# Fuzzy Set Conjoint Model in Describing Students' Perceptions on Computer Algebra System Learning Environment

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**Abstract**

There are many ways of analysing effectiveness of learning especially in computer learning environment. However, not much attention has been given to analyse it from fuzzy theory perspective. Due to the fuzziness in perception toward learning, this paper presents an application of fuzzy set conjoint model in analyzing students' perceptions on learning algebra and role of teacher in the presence of a computer algebra system (CAS). The study involves a survey which consists of eleven attributes in which six attributes were about learning algebra and another five attributes about role of teacher in a CAS learning environment. Data were collected from one hundred and sixty four students at a secondary school in Terengganu, Malaysia. Scores of perceptions were transformed into degree of similarity and level of agreement for attributes using fuzzy set conjoint model. It is found that attributes about learning indicate the highest degree of similarity at 0.77 with the level of 'disagree' while attributes about role of teacher measured at 0.81 degree of similarity with the level of 'strongly agree'. These measurements implicate the successful of fuzzy approach to evaluate perceptions toward learning algebra in CAS environment.

***Keywords:*** *fuzzy sets, conjoint model, perceptions, computer algebra system.*

## 1. Introduction

Computer algebra systems (CASs) have been proliferated in the algebra classes at the higher school level and tertiary education with the main purpose in enhancing learning outcomes. Aside from its actual efficacy in promoting learning, one of the most useful contributions of CAS is the extent to which it provokes in the relation to pedagogical principles and practice. One of the pedagogical considerations to be looked into is the way of teachers managing teaching process and how students react to the learning. In CAS environment it is notably accepted that teacher imparting knowledge not purely via physical computer screen but more importantly the way the students use cognitive domain rigorously. It is the same as the students deeply engage in algebraic operations with the help from CAS. Many are predicting that the CAS will bring about several benefits to the students and teachers. It has been suggested that the role of teacher must change in the sense that it is no longer sufficient for teachers merely to impart content knowledge. Also, students are expected to acquire knowledge efficiently and certainly are accumulated perceptions toward such the first ever technology that comes to their way of learning.

Study of perception in learning with computer technology is not uncommon and it is normal practice among researchers to analyse perceptions using descriptive and inferential statistics. Reference [1] used analysis of variance and least significance test to compare perceptions between semesters taught with and without multimedia. In a study conducted by [2], simple percentages were used to examine nursing students' perceptions of a Web-enhanced learning environment. Also, statistical inferences were employed by [3] in an analysis of computer knowledge and skill elements perceived by participants. Most recently, [4] employed descriptive statistics and univariate analyses of variance in studying the impact of students' perceived computer experience on behaviour and performance in an introductory information systems course. Since the perceptions and learning are very subjective in nature, an alternative way is practicable instead of descriptive and inferential statistics to analyze learning attributes.

Perceptions toward learning attributes are subjective indeed. Therefore, an alternative method based on fuzzy approach is possible. At this point, it is appropriate to fuzzy set theory being applied. The fuzzy set theory can represent the uncertainty or vagueness inherent in the definition of linguistic variables [5]. The situation of fuzziness and the role of linguistic variables will be explained further in this writing. In short, fuzzy system becomes an alternative method in explaining the fuzziness in the real world of vagueness. Reference [6] summarised the development in fuzzy sets theory into two different lines. The first line categorised fuzzy set theory as a formal theory when maturing became more sophisticated and specified. It was then enlarged by the original ideas and concepts as well as embracing classical mathematical areas such as algebra, graph theory, topology, and so on prior to generalising them. The second line categorised fuzzy set theory as a very powerful modelling language that can cope with a large fraction of uncertainties of real life situations. The nature of its generality, fuzzy sets theory can be well adapted to different circumstances and contexts.

