Twist analysis of piezoelectric laminated composite plates

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ABSTRACT

Recently scientists are showing interests in smart structures for their capability in controlling structural behavior and monitoring structural health. Twist control of helicopter rotors, micromirrors or shafts in torsional oscillation are the active areas of research where smart materials like piezoceramic crystal or shape memory alloy can play vital roles. In this paper analysis of twisting of piezoelectric laminated composite plates using higher order shear deformation theory has been presented.

Keywords: Piezoelectric laminated plate, twist, higher order shear deformation theory.

1. INTRODUCTION

The needs for structures with self-monitoring and self-controlling capabilities especially in aerospace applications have caused remarkable growth in the research and development of smart structures. A smart structure can be defined as a structure made up of purely elastic materials, called the substrate, integrated with surface mounted or embedded sensors and actuators that have the capability to sense and take corrective action [1]. Laminated composite plates are found to be suitable for use as substrates in the aerospace applications for its high strength-to-weight and stiffness-to-weight ratios [2]. Thus laminated composite plate with embedded piezoelectric material that can act both as sensor and actuator is a good choice for active control of structural shape. However, this kind of active structure still needs comprehensive analyses to be used successfully in applications.

There are few exact solutions available to describe the kinematic behavior of the piezoelectric laminated composite plates. Exact solution becomes very difficult when the configuration of the structures with embedded piezoelectric laminates becomes complex. An alternative solution would be to use finite element solutions. The work of Allik and Hughes [3] can be considered as one of the pioneer in the field of finite element formulation, which includes piezoelectric effects in structures. A two-dimensional quadrilateral piezoelectric plate element with one electrical degree of freedom per element was developed by Hwang and Park [4] to study vibration control of a piezoelectric laminated composite plate. The plate element has four nodes and each of the nodes has three mechanical degrees of freedom. Classical laminated plate theory (CLPT) with induced strain actuation and Hamilton’s principle were used to formulate the equation of motions. It was found active and passive control affect each other, thus should be considered simultaneously in designing efficient controlled structures.

The works done using CLPT were found only suitable to model thin plate. However to model thick plate, it is generally accepted that higher-order shear displacement theory (HSDT) can model more accurately, since through HSDT effects of transverse shear stresses can be captured. Ray et. al. [5] developed a finite element model for static analysis of a simply supported rectangular intelligent plate using the HSDT. An eight-noded two-dimensional quadratic quadrilateral isoparametric element was derived to model the coupled electromechanical behavior. The results obtained in these work were compared with exact solutions done by the same authors previously [6] and found to be in good agreement. Chattopadhyay and Seeley [7] developed a refined HSDT and used finite element method to analyze laminated composite plate surface bonded or embedded with piezoelectric layers. Non-linearities were introduced to the problem through the strain dependent piezoelectric strain coefficients and the assumed strain distribution through the thickness. Results obtained from this model were shown to agree well with published experimental results by Crawley and Lazarus [8].

All the above works are mainly concentrated on load displacement of piezoelectric plates under different actuation voltage. Twisting or torsional deformation has almost been disregarded so far. In the present research, piezoelectric material that has the capabilities to act as actuators and sensors due to direct and converse piezoelectric effect is chosen and integrated with a laminated composite plate to study twisting effect of the plate under different actuation voltage. Laminated composite plate is chosen as the substrate for its high strength-to-weight and stiffness-to-weight ratios. These characteristics make the laminated composite plate suitable to be used in many applications especially in aerospace applications. HSDT developed by Pervez [9] is used in order to accurately describe the kinematic behavior of the laminated composite plates and the piezoelectric layers. The elastic field and the electric field are coupled through the linear piezoelectric constitutive equations. In the finite element model, an eight-noded two-dimensional isoparametric element is used. Each node has seven mechanical degrees of freedom and one electrical degree of freedom.

2. MATHEMATICAL MODEL

Kinematics and Constitutive Relations

Figure-1 shows the geometry of a laminated composite plate with surface bonded piezoelectric actuator layers on the top and the bottom surfaces. The plate considered here has total thickness h, length a, width b and number of layers n.

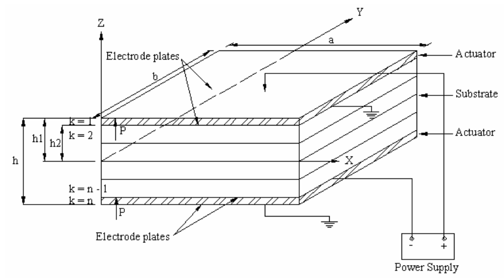

Figure 1 Geometry of a laminated composite plate with surface bonded piezoelectric layers.

