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PERGAMON

Plates

Shells

STABILITY AND IMPERFECTION SENSITIVITY OF SPHERICAL-TIP CONICAL SHELLS

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ABSTRACT

The instability under external pressure of conical end-caps with spherical-tips as end-closures of pressure vessels is studied in this paper, both analytically and experimentally. Spherical-tip conical end-closures of pressure vessels were electroformed with copper in such a way that continuity of the slope at the cone/sphere junction was maintained. The geometric imperfections of the axisymmetric end-closures were determined from measured geometric data. Theoretical buckling loads for these imperfect end-closures were determined from the solution of governing nonlinear differential equations of axisymmetric deformations using the method of multisegment integration. Asymmetric buckling loads for the conical frusta alone were also determined theoretically. Investigations show that spherical caps are very sensitive to imperfections of geometry rather than to the imperfection of thickness. Further it is observed that addition of even a small conical frustum at the base of a spherical end-closure reduces its imperfection sensitivity substantially. Spherical end-closures are found to buckle symmetrically at the tip, where large radius-type imperfection takes place and the combinations of spherical-caps and conical-frusta are found to buckle asymmetrically in the conical segments of the end-closures.

KEYWORDS

Stability, imperfection sensitivity, end-closure, shell.

INTRODUCTION

End-closures of cylindrical pressure vessels are in most cases spherical, ellipsoidal, plane or conical in shape. The end-closures are usually the most vulnerable section of pressure vessels both in terms of stresses and instability. Instability is becoming the most overriding criterion in designing externally pressurized vessels with time because of the availability of high strength materials and improvement in fabrication technology.

Seide (1959) showed that critical external pressure of a conical frustum is approximately equal to a certain factor times the critical external pressure of a cylinder having the same wall thickness, a radius equal to the average radius of curvature of the cone and a length equal to the slant length of the frustum. The factor is a function only of the ratio of the end radii of the cone and increases from 1.00 for the cylinder (1-r/R = 0) to 1.22 for a cone with (1-r/R) equal to 0.8 and then decreases to about 1.17 for a complete cone (1-r/R = 1). Thus, in line with these observations, overall buckling pressure of a cylindrical pressure vessel may be increased by closing the ends of the cylinder with a suitable conical cap. The average radius of a full cone is always less than that of any of its frustum so, instead of using a full cone, a conical frustum with a spherical tip attached in such a way that the slope is continuous at the cone/sphere junction may be a better solution to end-closures of cylindrical shells. Such spherical-tip conical shells are shown schematically in Fig. 1.

Though pressure vessels are made of combination of different shell geometries, until now, very few works are available on the stability of combination of shells (Galletly 1974, Aylward 1975, Ross 1987a, Ross 1987b, Khan 1995a , Khan 1995b) . In 1995, Khan et.al.(1995a) investigated axisymmetric stability behavior of conical end-caps and found that axisymmetric stability pressure of cap-cone combined end-closures are higher than that of the spherical caps. To study the suitability of cap-cone combinations, experiments were carried out with electroformed copper cap-cone shells and the results are compared with axisymmetric buckling load of the combinations. Asymmetric buckling load for the attached conical frustum of the combination has also been studied and presented in this paper.

ANALYSIS

The nonlinear governing equations of equilibrium for axisymmetric deformation of a shell of revolution applied to its deformed shape, as developed by Reissner (1949) and modified by Uddin (1969), are solved by the method of multisegment integration, developed by Kalnins et al (1967), using the computer code developed by Uddin (1986). Asymmetric buckling pressure is obtained from eigen value solution of linearized governing equations, derived from general equations with perturbation of variables, generally known as the stability equations.

EXPERIMENTAL PROGRAM

For determining the buckling pressure of combined cone-sphere end-closure, copper shell models with same apical angle and four different r/R ratio were fabricated by an electroforming process. The conical segment of the combined cap-cone end-closures, fabricated for experiment, had a semi apex angle of 30°. Geometric data of each model was then processed with a graphical package to find the geometry of the shells. In case of the cap-cone composite shells two geometric curves were fitted to the meridians, a straight line was fitted to the conical portion of the shell and a circular arc was fitted to the spherical cap of the shell meeting tangentially with the straight line .For the spherical cap end-closure (r/R=1.0) two circular arcs were fitted to the meridians . To determine the thickness of the shells, the shells were cut along three meridians after the experiment, and the thickness along these three meridians were measured with a precision micrometer. The accuracy of the micrometer was ± 0.001 inch. Description of the geometric parameters obtained from processed data are shown in Fig.2. Geometric data and average thickness data of the shells are given in Tables (1-2) .

