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# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

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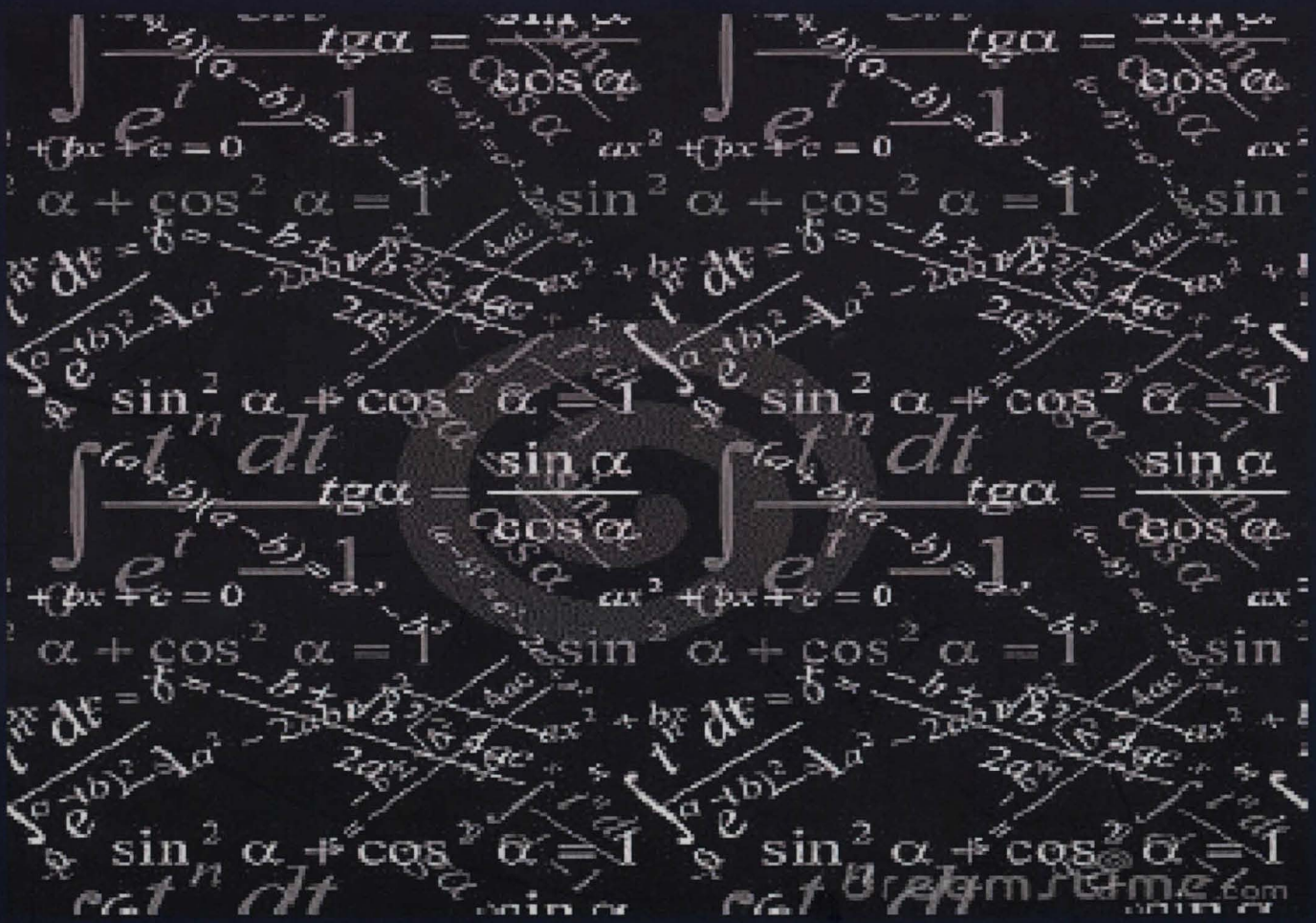


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: Mansoor Saburov

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# ON NONLINEAR DYNAMIC SYSTEM ARISING IN POTTS MODEL

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**Abstract**

From three-state Potts model we produce the recursion relations that provide us numerically exact phase diagrams of the Potts model. Each phase is characterized by a particular attractor and the phase diagram is obtained by following the evolution and detecting the qualitative changements of these attractors. These changements can either be continuous or abrupt, respectively, characterizing second or first order transitions. We present a few typical attractors.

***Keywords***: *Phase diagram; Potts model; nonlinear dynamical system; attractor.*

## Introduction

Statistical physics seeks to explain the macroscopic behavior of matter on the basis of its microscopic structure. This includes the analysis of simplified mathematical models [1]. The Potts model [3] was introduced as a generalization of the Ising model to more than two components (spins). Ising model considered only up and down spins [2] whereas Potts model incorporates more possibilities of spins and their interactions. The Potts model describes an easily defined class of statistical mechanics models. At the same time, its rich structure is surprisingly capable of illustrating almost every conceivable nuance of the subject. The Potts model encompasses a number of problems in statistical physics (see, e.g. [4]). A phase diagram of a model describes a morphology of phases, stability of phases, transitions from one phase to another and corresponding transitions line. A Potts model just as an Ising model on a Cayley tree with competing interactions [5-7], has recently been studied extensively because of the appearance of nontrivial magnetic orderings. The Cayley tree is not a realistic lattice; however, its amazing topology makes the exact calculation of various quantities possible. The aim of this paper is to study some typical attractors of nonlinear dynamic system generated by the Potts model with competing interactions.

## POTTS Model with Competing Interactions

Recall that the Cayley tree $\Gamma^k$ of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles with exactly $k + 1$ edges issuing from each vertex. Let denote