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VOLUME 1

$$x' = 2xy$$
$$y' = 2xz$$

INVESTIGATIONS ON PURE MATHEMATICS, FINANCE MATHEMATICS AND OPTICS

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$$\varphi_1(x, y, z) = z$$

$$\pi_1 = \begin{pmatrix} x & y & z \\ y & z & x \end{pmatrix}$$

$$z' = x^2 + y^2 + z^2 + 2yz$$

$$\pi_1 V_1 \pi_1 = V_{17}$$

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PHASE TRANSITION FOR ISING MODEL WITH TWO COMPETING INTERACTION ON CAYLEY TREE OF ORDER 4

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Abstract. *In this paper, we will study the phase transition phenomena on Cayley tree for Ising model of order four. In addition, the graph on order four is compared with graph of order two, and order three numerically. Cayley tree of order two and three has been done by Ganikhodjaev, Pah, and Wahiddin (2004) analytically. The recurrent equation in Cayley tree of order three is calculated and been produced by us.*

1 Recurrent equation for partition functions

Recurrent equation for a partition functions is one of the approaches to describe the limiting Gibbs measure in an equation on a Cayley tree graph. Another approaches to derive the equation based on Markov random fields on Cayley tree. However, in this project I consider the recurrent equation for a partition functions to derive the equation describing the Gibbs measure on Cayley tree.

Let Λ be a finite subset of *V*. Assume that $\Omega(\Lambda)$ is the set of all configurations onΛ, that is$\sigma(x), x \in \Lambda$.

Assigned that the probabilities to configurations, $\sigma(x)$proportional to

$$e^{-\frac{1}{kT}H(\sigma)}$$

where *k* is the universal constant and *T* is the temperature. Then, the probability measure on $\Omega(\Lambda)$ is given by

$$P(\sigma) = \frac{e^{-\frac{1}{kT}H(\sigma)}}{Z}$$

where the normalizing constant, *Z* which also called as partition function, is defined by

$$Z = \sum_{\sigma(\Lambda)\in\Omega(\Lambda)} e^{-\frac{1}{kT}H(\sigma)}$$

Now, let $\bar{\sigma}(V/\Lambda)$be a fixed boundary configuration. Then, the total energy of configuration $\sigma(\Lambda)\in\Omega(\Lambda)$ under condition $\bar{\sigma}(V/\Lambda)$ is defined just as in the Ising model. That is,

$$H(\sigma(\Lambda)\mid\bar{\sigma}(V/\Lambda)) = \sum_{\substack{<x,y>\\ x,y\in\Lambda}}\sigma(x)\sigma(y) - J_1\sum_{\substack{>x,y<\\ x,y\in\Lambda}}\sigma(x)\sigma(y) - h\sum_{x\in\Lambda}\sigma(x) - J\sum_{\substack{<x,y>\\ x\in\Lambda y\notin\Lambda}}\sigma(x)\bar{\sigma}(y) - J_1\sum_{\substack{>x,y<\\ x\in\Lambda y\notin\Lambda}}\sigma(x)\bar{\sigma}(y)$$

Then, the partition function $Z_\Lambda(\bar{\sigma}(V/\Lambda))$ in volume Λ under boundary condition, $\bar{\sigma}(V/\Lambda)$ is defined as

$$Z_\Lambda(\bar{\sigma}(V/\Lambda)) = \sum_{\sigma(\Lambda)\in\Omega(\Lambda)} \exp(-\beta H(\sigma(\Lambda)|\bar{\sigma}(V/\Lambda))$$

where $\beta = \frac{1}{T}$ is the inverse temperature.