Fuzzy set theory has been applied in many areas such as production, management and psychology as well. There has been substantial research in cognitive psychology showing that fuzzy sets are a good representation for linguistic variables. For example, [7] apply fuzzy membership function in learning achievement evaluation. Recently, [8] describe the application of a fuzzy set in measuring teachers' beliefs about mathematics. Details of the application of fuzzy sets in social application can be retrieved from [9]. The present study comes with a purpose to extend preferential theory to evaluate perceptions in learning processes. This paper was intended to add another application of fuzzy sets in cognitive psychology. Specifically, the purpose of this paper is to measure the degree of perceptions on learning algebra and role of teachers in a CAS environment by using fuzzy set conjoint model. The fuzzy set conjoint model is purely based on the fuzzy sets preference model. The measurement of each attribute was presented in form of degree of membership which believed to be able to reflect the degree of perceptions.

The rest of the paper is organized as follows. Section 2 presents related works on fuzzy preferences methods. Section 3 proposes a computational framework of fuzzy set conjoint model. Section 4 evaluates the students' perceptions in computer learning experiment. The paper ends with a conclusion in Section 5.

## 2. Related Works

There has been widely used of preferences in selecting services or products. Many models have been in placed thus far to analyse preferences. The preference models were widely used in new product design, marketing management, and market segmentation (see 10]; [11]). A decision is made after analysing preferences via numerous techniques. Decision is becoming more complex when preferences are based on multi-attributes products or services. One of the well known analyses especially in marketing and businesses that believed to be able in helping decision makers was conjoint analysis. Conjoint analysis was originally developed to study individuals' preferred levels and relative importance of the multiple attributes of market goods [12]. This technique was coined by [13] is based on the assumption that individuals can evaluate multi-attributes in such a way that their responses are approximately intervals in a measurement level. Because the customer requirements elicited from one customer group may have considerable conflicts with another, a comprehensive evaluation of multicultural factors among diverse customer needs is crucial. The multivariate technique used specifically to understand how respondents develop preferences for products or services. It is based on the simple premise that consumers evaluate the value of a service or product by combining the separate amounts of value provided by each attributes [14].

Recent research has used stated preference techniques such as stated choice and conjoint analysis to develop quantitative estimates of the relative importance on selected attributes of the services or products. Strength of stated choice and conjoint analysis methods is concentrated upon weights. Respondents are asked to express their preferences and provide importance weightings for a single attribute. At the same time, respondents are asked to rank, rate or choose among profiles that describe alternative configurations of the set of attributes under consideration. In recent years, conjoint analysis has been extended to study public attitudes and preferences concerning the provision and management of public goods (for example, [15]; [16]; [17]).

However, there are abundance of vague, uncertainty and very subjective nature in the element of preference. Despite the subjectivity in describing preferences, fuzzy sets theory offers an alternative mean to accommodate with the unclear boundaries and subjective in nature [18]. The fuzzy sets theory was developed in response to the need to have a mathematical measurement of preferences. Indeed, it was very fortunate that the fuzzy sets theory provides a framework that cope with uncertainty in language, that is, subjective uncertainty [19]. Therefore conjoint analysis was being extended to fuzzy set preference model mainly due to the imprecise meaning of preferences. The linguistic variables were prevalent in both psychological attributes (for example 'difficult') and the subject preferences (for example 'agree'). The vagueness of the rating 'agree' is inherent rather than being due to a lack of knowledge about the available rating. The fuzzy set for 'strongly agree' would consist of element pairs, each a domain variable and a degree of membership. A membership function maps each value of the domain variable to a degree of membership or belongingness in the set that are ranging from 0 to 1.

A fuzzy preference model requires fuzzy membership function for each of linguistic ratings on the measurement scale. Likert-type scale was used for all preference ratings, providing the subject with a balanced selection of 7 linguistic terms to indicate their preferences about learning under a computer algebra system environment. This scale has a central neutral evaluation with three positive and three negative evaluations. The underlying theory of fuzzy sets in the preference modelling can be retrieved further from [20]. Additionally, the appeal of using fuzzy sets in preference models comes from representing linguistic variables in a mathematical structure that closely corresponds to actual subject preferences. An overall preference for a statement can be decomposed into a combination of preferences for its constituent parts (attributes), which are combined using a combination function. A combination of preferences becomes the main underlying philosophy in the fuzzy set conjoint model

