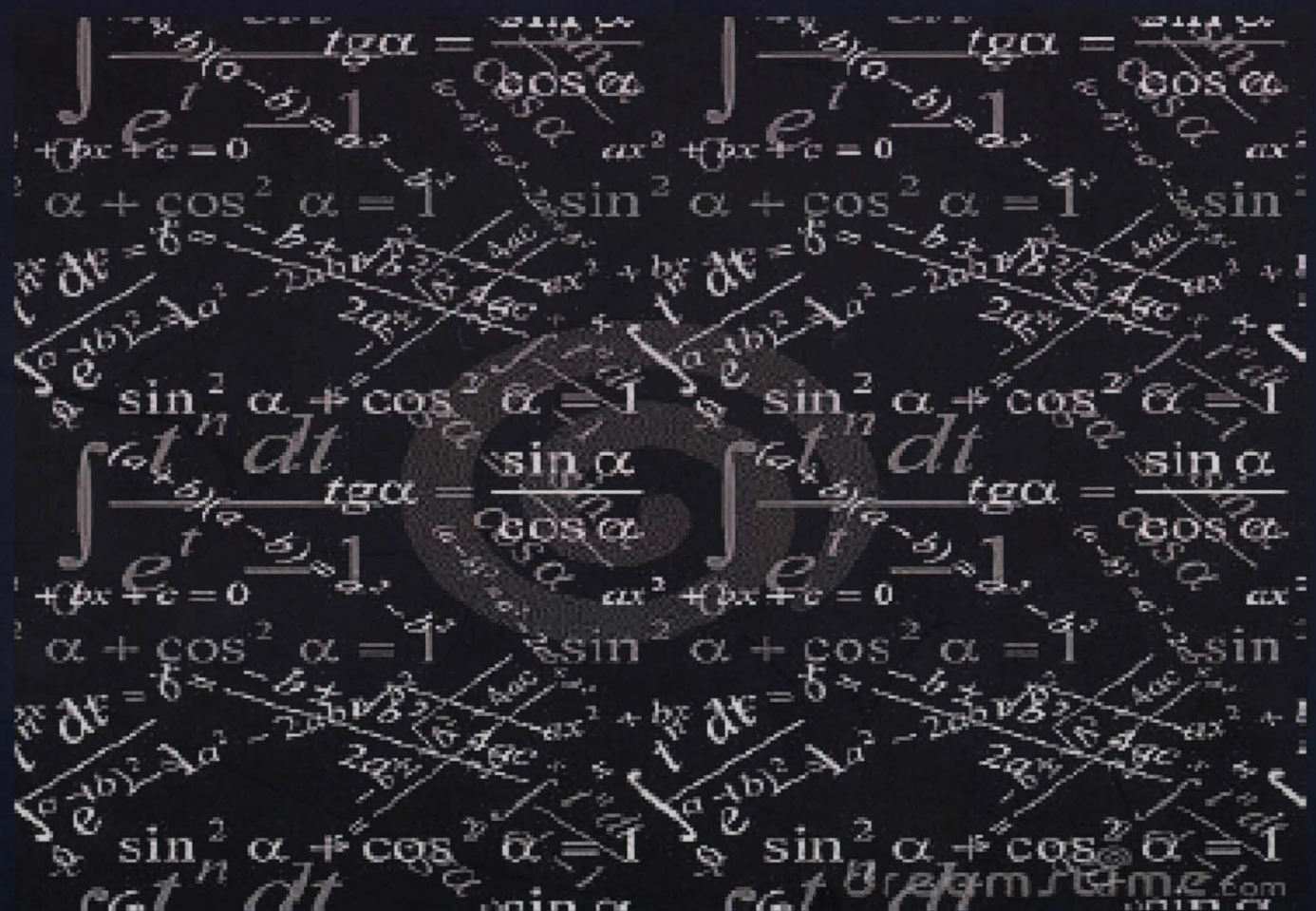




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

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## SINGLE POLYGON COUNTING FOR TWO FIXED NODES ON A CAYLEY TREE OF ORDER 2

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### Abstract

We have found an exact formula for a single polygon counting problem on Cayley tree of order 2, which the polygon contain 2 fixed vertices with a constraint. The solution is elementary based on Catalan numbers. Some identities of the solution have been established which associate with Catalan numbers.

**Keywords:** *Cayley tree, Catalan numbers, Single polygon counting*  
Mathematics Subject Classification : 82B20, 11B83.

### Introduction

In the study of computer science, tree model appears very often and attracted a lot of attention of researchers [1,2]. In a Cayley tree of order 2 [3], we denote  $V$  the set of all vertices of the Cayley tree, we fixed 2 vertices, namely  $x^0, x^* \in V$ . There is only a single shortest path connecting the two vertices as it is in a tree. Let  $l$  denote the number of the vertices along the shortest path, where  $l \in V$ . In this paper, we would like to find the number of different connected component containing these two fixed vertices as well as the vertices along the shortest path for a given  $n$  number of vertices. We call this connected component a "polygon" [4,5]. In network theory, the scenario stated above is identical to setting up 2 gateway with distance  $l$  nodes (shortest path), each gateway is allowed to expand in a rooted tree. These nodes are connected by using fiber optics or simple phone line. In this problem, each router is maximally connected to 3 network components (not end user). The problem statement is how many different ways we can arrange the network with given  $n$  routers (nodes). ation. For example, in figure below: Recall that a Cayley tree of order 2, denoted as  $\Gamma^2$ , is a graph with no cycles, each vertex emanates 3 edges. We denote the set of all vertices as  $V$  and denote the set of all edges as  $L$ , i.e.,  $\Gamma^2 = (V, L)$ . Let  $C_r$  be  $r$ -th Catalan numbers [6,7,8] which