



# A Method to Determine Radial Speed of Target from the FMCW Radar Signal

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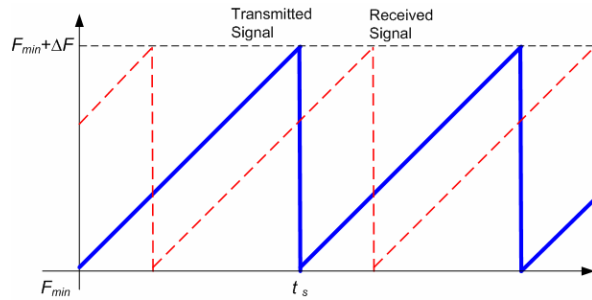
## ABSTRACT

An established method to detect a target from the FMCW radar signal is implemented by using the method of range FFT (Fast Fourier Transform). By repeating the same procedure many times at a certain azimuth angle, it is possible to additionally compute the radial speed of the suspected target. This paper describes a method to determine the radial speed of target by extracting the phase data from the repeated range FFTs. To verify this method a series of computer simulations was conducted at our research group.

**Keywords:** FMCW radar, radial speed, rate of change of phase

## 1. INTRODUCTION

In the frequency-modulated continuous wave (FMCW) radar, the transmitter frequency is changed as a function of time in a known manner. For a saw-tooth shaped FMCW radar signal the transmitter frequency is increased linearly with the time (fig. 1).



**Fig. 1:** Saw-tooth shaped FMCW radar signal. The frequency of the transmitted signal is linearly increased about  $\Delta F$  within the sweep time  $t_s$ .

If there is a reflecting target at a distance  $R$  it will cause a beat signal with the frequency [1,2]

$$f_R = \frac{2F_m R}{c}, \quad (1)$$

and the rate of frequency modulation  $F_m$

$$F_m = \frac{\Delta F}{t_s}.$$

By applying the Fourier transform (range FFT) we will get the spectrum of this beat signal. The distance

or range of the target can be determined by measuring the peak distance from the origin of the spectrum.

On the other hand, due to the Doppler effect, radar signal reflected by a moving target in radial direction will additionally show a frequency shift [1]. The relation between the frequency shift of radar signal and the radial speed of target is mathematically described as

$$f_d = 2v_r \frac{f_T}{c} \quad (2)$$

where

$v_r$  = radial speed of target,  
 $f_d$  = frequency shift due to Doppler effect,  
 $f_T$  = transmitted frequency, and  
 $c$  = speed of light.

In term of complex number notation the beat signal of a moving target with constant radial speed  $v_r$  at a range  $R$  can be formulated as

$$\mathbf{Y}(t) = A \cdot e^{j\varphi(t)} = A \cdot e^{j(\omega_R + \omega_d)t}$$

with the phase

$$\varphi(t) = (\omega_R + \omega_d)t \quad (3)$$

$$\omega_R = 2\pi f_R \text{ and } \omega_d = 2\pi f_d. \quad (4)$$

From the above equations (1 – 4) we can rewrite the phase of beat signal into



$$\varphi(t) = \frac{4\pi}{c}(F_m R + v_r f_T)t. \quad (5)$$

The rate of change of phase is derived from (5) to be

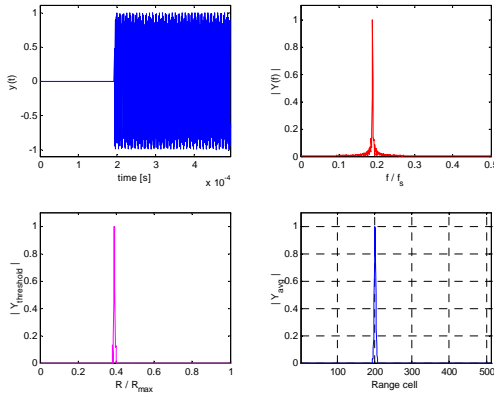
$$\omega = \frac{d\varphi(t)}{dt} = \frac{4\pi}{c}(F_m R + v_r f_T). \quad (6)$$

So for a given carrier frequency  $f_T$  the rate of change of phase  $\omega$  is determined by three parameters  $F_m$ ,  $R$  and  $v_r$ . If  $F_m$  and  $R$  at a certain time interval can be held constantly we will have a linear relationship between  $\omega$  and the target's radial speed  $v_r$ .

Indeed, this is a way to implement the FMCW radar.  $F_m$  is set to a constant value as we have selected the range scale. Then, the range  $R$  of target was subsequently calculated by performing the range Fourier transform. So, practically we know the values of  $F_m$  and  $R$  of targets at each point of time. Using the last relationship we can determine the radial speed of a target at a range  $R$ . In order to compute this radial speed we only need to repeat the range FFT many times at a certain azimuth angle.

## 2. METHOD

A series of computer simulations were conducted to verify the above relationship (6).



**Fig. 2:** Computer simulation step by step: generation of beat signal (top left), range-FFT (top right), thresholding (bottom left) and smoothing (bottom right). The peak distance from the origin determines the range  $R$  of target. (Note:  $f_s$  is the sampling frequency and  $R_{max}$  is the maximum range in the selected range scale.)