## 3. Fuzzy Set Conjoint Model

Conjoint analysis gives a degree of consensus agreement for each selected attribute using a combination function. A widely used function is a weighted sum of *T* attribute preferences, where the preference of alternative *m* is defined as

$$y(m) = \sum_{i=1}^{n} (w_i \times e_i(m)) + w_o \qquad (1)$$

, where a numeric value $e_i(m)$ is used for the evaluation of the *i*-th attribute (e.g. 'agree' represented by 3 on 1 to 7 scales). This combination function is known as the vector model, since overall preferences are a vector of attribute preferences. The vector preference model can be modified to use fuzzy sets by forming a linear combination of attribute evaluation sets. Reference [21] applied fuzzy set theory to develop an extended conjoint analysis which was known as fuzzy set preference model. Subject ratings are represented by the fuzzy set defined for linguistic rating, instead of the number associated with the rating. These fuzzy sets are combined in a linear preference model using attribute weights in an identical manner to the combination of crisp numbers in the vector model. The effect of using fuzzy sets to represent ratings can be directly measured, since all other aspects of the vector model are unchanged, such as the crisp important weights and the ordinal measurement property.

The fuzzy conjoint model is a fuzzified vector conjoint model from Equation (1) which adds two dimensions of information: multiple set elements defining a linguistic term and continuous membership of each element in these linguistics term.

A fuzzy set *R* was formed to represent the hierarchy of all respondents against the specific attributes. The approximate degree of membership for each element, $y_j$ ($y = 1,2,3,\ldots\ldots l$) in fuzzy set *R* is defined as

$$\mu_R(y_j, M) = \sum_{i=1}^{n} W_i \, . \mu_{Ri}(x_j, M) \qquad (2)$$

where

$\mu_{Ri}(x_j, M)$ : degree of membership of domain element $x_j$ in the subject's linguistic rating *R* of the *i*- th attribute of *M* for each element in the fuzzy set $R_i$ , $x_j = 1,2,3, \ldots,l.$

$R_i \in$ { very strongly agree (L1), strongly agree (L2), agree (L3), undecided (L4), Disagree (L5) , strongly disagree (L6), very strongly agree (L7)} by *i*-th respondent , $i = 1, 2,3,\ldots, 23$ against attribute *M.*

$W_i$ : weight for *i*-th respondent and for $W_i = \dfrac{w_i}{\sum_{k=1}^{n} w_k}$, as

$w_i$ is a score of linguistic values given by *i*-th respondent.

$l$ : number of linguistic values used (in this study, $l = 7$).

$\mu_R(y_j, M)$ : approximate overall degree of membership of the linguistic value *R* for all factor *M* attributes based on domain value $y_j = 1, 2, 3, 4, 5, 6.$

*M* : factor attributes.

*n* : number of respondents.

### 3.1 Defining Linguistic Variables

Reference [5] introduces the application of linguistic variables. In this study, there were seven linguistic variables used and defined as Lk = {very strongly agree, strongly agree, agree, undecided, disagree, strongly disagree, very strongly disagree}. The fuzzy sets represented for each linguistic value, Lk (k= 1, 2, 3, 4, 5, 6, 7) were defined as follows:

Very strongly agree
L1 = {1/1, 0.8/2, 0.5/3, 0.2/4, 0/5, 0/6, 0/7},
Strongly agree
L2 = {0.7/1, 1/2, 0.6/3, 0.4/4, 0/5, 0/6, 0/7},
Agree
L3 = {0.4/1, 0.6/2, 1/3, 0.6/4, 0.4/5, 0/6, 0/7},
Undecided
L4 = {0/1, 0.3/2, 0.7/3, 1/4, 0.7/5, 0.3/6, 0/7},
Disagree
L5 = {0/1, 0.2/2, 0.4/3, 0.6/4, 1/5, 0.6/6, 0.4/7},
Strongly disagree
L6 = {0/1, 0/2, 0/3, 0.4/4, 0.6/5, 1/6, 0.7/7},
Very strongly disagree
L7 = {0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7}.

