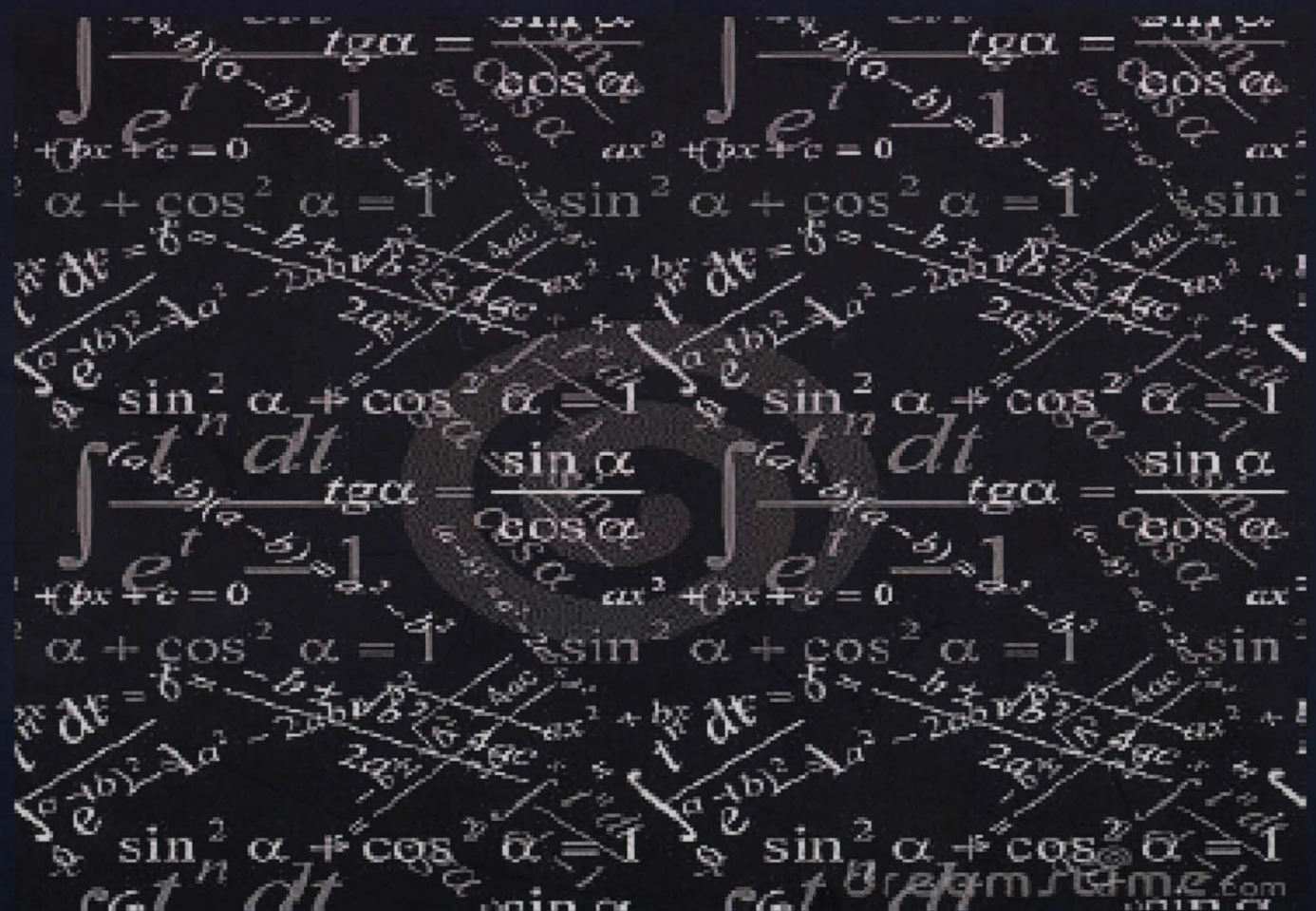




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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# STRONGLY LOCALIZED MODELS IN TWO-COMPONENT DISCRETE MEDIA WITH CUBIC-QUINTIC NONLINEARITY

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## Abstract

The nonlinear localized modes in the systems described by two coupled discrete nonlinear Schrödinger equations (DNLS) with cubic-quintic nonlinearity and associated inter-component couplings is considered. In the limit of weak linear inter site coupling (anti-continuum limit) conditions for the onset of localized modes of different type are revealed and the stability of these modes are checked. Analytical predictions are supported by numerical simulations of the underlying coupled cubic-quintic DNLS. Relevance of obtained results to dense Bose-Einstein condensates in deep optical lattices, when three body processes are essential, is discussed. Bimodal light propagation in waveguide arrays fabricated from optical materials with non-Kerr nonlinearity is discussed as another possible physical realization for the considered model.

## Introduction

Nonlinear discrete equations appear naturally in the description of a large variety of physical systems. One of the most generic and important examples of discrete system is the DNLS [1]. It is well known that in this system the existence of stable self localized excitations (discrete solitons) are possible as a result of interplay between nonlinearity and discrete dispersion or diffraction. Properties of discrete solitons in the DNLS equation with the simplest, cubic, nonlinearity have been studied in detail, including three dimensional settings, and are now well understood [2]. These solitons were experimentally observed in arrays of nonlinear optical waveguides. They also correspond, in the DNLS approximation, to the intrinsic localized modes in more sophisticated dynamical lattices [3]. Nonlinear Schrödinger (NLS) equations with more complex nonlinearities were studied in detail in continuum models. As well as their cubic counterparts, such models are of interest by themselves, and may also have direct applications. In particular, glasses and organic optical media whose dielectric response features the cubic-quintic (CQ) nonlinearity, i.e., a self-defocusing quintic correction to the self-focusing cubic Kerr effect, are known. Properties of solitons in the NLS