Schematic diagram of the experimental setup with a shell model placed inside the pressure chamber is shown in Fig.3 .The experimental model was filled with water and then was attached to the base-attachment with six fastening bolts. A tripod was placed inside the pressure chamber to support the shell-base assembly keeping the tip of the model down and the base attachment up so that water inside the model could not come out through the tube attached to the base without the application of pressure, external to the model.

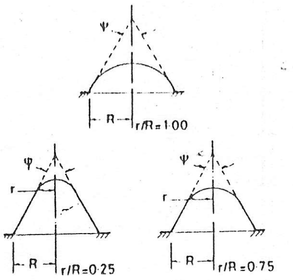

Fig.1 End-closures with different r/R ratio.

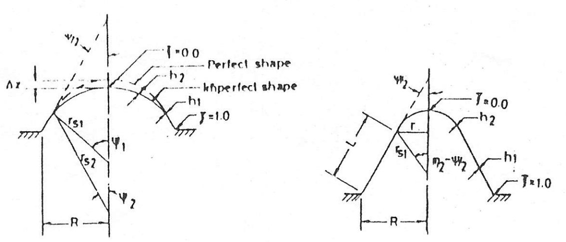

Fig.2 Geometry of Spherical end-closure and Spherical-tip conical end-closure.

External pressure was applied to the models of end-closure by pumping in water by a hand pump attached to the pressure chamber. The external pressure was gradually increased by steps and at every step increase in pressure was recorded. At every step of external pressure, the internal volume change of the model was recorded by the volume-meter by measuring the amount of water coming out from the inside of the model. Pressure was increased up to a limit when either the manometer reading remained constant with a large change in volume or the manometer reading dropped suddenly with a large change in volume. The limiting pressure was observed to correspond either to a large deformation of the spherical tip or of the conical base of the end-closure. This limiting pressure was considered as the buckling load for the corresponding end-closure.

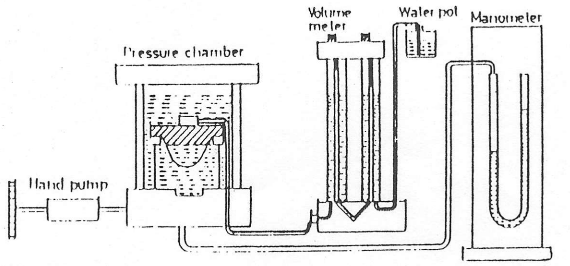

Fig.3 Schematic diagram of experimental set-up.

RESULTS AND DISCUSSION

Buckling test was performed for four different sets of shells, described earlier. Photographs of the electroformed models before and after experiment are shown in Fig.4. The four sets of models are designated as CS100, CS80, CS60 and CS30, corresponding to r/R ratio of 1.00, 0.80, 0.60, and 0.30, respectively.

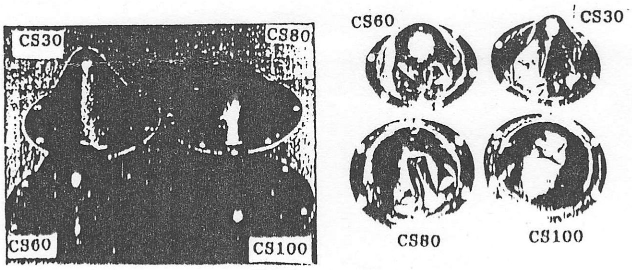

Fig.4 Models before and after experiment.

Both the theoretical and experimental buckling loads are presented in Tables (3-4) for all the four sets of end-closures. Experimental load versus volumetric change curves for the four sets of end-closures are presented in Fig.5. During the experiment, all the end-closures except CS100 were observed to buckle asymmetrically in their conical segments. The CS100 end-closures, that is, the pure spherical caps, buckled axisymmetrically only at the tip.