### 3.2 Measuring Degree of Similarity

For each attribute, *M*, the calculation of degree of similarity between the fuzzy set representing the whole respondents with every fuzzy set that represented by seven linguistic values (Lk, k= 1, 2, 3, 4, 5, 6, 7) was measured using the sum of a distance formula. The degree of similarity was defined as the Euclidean distance between fuzzy sets R and $L_k$. The formula for the similarity of two sets is

$$Sim(R,Lk)= \frac{1}{\left[1+\sqrt{\sum_{j=1}^{7}(\mu_R(j,M)-\mu_{L_K}(j))^2}\right]} \qquad (3)$$

where $\mu_{L_K}(j)$ is the fuzzy set defined for linguistic rating and $\mu_R(j,M)$ is the calculated set for product *m* from Equation (2).

### 3.3 Measurement Procedures

There were eleven attributes to be considered and analyzed in this study. The computations were executed based on the following steps.

Step 1: Obtain students' responses, $R$ $( i = 1,2,\ldots,n)$

Step 2: Find weights for each respondent, $W_i = \dfrac{w_i}{\sum_{k=1}^{n} w_k}$,

Step 3: Obtain membership of each element in fuzzy set, *R* using equation (2).

Step 4: Obtain similarity measures between fuzzy set *R* and fuzzy set, $L$, $k = 1,2,\ldots 7$ using Equation (3).

Step 5: Select the linguistic L which registered the highest degree of similarity.

## 4. An Experiment

One hundred and sixty four Form Four students from a secondary school in the east coast of Peninsular Malaysia participated in this study. The students were located in a multimedia laboratory equipped with 30 workstations, one workstation for teachers, a Liquid Crystal Display (LCD) projector, a screen and a white board. Every workstation had been installed with CAS software.

The students spent one forty minute session using a CAS to investigate the maximum (or minimum) values of quadratic functions. Students were presented with a combination of a Math notebook document, the activity notebook and a paper study guide booklet.

After their investigations were completed, they were then taught how to determine the maximum (or minimum) values of a quadratic function using a traditional approach in learning algebra. In another words, the students were studying the same knowledge but using a different medium. After completing the lessons, they were asked to respond to a survey. A simple eleven attributes survey was used to establish the students' perceptions about the procedural knowledge approach in learning algebra. The survey comes with seven scales of agreement (Lk) ranging from 'very strongly agree' (1) to 'very strongly disagree' (7) for each attributes. The six attributes At1, At2, At3, At4, At5, and At6 were the general outlook of learning algebra in CAS environment. Another five attributes At7, At8, At9, At10, and At11 were the statement about roles of teacher in CAS environment. Six class sessions were held to accommodate the complete student cohort. Data collected were then analysed with fuzzy conjoint model. A sample of student's exploration in activity notebook and the details of the attributes can be seen from [22]; [23].

The computations were conducted using a computational data analyse software based on the procedures as described in Section 3. The six attributes about learning algebra and their corresponding linguistic variables of degree of agreement were obtained. In addition to this, each attribute comes with their degree of similarity to reflect the strength of level of agreement. The perceptions of the first six attributes can be seen in Fig. 1.

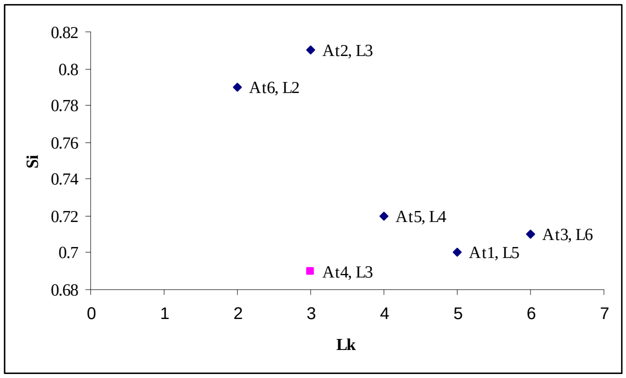


Fig. 1 Degree of similarity, Si and level of agreement, Lk for attribute 1 to attribute 6

It can clearly be seen that At2 has the highest degree of similarity (0.81) with level of agreement at L3. On the other extreme, At4 has the lowest degree of similarity (0.69) with the same level of agreement as At2.

Another five attributes which explaining role of teachers and their positions in degree of similarity and level of agreement can be seen in Fig. 2. The At7 and At8 have scored the same degree of similarity (0.76) but with different level of agreement.