Fig. 2 shows the principal steps of the conducted computer simulations consisting of:

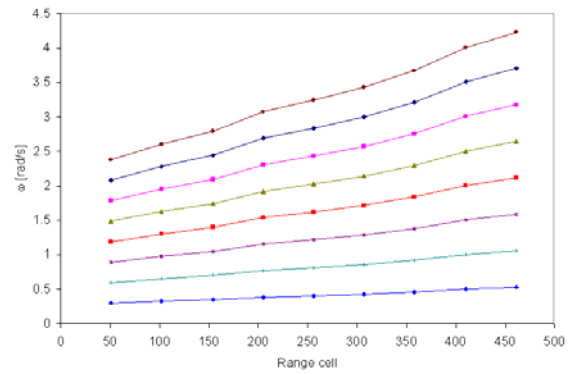
- generating beat signal of targets,
- performing range FFT of the beat signal,
- filtering (thresholding, averaging),
- repeating step 2 and 3 for 512 times,

- detecting the signal peaks, and
- determining the radial speed of targets.

A set of basic considerations have been made in conducting the computer simulations [3,4]. These are listed in the following.

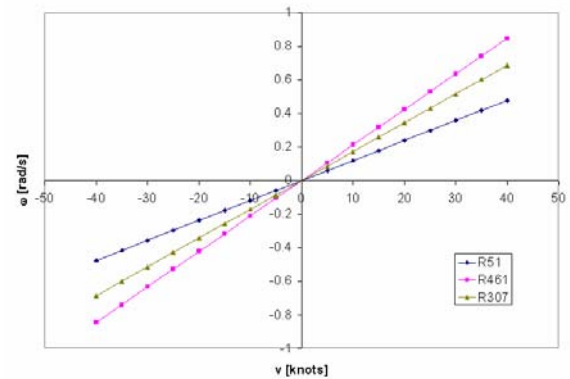
- Targets are moving in radial direction (+/-) with the speed 5 - 40 knots.
- Number of repetition of range FFT is selected to be 512.
- Range scale varies from 1 to 8 (maximum range 40 NM).
- Each range scale is divided into equidistant 512 cells.
- Bandwidth of the beat signal is 1 MHz.
- The sampling frequency is 10 MSPS.
- The sweep time  $t_s$  is given about 0.5 ms.

## 3. SIMULATION RESULTS



**Fig. 3:**  $\omega$ - $R$  curve for various speeds of targets in the range scale 5.

Fig. 3 shows the  $\omega$ - $R$  curves for various speeds of targets in the range scale 5.



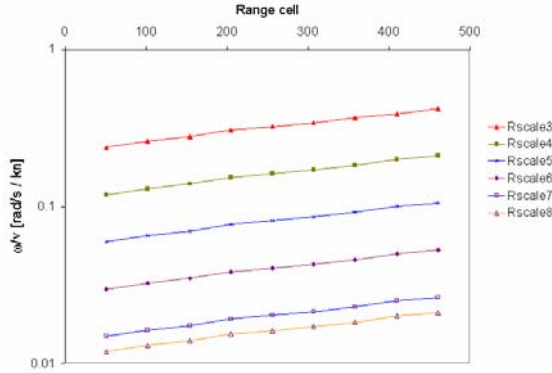
**Fig. 4:**  $\omega$ - $v$  curve for various  $R$  parameter ( $R = 51$ ,  $R = 307$  and  $R = 461$ ).

The target speeds were increasing from blue (bottom) to brown (top). All curves shown in the above figure do not across the vertical axis at the origin. According to previously derived relationship (6) the intersections



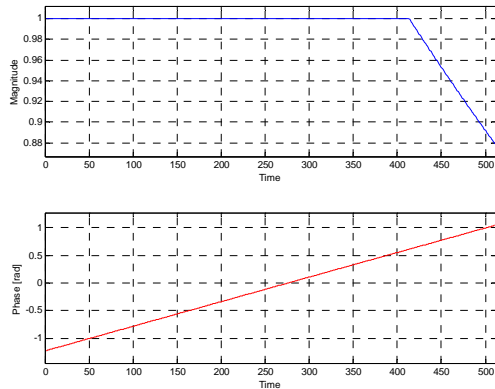
with vertical axis are linear proportional to the target radial speed  $v_r$ .

In fig. 4 we can see  $\omega$ - $v$  curves for various  $R$  parameter ( $R = 51$ ,  $R = 307$  and  $R = 461$ ). It is shown that the parameter  $R$  determines the slope of the curve (the larger  $R$  the larger slope).



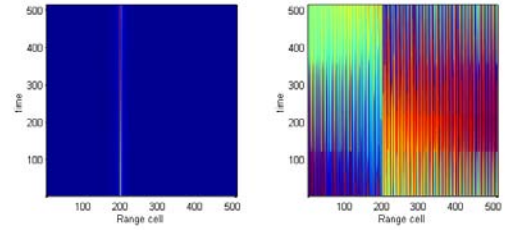
**Fig. 5:**  $\omega$ - $v$  ratio curve for various range scales.

