

**Advances**  
**in**  
**Aircraft Structures**

**Editor**

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# Chapter 26

## Accurate Geometric Stiffness Matrix Formulation of Beam Finite Element

Erwin Sulaeman

### Abstract

*Alternate estimation of beam buckling load is presented in this Chapter. The standard procedure using a variational principle approach is usually utilized a cubic polynomial approach for the beam displacement shape such that the geometric stiffness matrix is a function of the geometric length of the element only. In the present work, the geometric stiffness matrix is developed as function geometric length and pre-assumed buckling load such that the accuracy of the buckling load can be improved by performing iteration. If the pre-assumed buckling load is set to zero, the present work will yield to the standard procedure. Therefore the present work offers a procedure to increase the accuracy of buckling estimation by performing iteration using the buckling load estimated by previous iteration as a good estimate for the pre-assumed buckling load.*

*Keywords: Buckling, geometric stiffness matrix, non-prismatic beam, finite element method.*

### 1. Standard Procedure for Geometric Stiffness Matrix

#### Formulation

Gallagher and Lee (1970) derived an approximation to the bending stiffness and geometric stiffness matrix  $[K_g]$  shown in Eq. (1) based on a cubic function displacement. It has been known that based on this assumption, the buckling load