The higher-order shear displacement fields considered in this research work are [9]:

$$u(x, y, z, t) = u_o(x, y, t) + z\theta_x + z^3\zeta_x \tag{1a}$$

$$v(x, y, z, t) = v_o(x, y, t) + z\theta_y + z^3\zeta_y \tag{1b}$$

$$w(x, y, z, t) = w_o(x, y, t) \tag{1c}$$

where u, v, w are the displacements of a generic point (x, y, z) in x, y and z directions respectively; u_o, v_o, w_o are the displacements of mid-plane in x, y and z direction respectively; z is the coordinate in thickness direction; θ_x, θ_y are the rotations of normals to reference surface about the y and x axes respectively; and ζ_x, ζ_y are the third order displacements or warping functions.

The strain associated with the displacement fields can be expressed as follows [10]:

$$\{\varepsilon\} = \left[Z'\right]\{\bar{\varepsilon}\} \tag{2a}$$

where,

$$\{\varepsilon\} = \{\varepsilon_{xx} \ \ \varepsilon_{yy} \ \ \gamma_{yz} \ \ \gamma_{xz} \ \ \gamma_{xy}\}^T \tag{2b}$$

$$\{\bar{\varepsilon}\} = \left[\frac{\partial u_o}{\partial x} \ \ \frac{\partial v_o}{\partial y} \ \ \frac{\partial \theta_x}{\partial x} \ \ \frac{\partial \theta_y}{\partial y} \ \ \frac{\partial \zeta_x}{\partial x} \right.$$
$$\frac{\partial \zeta_y}{\partial y} \ \ \theta_y + \frac{\partial w_o}{\partial y} \ \ 3\zeta_y \ \ \theta_x + \frac{\partial w_o}{\partial x} \ \ 3\zeta_x$$
$$\left. \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} \ \ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \ \ \frac{\partial \zeta_x}{\partial y} + \frac{\partial \zeta_y}{\partial x} \right]^T \tag{2c}$$

$$\left[Z'\right] = \begin{bmatrix} 1 & 0 & z & 0 & z^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & z & 0 & z^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & z^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & z^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & z & z^3 \end{bmatrix} \tag{2d}$$

Here $\varepsilon_{xx}, \varepsilon_{yy}$ and γ_{xy} are the in plane strain components. γ_{yz} and γ_{xz} are the transverse shear strain components.

In the present research, quasi-static loading and plane stress formulations are assumed. The constitutive equations for the kth orthotropic layer expressed in the material coordinate system are given as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix}^k = \begin{pmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & R_{44} & 0 & 0 \\ 0 & 0 & 0 & R_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{pmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}^k \tag{3}$$

For the piezoelectric layers, the linear piezoelectric constitutive equations that couple the elastic field and electric field are given as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix}^k = \begin{pmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & R_{44} & 0 & 0 \\ 0 & 0 & 0 & R_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{pmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}^k - \begin{pmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^k \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}^k \tag{4a}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}^k = \begin{pmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{pmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix}^k + \begin{pmatrix} p_{11} & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{33} \end{pmatrix}^k \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}^k \tag{4b}$$

The electric fields are given as [11]:

$$\begin{Bmatrix} E_{xx} \\ E_{yy} \\ E_{zz} \end{Bmatrix}^k = -\begin{Bmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial \phi}{\partial z} \end{Bmatrix}^k \tag{5}$$

where ϕ^k is the electric potential.

The electric potential is assumed to have linear variations across the thickness of the actuator/sensor layers as:

$$\phi^k(x, y, z) = \frac{z - h_{k+1}}{h_k - h_{k+1}} \phi_o^k(x, y) \tag{6}$$

where ϕ_o^k can be treated as the generalized electric potential similar to the generalized displacement at any point on the surface of the actuator and the sensor layers.