The analytical results for the spherical-tip conical end-closures, presented in Table 3, are the axisymmetric buckling loads of the compound end-closures and asymmetric buckling loads of their conical segments. In case of the pure spherical end-closure the analytical results are the axisymmetric buckling loads of imperfect experimental models, generated from measured geometric data and of the perfect spherical cap with thickness equal to the minimum thickness of the corresponding imperfect model cap.

TABLE 1

Geometric data of spherical-tip conical end-closures

End-closure type	Model No.	$\Psi/2^o$	L mm	R mm	r_{s1} mm	r mm	h_1 mm	h_2 mm
CS30	1	29.886	81.413	58.046	20.16	17.479	0.089	0.127
	2	30.174	82.391	57.900	19.08	16.494	0.089	0.101
	3	30.029	81.486	57.800	19.66	17.185	0.089	0.114
CS60	1	30.174	46.436	58.139	40.06	34.639	0.109	0.0889
	2	30.029	45.700	58.381	40.44	35.014	0.132	0.088
	3	29.887	45.884	58.045	40.41	35.036	0.140	0.102
	4	30.029	46.395	57.803	40.10	34.713	0.135	0.102
CS80	1	30.174	15.987	57.965	57.78	49.890	0.102	0.097
	2	30.03	16.170	58.380	57.67	49.930	0.084	0.084
	3	30.174	16.090	57.965	57.59	49.530	0.094	0.076

TABLE 2

Geometric data of pure spherical end-closure CS100

Shell No.	$\Psi/2^o$	Ψ^o_1	Ψ^o_2	R mm	r_{s1} mm	r_{s2} mm	h_1 mm	h_2 mm	Δz mm
1	31.887	12.315	8.498	58.175	68.39	98.71	.102	.089	.49
2	31.365	13.127	9.678	58.14	68.09	91.98	.102	.076	.47
3	31.125	12.699	9.651	58.014	67.77	88.82	.108	.094	.40
4	31.92	11.565	9.087	58.256	68.63	87.12	.132	.109	.30
5	31.67	12.409	8.493	58.18	68.36	100.10	.109	1.102	.421

Comparison of experimental and analytical results of Table 3 show that, analytical axisymmetric buckling loads of the compound end-closures are always much higher than their corresponding experimental results. But the analytical asymmetric buckling loads are found to be very close to their experimental results. The difference between the experimental and theoretical asymmetric buckling loads is observed to decrease with increasing length of the conical segment of the end-closure.

Irrespective of the technique of fabrication, the tips of pure spherical end-closures are always found to be imperfect both in geometry and in thickness. The geometric imperfections that mostly occur at the tip are of flat spot type, that is, the tip is generally a cap of larger radius of curvature than that of the sphere. The present analysis shows that this type of imperfection is very buckling sensitive. Even analytical solution for this type of imperfect tip spherical end-closures gives buckling load 1.25 to 1.5 times lower than that for perfect geometry of the tip. The analytical and experimental results of pure spherical caps, CS100, presented in Table-4, are thus in wide disagreement. Inclusion of both geometric and thickness imperfections in the analytical investigation gives 2 to 3 times higher load than that obtained experimentally. Even for a very small deviation in geometry of the spherical end-closure, the analytical buckling loads (shown in Table-4 shell No. 4 and 5) are found in wide disagreement with the experiment. This reveals that not only the imperfection of the tip geometry but even a small imperfection of the overall geometry of the end-closure is highly sensitive to their instability loads.

