Nasir Ganikhodjaev
Farrukh Mukhamedov
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VOLUME 1

$$x' = 2xy$$
$$y' = 2xz$$

INVESTIGATIONS ON PURE MATHEMATICS, FINANCE MATHEMATICS AND OPTICS

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$$\varphi_1(x, y, z) = z$$

$$z' = x^2 + y^2 + z^2 + 2yz$$

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Contents

Part II Finance Mathematics Concentration

Part III Optics Concentration

STUDY OF ROLES OF EXTERNAL MAGNETIC FIELD ON ISING AND POTTS MODEL

Mohd Hirzie Mohd Rodzhan
Prof. Dr. Nasir Ganikhodjaev

Abstract. *The phase diagram of the model describes morphology of phases, stability of phases, different transitions from one phase to another and corresponding transition lines. We study the phase diagram for the Ising and Potts model on a Cayley tree with nearest-neighbour interaction J in the presence of an external magnetic field h. To perform this study, an iterative scheme similar to that appearing in real space renormalization group frameworks is established. The phase diagram is fully determined in the $T/J - h/J$ space for all values and signs of J and h. These phase diagrams explicitly demonstrates relations between parameters J, h and T, where as in previous works of Peruggi et al such relations were implicit.*

1 Introduction

1.1 Ising model, Potts model and phase diagram

Statistical physics seeks to explain the macroscopic behaviour of matter on the basis of its microscopic structure. This includes the analysis of simplified mathematical models (Baxter 1982, Georgii 1988). The Potts model was introduced as a generalization of the Ising model to more than two components (:.pins) (Potts 1952).Ising model considered only up and down spins where as Potts model incorporates more possibilities of spins and their interactions (Baxter 1982). The Potts model describes an easily defined class of statistical mechanics models. At the same time, its rich structure is surprisingly capable of illustrating almost every conceivable nuance of the subject. The Potts model encompasses a number of problems in statistical physics (Wu 1982).

A phase diagram of a model describes a morphology of phases, stability of phases, transitions from one phase to another and corresponding transitions line. A Potts model just as an Ising model on a Cayley tree with competing interactions has recently been studied extensively because of the appearance of nontrivial magnetic orderings (Ganikhodjaev 2008, Mariz 1985, Vannimenus 1981). The Cayley tree is not a realistic lattice; however, its amazing topology makes the exact calculation of various quantities possible. In this project, we present 2 phase diagrams, Ising and Potts model on a Cayley tree with nearest-neighbour interactions J in the presence of an external magnetic field h and furthermore, we discuss the role of external magnetic field on both models, for the phase diagrams are fully determined in the $h/J - T/J$ space for all values and signs of h and, J.

1.2 Ising and Potts model with external magnetic field

A Cayley tree Γ^k of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles with exactly $k+1$ edges issuing from each vertex. Let denote the Cayley tree as $\Gamma^k = (V, \Lambda)$, where V is the set of vertices of Γ^k, Λ is the set of edges of Γ^k. Two vertices x and y, $x, y \in V$ are called nearest neighbours if there exists an edge $l \in \Lambda$ connecting them, which is denoted by $l = \langle x, y \rangle$.The distance $d(x, y), x, y \in V$, on the Bethe lattice, is the number of edges in the shortest path from x to y. For a fixed $x^0 \in V$ we set $V_n = \{x \in V | d(x, x^0) \leq n\}$ and L_n denotes the set of edges in V_n. We will consider a semi-infinite Cayley tree Γ_+^2 of order 2 i.e.