3. FINITE ELEMENT FORMULATION

The overall plate is meshed into a finite number of elements by an eight-noded two-dimensional isoparametric element. Referring to Eq.(1), the generalized displacement vector for node i (i=1,2,….,8) of the element can be written as:

$$\{u_i\} = \{u_{oi} \ v_{oi} \ w_{oi} \ \theta_{xi} \ \theta_{yi} \ \zeta_{xi} \ \zeta_{yi}\}^T \tag{7}$$

From the above equation, the nodal generalized displacement vector of a typical eight-noded element e can be expressed as:

$$\{u^e\} = \{(u_1)^T \ (u_2)^T \ .... \ (u_7)^T \ (u_8)^T\}^T \tag{8}$$

Thus, the generalized displacement vector at any point within the element can be obtained by interpolating the nodal generalized displacement as:

$$\{u\} = [N]\{u^e\} \tag{9a}$$

where the shape function matrix is given as:

$$[N] = [[N_1] \ [N_2] \ .... \ [N_7] \ [N_8]] \tag{9b}$$

and $[N_i] = n_i[I]$. Here $[I]$ is an identity matrix of size 7x7 and $n_i(i = 1,2,3......,8)$ is the shape function of natural coordinates (ζ, η) associated with the i th node [11].

Referring to Eq.(2a), by substituting Eq.(9a) the generalized strain vector $\{\bar{\varepsilon}\}$ at any point within the element can be express as:

$$\{\bar{\varepsilon}\} = [B]\{u^e\} \tag{10a}$$

where, the nodal strain-displacement matrix is given as:

$$[B] = [[B_1] \ [B_2] \ .... \ [B_7] \ [B_8]] \tag{10b}$$

The elements of each submatrix $[B_i]$ of $[B]$ are given explicitly in Mahmood [12].

Similarly, electric potential can also be generalized at any point within the element as:

$$\{\phi_o^k\} = \lfloor N_p \rfloor \{\phi_o^{ke}\} \tag{11a}$$

where,

$$\{\phi_o^{ke}\} = \{\phi_{o1}^{ke} \ \phi_{o2}^{ke} \ .... \ \phi_{o7}^{ke} \ \phi_{o8}^{ke}\}^T \tag{11b}$$

$$\lfloor N_p \rfloor = \lfloor N_1 \ N_2 \ .... \ N_7 \ N_8 \rfloor \tag{11c}$$

and $\{\phi_{oi}^{ke}\}(i = 1,2........8)$ is the electric potential at the ith node.

Using Eq.(11a), electric filed in Eq.(5) can be written as follows:

$$\{E^k\} = \lfloor Z_p^k \rfloor \lfloor B_p \rfloor \{\phi_o^{ek}\} \tag{12}$$

where $\lfloor Z_p^k \rfloor$ and $\lfloor B_p \rfloor$ are given explicitly in Mahmood [12].

Variational principle in conjunction with Eq.(7) to (12) yields the following two sets of matrix equations:

$$\lfloor K_{uu}^e \rfloor \{u^e\} + \lfloor K_{u\phi}^e \rfloor \{\phi_o^{ek}\} = \{F_S^e\} \tag{13a}$$

$$\left\lfloor K_{\phi u}^{e} \right\rfloor \left\{u^{e}\right\} + \left\lfloor K_{\phi\phi}^{e} \right\rfloor \left\{\phi_{o}^{ek}\right\} = \left\{Q_{S}^{e}\right\} \tag{13b}$$

where,

$$\left[K_{uu}^{e}\right] = \int_{-1}^{+1}\int_{-1}^{+1} [B]^{T} \left(\sum_{k=1}^{k=n} \int_{h_{k+1}}^{k} \left[Z'\right]^{T} \left[\overline{Q}\right] \left[Z'\right] dz \right) [B] \det J d\varsigma d\eta \tag{13c}$$

$$\left[K_{u\phi}^{e}\right] = \int_{-1}^{+1}\int_{-1}^{+1} [B]^{T} \left(\sum_{k=1}^{k=n} \int_{h_{k+1}}^{k} \left[Z'\right]^{T} \left[\overline{e}\right] \left[Z_{p}^{k}\right] dz \right) \left[B_{p}^{k}\right] \det J d\varsigma d\eta \tag{13d}$$

$$\left[K_{\phi u}^{e}\right] = \int_{-1}^{+1}\int_{-1}^{+1} \left[B_{p}^{k}\right]^{T} \left(\sum_{k=1}^{k=n} \int_{h_{k+1}}^{k} \left[Z_{p}^{k}\right]^{T} \left[\overline{e}\right] \left[Z'\right] dz \right) [B] \det J d\varsigma d\eta \tag{13e}$$

$$\left[K_{\phi\phi}^{e}\right] = \int_{-1}^{+1}\int_{-1}^{+1} \left[B_{p}^{k}\right]^{T} \left(\sum_{k=1}^{k=n} \int_{h_{k+1}}^{k} \left[Z_{p}^{k}\right]^{T} \left[\overline{p}\right] \left[Z_{p}^{k}\right] dz \right) \left[B_{p}^{k}\right] \det J d\varsigma d\eta \tag{13f}$$

$$\left[F_{S}^{e}\right] = \iint_{S1} [N]^{T} [Z]_{z=h1}^{T} \left\{\overline{T}\right\} dS \tag{13g}$$

$$\left[Q_{S}^{e}\right] = \iint_{S2} \left[N_{p}\right]^{T} \left\{\overline{q'}\right\} dS \tag{13h}$$