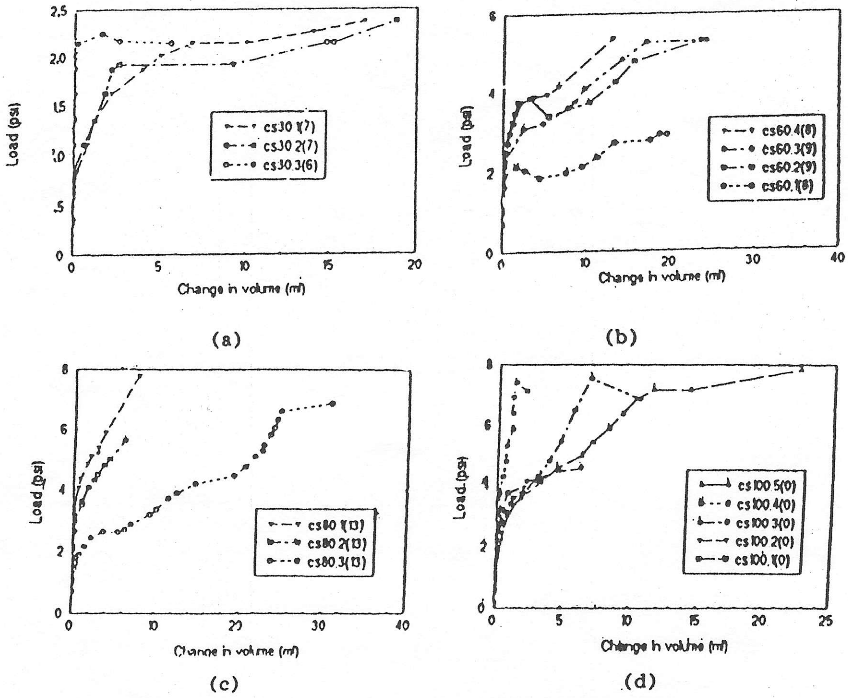

Fig.5 Load vs. volumetric displacement of different end-closures. (a) CS30 (b) CS60 (c) CS80 (d) CS100.

TABLE 3

Experimental and analytical buckling loads of spherical tip conical end-closures. E for copper is 97.86 Gpa

End-closure type	Model No.	Experimental Critical Load P/E	Asymmetric Critical Load for Cone P/E		Axisymmetric Critical Load for Experimental Geometry P/E
			Ends Hinged	Ends Fixed	
CS30	1	1.69×10^{-7}	1.94×10^{-7}	1.94×10^{-7}	26.2×10^{-7}
	2	1.60×10^{-7}	1.91×10^{-7}	1.91×10^{-7}	26.2×10^{-7}
	3	1.69×10^{-7}	2.03×10^{-7}	2.04×10^{-7}	26.3×10^{-7}
CS60	1	2.75×10^{-7}	5.005×10^{-7}	5.195×10^{-7}	18.7×10^{-7}
	2	3.835×10^{-7}	5.082×10^{-7}	5.199×10^{-7}	27.0×10^{-7}
	3	3.82×10^{-7}	5.928×10^{-7}	6.071×10^{-7}	31.6×10^{-7}
	4	3.857×10^{-7}	5.461×10^{-7}	5.88×10^{-7}	20.0×10^{-7}
CS80	1	6.393×10^{-7}	7.346×10^{-7}	7.843×10^{-7}	15.9×10^{-7}
	2	4.786×10^{-7}	4.142×10^{-7}	4.445×10^{-7}	10.5×10^{-7}
	3	5.00×10^{-7}	5.513×10^{-7}	5.937×10^{-7}	13.0×10^{-7}

Table 4

Experimental and analytical buckling loads of pure spherical end-closure CS100. E for copper is 97.86 Gpa

Model No.	Experimental Critical Load P/E	Axisymmetric Critical Load with Experimental Geometry P/E	Axisymmetric Critical Load for Perfect Geometry with Lowest Thickness P/E
1	3.236×10^{-7}	7.25×10^{-7}	10.15×10^{-7}
2	3.035×10^{-7}	6.04×10^{-7}	$8.456 \times ^{-7}$
3	5.478×10^{-7}	8.11×10^{-7}	11.354×10^{-7}
4	5.393×10^{-7}	15.5×10^{-7}	21.7×10^{-7}
5	5.928×10^{-7}	8.99×10^{-7}	12.586×10^{-7}

Analytical and experimental results of the CS80 shells, show that they are quite insensitive to both geometric and thickness imperfections. None of the CS80 shells initiated buckling at the tip, where lies all sorts of imperfections. These shells can sustain much higher pressure in comparison to other models having the same thickness.

CONCLUSION

The results presented herein show that spherical-tip conical end-closures with r/R ratio around 0.80 are the least imperfection sensitive and can sustain the highest prebuckling external load. Thus a spherical-tip conical shell with r/R of about 0.80 is a better end-closure of pressure vessels under uniform external pressure in comparison to the conventional ones.

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