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VOLUME 1

$x' = 2xy$

$y' = 2xz$

INVESTIGATIONS ON PURE MATHEMATICS, FINANCE MATHEMATICS AND OPTICS

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$\varphi_1(x, y, z) = z$

$\pi_1 = \begin{pmatrix} x & y & z \\ y & z & x \end{pmatrix}$

$z' = x^2 + y^2 + z^2 + 2yz$

$\pi_1 V_1 \pi_1 = V_{17}$

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THE FIRST RETURN TIME AND DIMENSION

Ashraf Mohamed Nawi
Prof. Dr. Nasir Ganikhodjaev

Abstract. *This project paper subjects to study the first return time and the dimension of the dyadic and baker's transformations. The main problem here is the study of limit in order to find the dimension. Then show the important of suitable condition in this case. This manual provides a full explanation of the dimension and portrays the graphs for supplementary understanding.*

1 Introduction

In dynamical systems, for measure preserving transformation on metric space there been study about the time needed by typical orbit to return back close to its initial point or the appearing of the neighbourhood of initial point. After that, the notion of dimension of a real vector space is generalized by Hausdorff dimension. In particular cases, the Hausdorff dimension of a single point is zero, the Hausdorff dimension of line is one, the Hausdorff dimension of plane is two, etc. There are still many irregular sets that have non-integer Hausdorff dimension. This concept was initiated by the mathematician Felix Hausdorff in 1918.

Naturally, the dimension any set is the number of independent parameters needed to portray a point in the set. One of mathematical concept may be closely modelled this raw idea is that of topological dimension of a set. As illustration, any point in the plane is be describe by two independent parameters, so one can assume, the plane is 2-dimensional. As one would be expect that topological dimension is always natural number.

On the other hand, the topological dimension behaves in quite surprising behaviour on certain highly irregular sets such as fractals. As the example, the Cantor set has topological zero, but in some reason it behaves like a higher dimensional space. Hausdorff dimension gives another approach to define dimension, which considering the metric.

In L. Barreira et al (2001) and (2002), there have been mention that for many shapes of Hausdorff dimension are considered in mathematics, physics or other fields, their invariant measure have integer dimensions. Nevertheless, sets with non-integer Hausdorff dimension are important and prevalent. Benoit Mandelbrot, a populariser of fractals, he advocates that most shapes found in nature are fractals with non-integer dimension.

The dyadic transformation and baker's transformation can be considered as example of fractals. Then, the investigation their dimension by using Hausdorff dimension gives the new understanding of fractal cases in Hausdorff dimension. The computation of dimension consider their first return times in dynamical systems, i.e. the first time required by a trajectory to revisit journey to the neighbourhood of the starting position given.

By considering these transformations under the dynamical system are ergodic and those motions take place in a compact and finite dimensional space. This consideration case will be general enough to cover many varies of applications. The return times and invariant measures are linked by a variety of results that stand on pillars of the classical theorems of Poincaré and Kac, see Petersen et al.

There will be sure that the first return time of any initial point to the set is surely finite with respect to any invariant measure. Before goes to details, the measured dimensions are defined independently of the dynamics. While for any initial point has differ in the first return times. Accurately, the studying of relations holding in the two difference dimension is