In the above equations $\left\lfloor K_{uu}^{e} \right\rfloor$ is the elastic stiffness matrix, $\left[K_{u\phi}^{e}\right] = \left[K_{\phi u}^{e}\right]^{T}$ is the coupling stiffness matrix between mechanical and electrical effects, $\left\lfloor K_{\phi\phi}^{e} \right\rfloor$ is the dielectric stiffness matrix, $\left\lfloor F_{S}^{e} \right\rfloor$ is the mechanical force as a result of the surface force and $\left\lfloor Q_{S}^{e} \right\rfloor$ is the electric force as a result of the applied surface charge on the actuators.

4. VALIDATION OF THE MATHEMATICAL MODEL

The results obtained using the mathematical model and the computer code developed in this research work were validated comparing with those available for transverse deflection of laminated plates [13] and piezoelectric beams [14]. In both the cases maximum error were found to be limited to less than 0.4%. It is to be noted here results obtained in [13] and [14] are based on First Order Shear Deformation Theory (FSDT), whereas the present analysis uses HSDT.

Twist Analysis

This research is mainly targeted to analyze twist activation in a piezoelectric laminated plate under different actuation voltage. To do so, segmented actuators are placed on a cantilever plate in the configuration as shown in Figure-2. Several set of activation schemes as listed in Table.1 are performed on the cantilever plate to further analyze this capability. Figure-3 to Figure-8 present shapes of the cantilever plate produced under these activation schemes. Referring to Figure-3 and Figure-4, activating either TP or BT scheme produces an asymmetric twist in the cantilever plate.

It is found that activating the TP-BT scheme produces a symmetric twist in the cantilever plate as shown in Figure-5. Unlike in the case of TP and BT schemes where the plate centerline deflects, the centerline of the cantilever plate under TP-BT scheme remains flat as can be seen in Figure-6 and Figure-7. Figure-8 presents further investigation on symmetric twist, where the actuator voltage is varied from 0 V to 300 V. The degree of twist is found to increase as the actuator voltage increases.

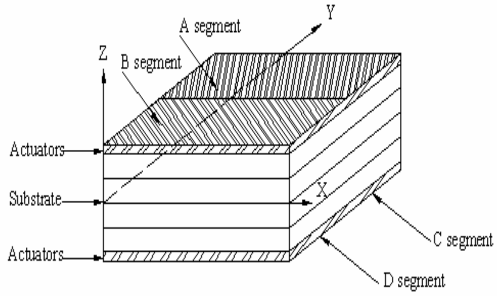

Figure 2 Segmented actuator configuration to produce twist.

Table 1 Set of activation schemes to produce twisting.

Scheme	Actuator segment			
	A	B	C	D
TP	300 V	0 V	-300 V	0 V
BT	0 V	300 V	0 V	-300 V
TP-BT	300 V	-300 V	-300 V	300 V

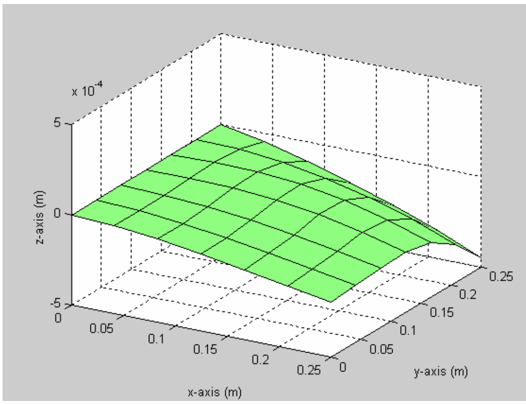

Figure 3 Shape of a cantilever plate under TP scheme of activation under constant actuator voltage of 300 V.

