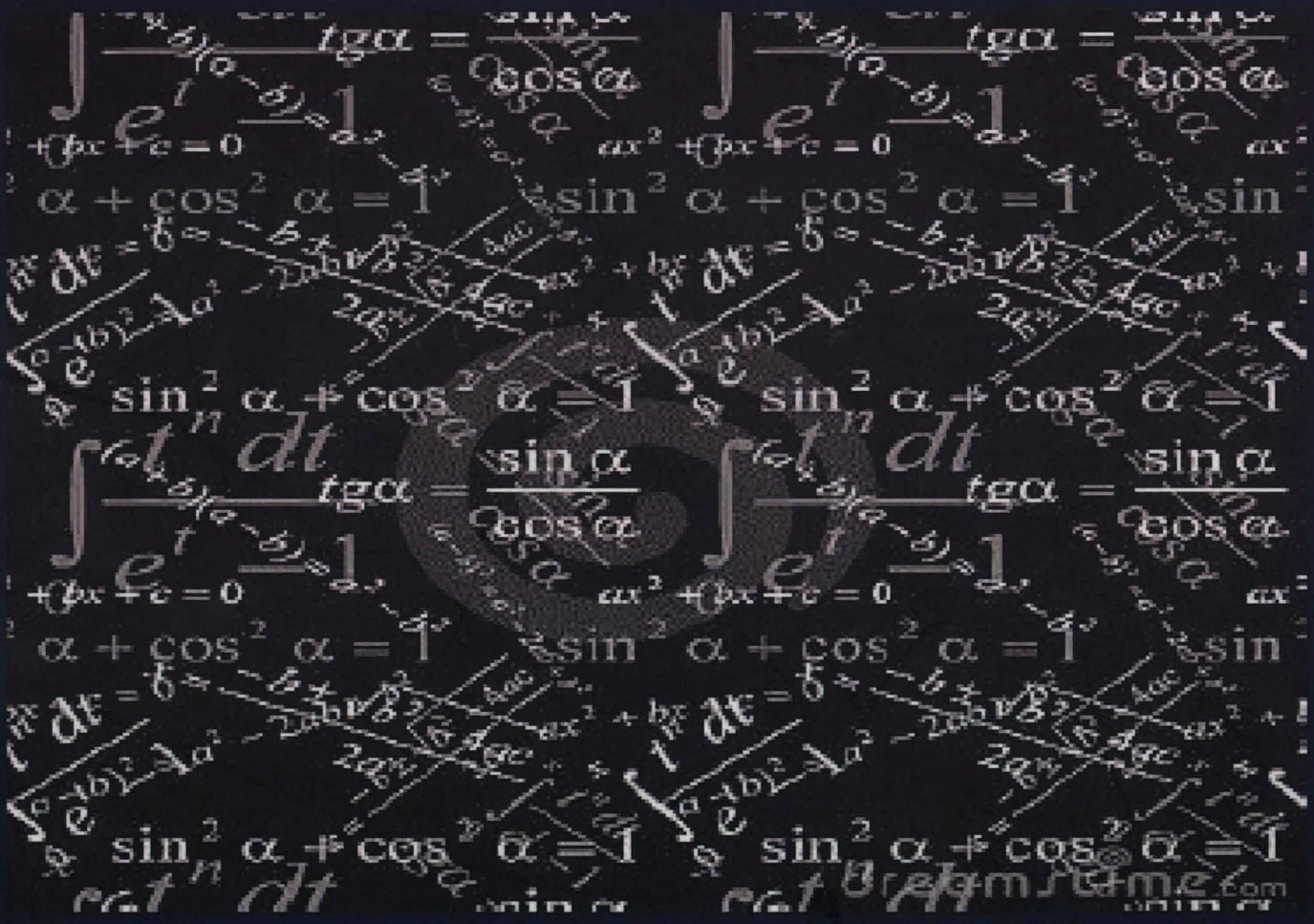
الجامعة الإسلامية العالمية ماليزيا
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA
يونيرسيتي إسلام انتارابغسا ملسيا

VOLUME 2

RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of Computational and Theoretical Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov
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Vol. 2

IIUM Press

Published by:
IIUM Press
International Islamic University Malaysia

First Edition, 2011

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov
Recent Achievements in Dynamical Systems
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ISBN: 978-967-418-201-4

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by :
IIUM PRINTING SDN. BHD.
No. 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

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ISING MODEL ON A GENERAL CAYLEY TREE WITH COMPETING NEXT-NEAREST-NEIGHBOR INTERACTIONS

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Abstract

We study the Ising model on a general Cayley tree of arbitrary order and produce the phase diagram with competing interactions prolonged next-nearest-neighbour J_p and one-level k-tuple next-nearest-neighbour J_o . Vannimenus proved the existence of modulated phase in the phase diagram of Ising model with competing nearest-neighbour interaction J_1 and prolonged next-nearest-neighbour interactions J_p, as found for similar models on periodic lattices. Later Mariz et al generalized this result for Ising model with $J_o \neq 0$. For a given lattice model on a Cayley tree, i.e., $J_p \neq 0$; $J_o \neq 0$ with $J_1 = 0$, we describe the general equation, phase diagram and clarify the role of nearest-neighbour interaction J_1. In the presence of nearest-neighbour interaction J_1, Vannimenus demonstrated that for arbitrary random initial data one can reach the same phase diagram. We show that in the case $J_1 = 0$ the set of all possible initial data can reach different phase diagrams.

***Keywords**: Lattice models, Cayley tree, prolonged next-nearest neighbour, one-level next-nearest neighbour, modulated phase*

Introduction

Statistical mechanics well known attempts is to predict the properties of complex system containing many interesting components undergoing random thermal motion. These properties might be molecules in a gas, atoms in a magnet, polymer in solution or other interesting physical system. Lattice models are a big industry within statistical mechanics. These models have a variable at each site of a regular grid, and a Hamiltonian or evolution law for these variables. The Ising model is the most extensively studied lattice model in statistical mechanics. The model consists of a set of magnetic spins arranged on a regular square lattice. It has a lattice of N sites i with a single, two states degree of freedom s_i, on each site that may take values ± 1 [11].

In the case of the Ising model with competing nearest-neighbour (NN) interactions J_1 and prolonged next-nearest-neighbour interactions J_p Vannimenus [8] was able to find new modulated phases, in addition to the expected paramagnetic and ferromagnetic ones. From this result follows that