$\omega$ - $v$  ratio curves for various range scales are shown in fig. 5. The horizontal axis is the cell number (which in turn proportional to the range  $R$ ). Here we can see the linear relationship between the normalized  $\omega/v$  and the range  $R$  (or cell number) for various range scales.



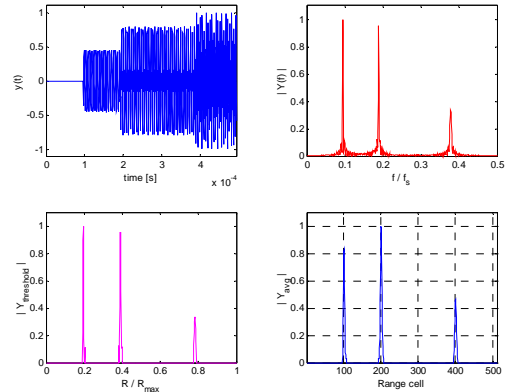
**Fig. 6:** Curve of time-varying magnitudes (top) and phase (bottom) of a series of range-FFT conducted at a certain cell in which a target is detected.

Fig. 6 shows time-varying magnitudes (top) and phase (bottom) curve of a series of range-FFT at a certain cell where a target is detected. The radial speed of a target at this cell can be determined by calculating the rate of change of its phase. This rate of change of phase was obtained by computing the curve's slope. Positive slope of the phase curve indicates that the target is moving away from the origin of radar.



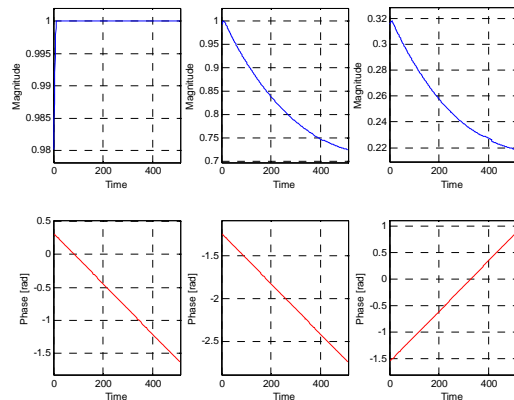
**Fig. 7:** The same result from fig. 6 can be represented as a pair of two dimensional  $A$ - $t$  and  $\phi$ - $t$  image for time-varying magnitudes (left), and phases (right) respectively, of a series of range FFT.

The same result from fig. 6 can also be represented as a pair of two dimensional  $A$ - $t$  and  $\phi$ - $t$  image for time-varying magnitudes, and phases respectively, of a series of range FFT (fig. 7). The horizontal axis represents the range  $R$  (or cell number) and the vertical axis represents the time scale. The values of magnitudes and phases are shown in false colors.



**Fig. 8:** Result of a simulation in the case of three targets detected at range cells 100, 200 and 400.

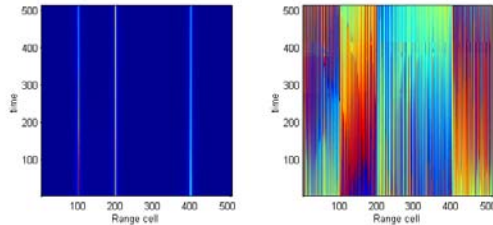
Based on the same principles we can extend the computer experiments for multiple targets. The result of a simulation in the case of three targets detected at range cells 100, 200 and 400 is presented in fig. 8. Starting from this range-FFT we can determine the radial speed of each target using the same method shown in the previous discussion.



**Fig. 9:** Time-varying magnitude (top) and phase (bottom) of three targets at cells 100, 200 and 400.



The time-varying magnitude and phase plots of three targets resulted from the previous simulation is shown in fig. 9. The horizontal axes represent the number of repetition of the performed range-FFT. Based on the slopes of phase plots we can see that the first targets were approaching whereas the last target was moving away from the origin of the radar.



**Fig. 10:** A pair of two-dimensional  $A-t$  (left) and  $\phi-t$  (right) images resulted from fig. 9.

If we represent the result as a pair of 2D  $A-t$  and  $\phi-t$  images then we will get similar results (fig. 10). In this representation mode three targets are recognized as three quasi vertical bright lines at the  $A-t$  image, which in turn consistent with the result shown in fig. 7.

In comparison to the Doppler-FFT method, we need to complete the Fourier transform only once per sweep time. Due to this fact the determination of radial speed of target using this method is more time efficient.

#### 4. CONCLUSION

In this paper a relative simple method to determine the radial speed of a target from FMCW radar signal is presented. This method was implemented by extending and repeating the range FFT many times conducted at a certain azimuth angle in which a target has been detected. Then the radial speed of target is derived from the phase data of a series of this range FFT. From previous discussion it is shown that most calculations needed in the processing stages are linear operations. Furthermore, since the FFT operation is performed only once per sweep the processing time can be significantly reduced.

#### REFERENCE

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