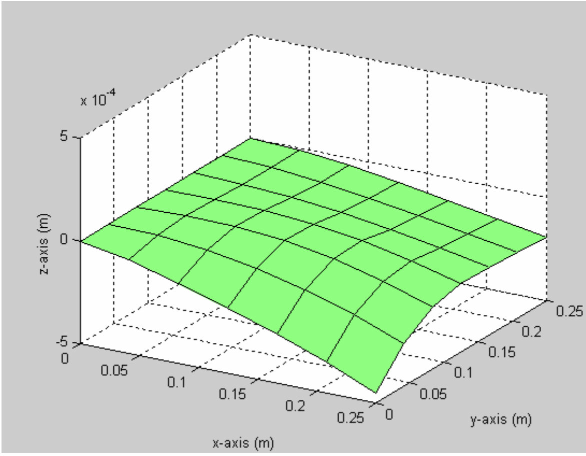

Figure 4 Shape of a cantilever plate under BT scheme of activation under constant actuator voltage of 300 V.

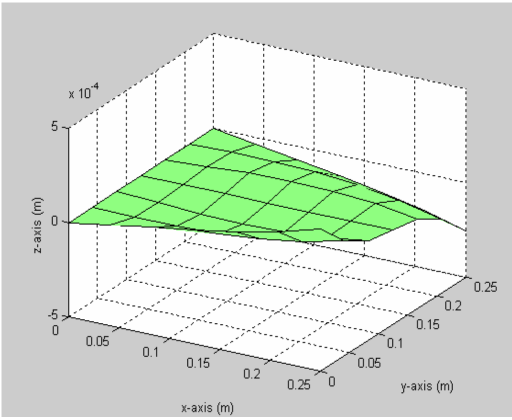

Figure 5 Shape of a cantilever plate under TP-BT scheme of activation under constant actuator voltage of 300 V.

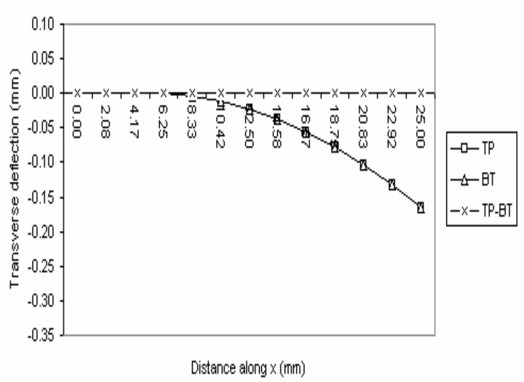

Figure 6 Effect of segmented actuator on centerline (y = b/2) deflection of a cantilever plate under a constant actuator voltage of 300 V.

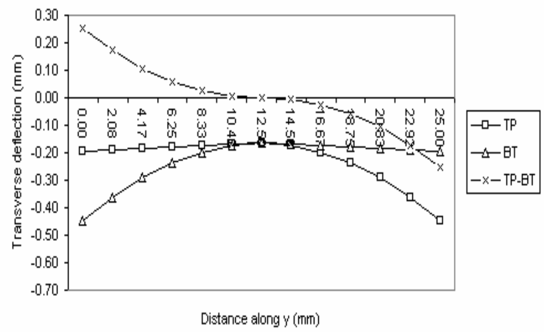

Figure 7 Effect of segmented actuator on free end (x = a) deflection of a cantilever plate under a constant actuator voltage of 300 V.

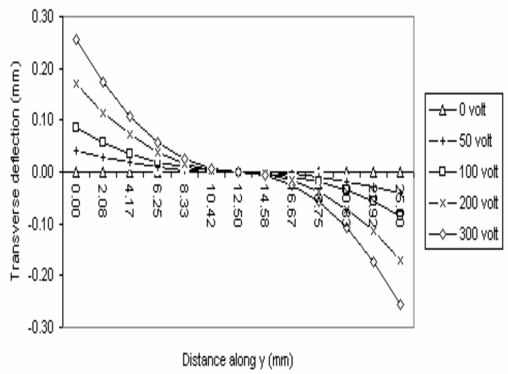

Figure 8 Effect of different actuator voltage on free end (x = a) deflection of a cantilever plate under TP-BT scheme of activation.

5. CONCLUSIONS

It is found that, symmetric as well as antisymmetric twist can be introduced in a cantilever plate by activating segmented actuators under different scheme of activation. The degree of the twist can be increased by just increasing the applied voltage to the segmented actuators. This capability presents a good opportunity to be used in individual blade control (IBC) of helicopters. A scaled down scheme of such a plate may be used in micromirror orientation for operating optical switches.

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