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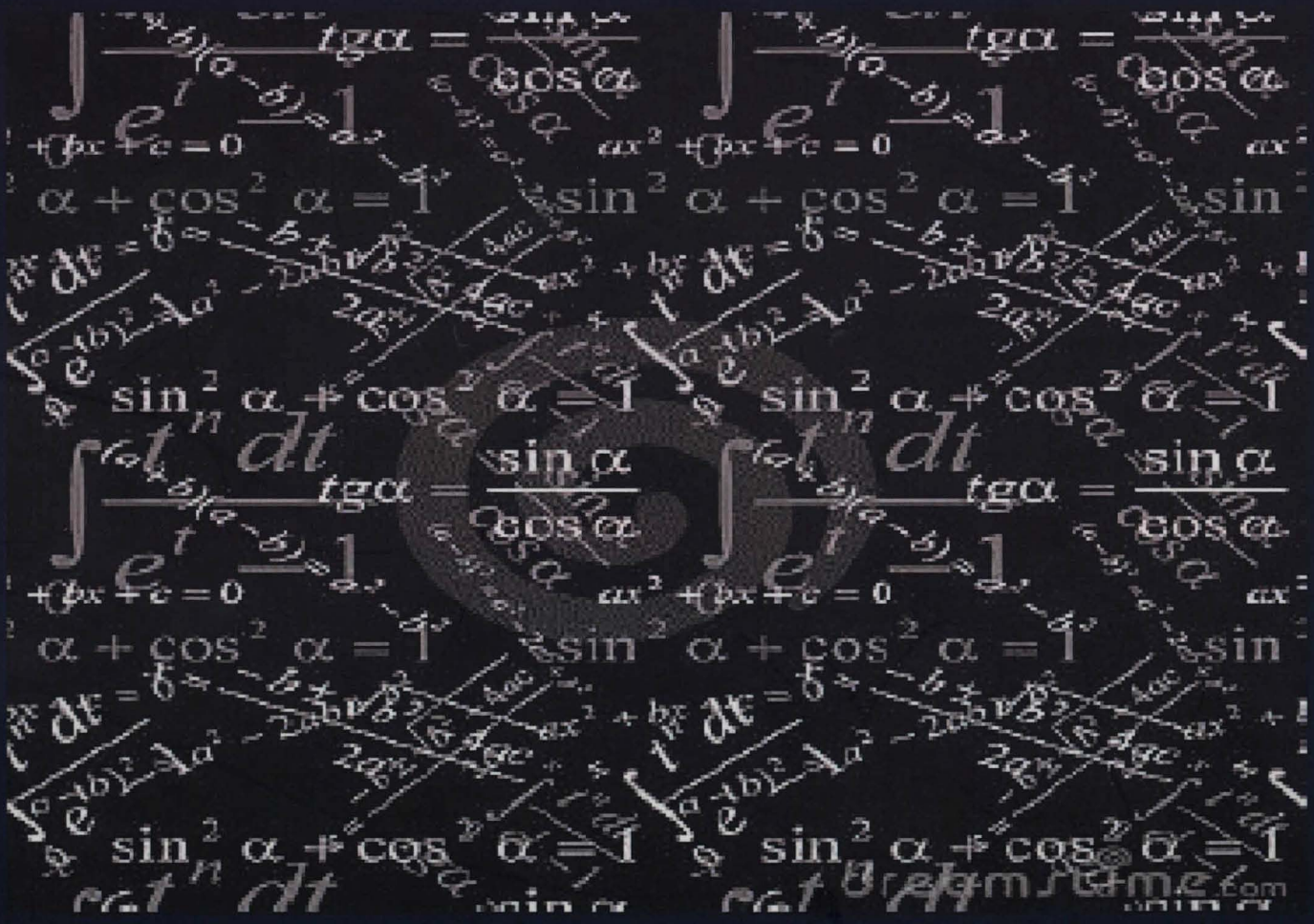

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FREE ENERGY OF THE ISING MODEL WITH COMPETING INTERACTIONS ON A CAYLEY TREE

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Abstract

In the present paper the Ising model with competing binary J and J_1 interactions with spin values ± 1, on a Cayley tree is considered. We study translation-invariant Gibbs measures and corresponding free energies ones.
Keywords: *Ising model; free energy; Gibbs measure.*

Introduction

Nowadays the investigations of statistical mechanics on non-amenable graphs is a modern growing topics ([10]). One of the non-amenable graph is a Cayley tree. The Cayley tree is not a realistic lattice, however, its amazing topology makes the exact calculation of various quantities possible [10]. It is believed that several among its interesting thermal properties could persist for regular lattices, for which the exact calculation is far intractable. Here we mentions that in 90's a lot of research papers were devoted to studying of the classical the Ising model, with two spin values ± 1, on such Cayley tree (see [15], [2],[3],[4],[5],[8]). In the present paper we extend some results of the paper [7], where it has been investigated the Ising model with competing interactions, with spin values ± 1, on a Cayley tree. In this paper using the methods of [11] we will calculate critical curve such that there is a phase transitions above it, and a single Gibbs state is found elsewhere (cp.[7]). We also find ground states of the model. Besides, we will find a form of the free energy of the model under consideration. This gives us to study some asymptotics one.

Preliminaries

Recall that the Cayley tree Γ^k of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, such that each vertex of which lies on $k+1$ edges. Let $\Gamma^k = (V, \Lambda)$, where V is the set of vertices of Γ^k, Λ is the set of edges of Γ^k.