	0.1158	0.9933				
10	-0.0149	-0.0851	0.1133	0.9898				
11	-	-	0.1346	0.9909				
12	-0.0189	-0.1104	0.1252	0.9858				

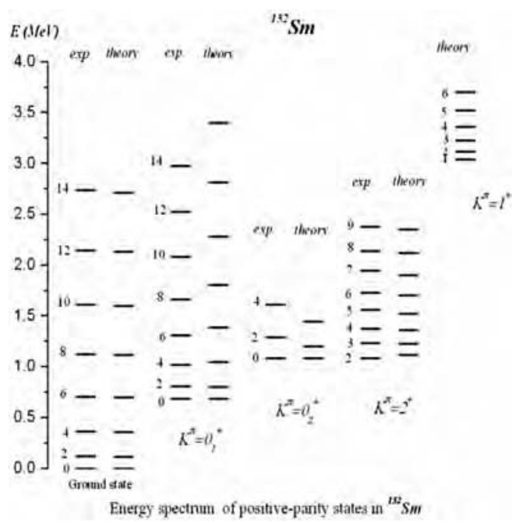

Fig. 2 — Energy spectrum of positive-parity states in ^{152}Sm

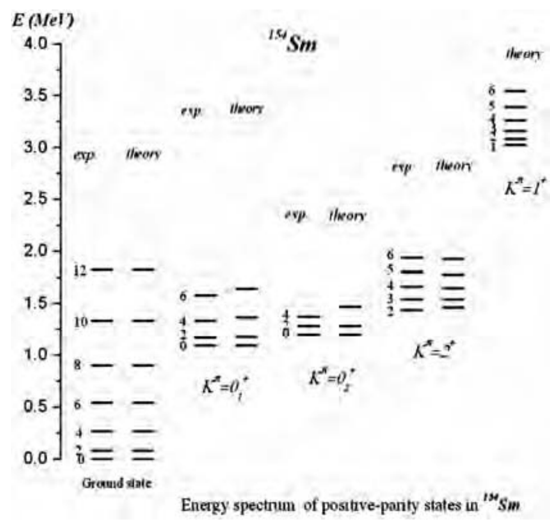

Fig. 3 — Energy spectrum of positive-parity states in ^{154}Sm

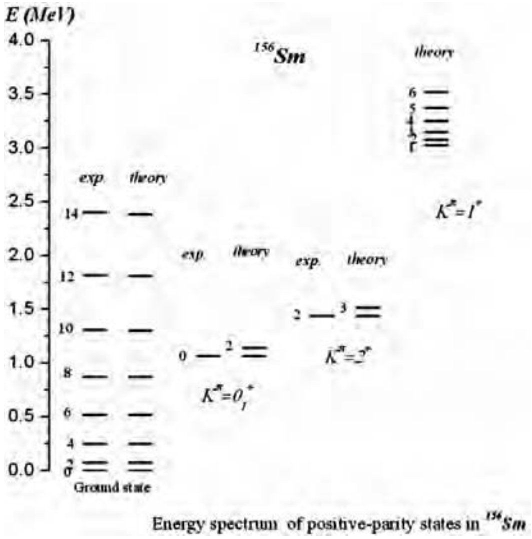

Fig. 4 — Energy spectrum of positive-parity states in ^{156}Sm

respectively in comparison with the experimental energies[5-8] From Figs 2-4, we see that energy difference $\Delta E(I)=E^{theor}(I)-E^{exp}(I)$ of the β_1-band increases with the increase in the angular momentum *I*. At high spin, *I* the non-adiabaticity of energy rotational bands occurs. Two states with same spin, *I* and parity, π from different bands cross in that region causes Coriolis mixing. We predict the existence of s-band states to perturb the pure β_1- band states. Other than this deviation, the theoretical positive-parity states energy spectra are in best agreement with the experimental data. Few new states and collective I^+ bands are predicted.

4 Conclusions

This work is based on the phenomenological model[3,4], which shows the deviation of the energy spectrum of positive parity states in even-even deformed nuclei from the adiabatic theory. Energy spectra for the isotopes $^{152-156}$Sm were calculated and the results showed are in good agreement with the experimental data. At high spin *I*, the law[1] of $E(I)\sim I(I+1)$ is violated. The calculations are done by taking into account the Coriolis mixing of positive parity states. The mixing components of the states is represented by the calculated values of Coriolis mixing coefficients, $\varphi^{I}_{\nu,K}$. The value of the mixing component explained why deviation has occurred. With the agreement between the theoretical and experimental data, few states that never been observed experimentally are predicted.

Acknowledgement

The authors would like to thank Prof P N Usmanov for useful discussion, also to the University of Malaya and Fundamental Research Grant Scheme, RG120/10AFR for financial support.

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