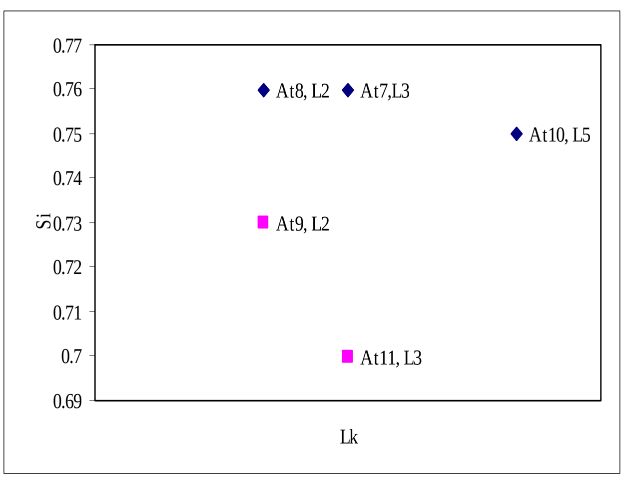


Fig. 2. Degree of similarity, Si and level of agreement, Lk for attributes 7 to 11

Levels of agreement were different among the attributes which saw the At1 to At6 of learning algebra recorded the degree of agreement ranged from 0.69 to 0. 81. The different fashion of distributions for level of agreement and degree of similarity is shown by the At 7 to At11. The latter attributes were explaining the role of teachers in learning.

Degree of similarity for each attribute was solely reflect the specific attribute and it is far better if the multiple attributes are being integrated into one single consensus level of agreement using a fuzzy set preference model. Based on the capability of the fuzzy set conjoint model in seeking one single level of agreement from multiple attributes, another cycle of procedures of fuzzy set preference model was executed for two groups of attributes. The results are presented in Figure 3.

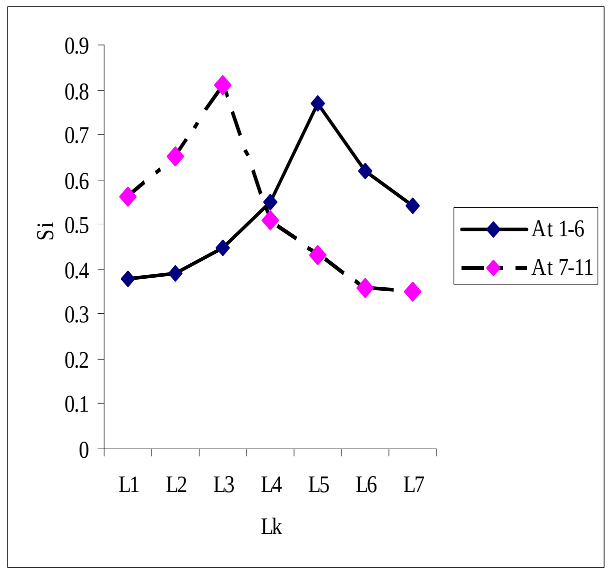


Fig. 3. Level of agreement with their respective degree of membership for the two groups of attributes

The At1 to At6 and At7 to At11 have recorded the different level of agreement and degree of similarity as well. The differences were obvious with two groups of attributes have peaked at different level of agreement. It implicates that students perceived differently toward the general outlook of teaching and the role of teacher in CAS learning environment

## 5.0 Conclusions

In the present study, the preferences of users and conjoint analyses were form as a basis in describing preferences or to be exact in this case is the perceptions on learning attributes from students' perspectives. Fuzzy set conjoint model has shown its superiority to rank alternative configurations of the learning attributes from most preferred to least preferred in form of level of agreement and degree of similarity. Perceptions of students were hierarchically ordered into linguistic variables accompanied with their respective strengths. Findings from the study suggest that students viewed a certain level of agreement in order to provide greater opportunities for learning algebra in computer environment. A fuzzy set preference model has successfully measured the students' perceptions in form of degree of memberships and perhaps could be used as meaningful input specifically in a computer technology learning environment. The multi-attributes fuzzy evaluation approach in learning algebra under computerised technology seems to have worked and hence is very useful in providing information for technology education decision makers. Further applications of fuzzy set theory especially in social sciences could possibly be explored and perhaps would offer more viable and meaningful results.

