Nasir Ganikhodjaev
Farrukh Mukhamedov
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VOLUME 1

$$x' = 2xy$$
$$y' = 2xz$$

INVESTIGATIONS ON PURE MATHEMATICS, FINANCE MATHEMATICS AND OPTICS

Proceedings of the Department of Computational and Theoretical Sciences Kulliyyah of Science, IIUM

$$\varphi_1(x, y, z) = z$$

$$z' = x^2 + y^2 + z^2 + 2yz$$

الجامعة الإسلامية العالمية ماليزيا
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA
يونيورسيتي إسلام انتارابغسا ملايسيا

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IIUM Press

Published by.
IIUM Press
International Islamic University Malaysia

First Edition, 2011

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Nasir Ganikhodjaev, Farrukh Mukhamedov & Pah Chin Hee Investigations on Pure Mathematics, Finance Mathematics and Optics

ISBN: 978-967-418-198-7

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by
IIUM PRINTING SDN.BHD.
No 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

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ON ξ^a-QUADRATIC STOCHASTIC OPERATORS AND THEIR CLASSIFICATIONS

Wan Nur Fairuz Alwani Wan Rozali
Assoc. Prof. Dr. Farrukh Mukhamedov

Abstract. *We introduce a new class of quadratic stochastic operators (QSO) which is called ξ^a-QSO. In two dimensional simplex, we classify such operators into two non-isomorphic classes.*

1 Introduction

It is known that there are many systems which are described by nonlinear operators. One of the simplest nonlinear case is quadratic one. Quadratic dynamical systems have been proved to be a rich source of analysis for the investigation of dynamical properties and modeling in different domains. One of such operators is quadratic stochastic operator which naturally arises in modeling of a population dynamics [1]. During many years this theory has developed, and appeared in lots of papers (see e.g. [3–5]). In recent years it has again become of interest in connection with numerous applications to many branches of mathematics, biology and physics.

A quadratic stochastic operator (QSO) has meaning of a population evolution operator, which arises as follows.By considering a population consisting of m species we let $x^0=(x_1^0,...,x_m^0)$ be the probability distribution of species in the initial generations, and $P_{ij,k}$ the probability that individuals in the ith and jth species interbreed to produce an individual k. Then the probability distribution $x'=(x_1',...,x_m')$ (the state) of the species in the first generation can be found by the total probability such that

$$x_k'=\sum_{i,j=1}^{m}P_{ij,k}x_i^0x_j^0,\ k=1,...,m.$$

This means that the association $x^0\to x'$ defines a map V called the evolution operator. The population evolves by starting from an arbitrary state x^0, then passing to the state $x'=V(x)$ (in the next "generation"), then to the state $x''=V(V(x))$, and so on. Thus states of the population described by the following dynamical system

$$x^0,\ x'=V(x),\ x''=V^2(x),\ x'''=V^3(x),...$$

Note that V (defined by (1)) is a non linear (quadratic) operator, and it is higher dimensional if $m\geq 2$. Higher dimensional dynamical systems are important but there are relatively few dynamical phenomena that are currently understood.

In this paper, we introduce a new class of quadratic stochastic operators called ξ^a-QSO which are defined through coefficient of the operator from measure-theoretic point of view .We consider ξ^a-QSO on 2-D simplex and classify them with respect to conjugacy and their numeration of the coordinates. It shall show such operators will be classified into two classes K_1 and K_2.