Nasir Ganikhodjaev
Farrukh Mukhamedov
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VOLUME 1

$$x' = 2xy$$
$$y' = 2xz$$

INVESTIGATIONS ON PURE MATHEMATICS, FINANCE MATHEMATICS AND OPTICS

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$$\varphi_1(x, y, z) = z$$

$$z' = x^2 + y^2 + z^2 + 2yz$$

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SOME APPLICATION OF ERGODIC THEORY IN NUMBER THEORY

Mohd Hirzie Mohd Rodzhan
Prof. Dr. Nasir Ganikhodjaev

Abstract. *For a given sequence,* $\{a^n\}$ *where* $\forall a \in \mathbb{N} \setminus \{1,10,10^2,10^3,\dots\}$, *distribution of first digit DFD) could be calculated from this sequence. In this project, as* $n \to \infty$, *DFD for this sequence will be formulate and proven by applying definition of Uniform Distribution Modulo One. In addition, this distribution confirms Benford's distribution of first digit or well known as Benford's Law.*

1 Introduction of Benford's Law

First Digit Law, well-known as Benford's Law was introduced by Frank Benford, an American physicist in 1936. In a certain paper, this law has been called as Newcomb-Benford's Law, since the law had been discovered by Simon Newcomb (1881). Simon Newcomb, an astronomer published a 2-page article in the American Journal of Mathematics describing his observation of books of logarithms (using for the calculations at that time) in the library', at the earlier pages, numbers starting with I much more than other numbers and numbers starting with 9 is less.

He inferred that researchers, mathematicians, biologists, sociologists as well as physicists, were looking up numbers starting with 1 much more often than numbers starting with 2, then numbers starting with2 much more often than numbers starting with 3 and so on' This led him to conclude that the probability that a number has first significant digit, $d(1 \leq d \leq 9)$ is $\text{Prob}(d) = \log_{10}(1 + 1/d)$. In addition, Newcomb's conjecture stated that the first digit is 1 about 30.1% of the time and 9 is only about 4.6%o of the time.

The proposed law was discovered again by Frank Benford (1938). He provided empirical evidence by analyzing the frequency of significant digits from twenty different tables including such diverse data such as surface areas of 355 rivers, specific heat of thousands of chemical compounds and square-root tables.

In more interesting cases, Benford's law has been applied in real life applications. Some applications of Benford's Law had been discussed by Schatte (1988). He had mentioned on how Benford's law contributes in the development of computer science. The computer design that minimizes expected storage space is base 8 and other researchers have started exploring the use of logarithm computers to speed up calculations. In other paper published by Hill (1995) suggested Benford's law could be as test of reasonableness of forecasts of a proposed model. Furthermore, Nigrini (1999) had mentioned in his paper that Benford's law could be used to test the accuracy of measuring equipment.

1.1 Distribution of First Digit (DFD), r, $(r = 1, 2, \dots, 9)$