## References

[1] K.A. Beerman, Computer based multimedia, New direction in teaching and learning. Journal of Nutritional Education, 28, 1996, pp.15-18.

[2] D.K Creedy, M. Mitchell, P.S., Sykes, and M. Cooke, Evaluating a Web-Enhanced Bachelor of Nursing Curriculum: Perspectives of Third-Year Students. Journal of Nursing Education, 46(10), 2007, pp. 460-468.

[3] J.L Davis, and H. Davis, Perceptions of career and technology and training and development students regarding basic personal computer knowledge and skills. College Student Journal, 41(1), 2007, pp.69-79.

[4] D.J. Ballou, and B.R. Huguenard, The Impact of Students' Perceived Computer Experience on Behavior and Performance in an Introductory Information Systems Course, Journal of Information Systems Education, 19(1), 2008, pp. 87- 98.

[5] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Part 1 and 2, Information Sciences 8, 199- 249, 1975,pp.301- 357.

[6] H.J. Zimmerman, Fuzzy Sets Theory and its applications (2nd revised ed.). Boston: Kluwer Academics Publishers, 1991.

[7] S. Weon, and J.Kim, Learning achievement evaluation strategy using fuzzy membership function, 31st ASEE/IEEE Frontiers in Education Conference, Reno, NV., 2001.

[8] A. Lazim, Procedural knowledge in the presence of a Computer Algebra System (CAS): Rating the drawbacks using a multi-factorial evaluation approach. International Journal for Technology in Mathematics Education, 14 (1), 2007, pp.14-20

[9] A Lazim., W.A, Wan Salihin and M.T. Abu Osman Fuzzy Sets in Social Sciences: An Overview of Related Research. Jurnal Teknologi, 41(E), 2004, pp.43-54.

[10] P.Z Green, and V. Srinivasan Conjoint analysis in market research: New developments and directions, Journal of Marketing, 54, 1990, pp. 3-19.

[11] D.R., Wittink, and P. Cattin, Commercial use of conjoint analysis: An update, Journal of Marketing, 53, 1989, pp.91-96.

[12] J. J. Louviere, Analyzing decision making: Metric conjoint analysis. Newbury Park, CA: Sage Publications, Inc. ,1988.

[13] P.Z. Green and V. Srinivasan, Conjoint analysis in consumer research: Issues and outlook, Journal of Consumer Research, 5, 1978, pp.103–123.

[14] D.H. Stamatis, Six Sigma and Beyond: Statistics and Probability, Volume III. New York: St Lucie Press, 2003.

[15] D. Dennis, Analyzing public inputs to multiple objective decisions on national forests using conjoint analysis, Forest Science, 44, (2), 1998. pp.1-429.

[16] K.Kneeshaw, , J., Vaske, A., Bright, & J.Absher, Situational influences of acceptable wildfire management actions. Society and Natural Resources, 17, 2004, pp. 477-489.

[17] T. Stevens, R.Belkner, , D., Dennis, D., Kittredge, & G.Willis, Comparison of contingent valuation and conjoint analysis in ecosystem management. Ecological Economics, 32, 2000, pp. 63-74.

[18] L.A. Zadeh, Fuzzy Set, Information and Control 8, 1965, pp. 338 – 353.

[19] M. Mukaidono, Fuzzy Logic for Beginners. Singapore: World Scientific Publishing, 2001.

[20] I.B Turksen, and I.A. Willson, A fuzzy set model for market share and preference prediction. European Journal of Operational Research 82, 1995, pp.39-52.

[21] I.B Turksen, and I.A.Willson, A fuzzy set preference model for consumer choice. Fuzzy Sets and Systems 68, 1994,

[22] A. Lazim, Construction of conceptual knowledge of quadratic functions in QFtica: A fuzzy approach and multiple representations. Unpublished Ph.D thesis, University College of Science and Technology Malaysia. Kuala Terengganu, Malaysia, 2004.

[23] A. Lazim,. and M.T. Abu Osman, Measuring Teachers' Beliefs about Mathematics: A fuzzy Set Approach, International Journal of Social Sciences, 4(1), 2009, pp. 39-43.

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