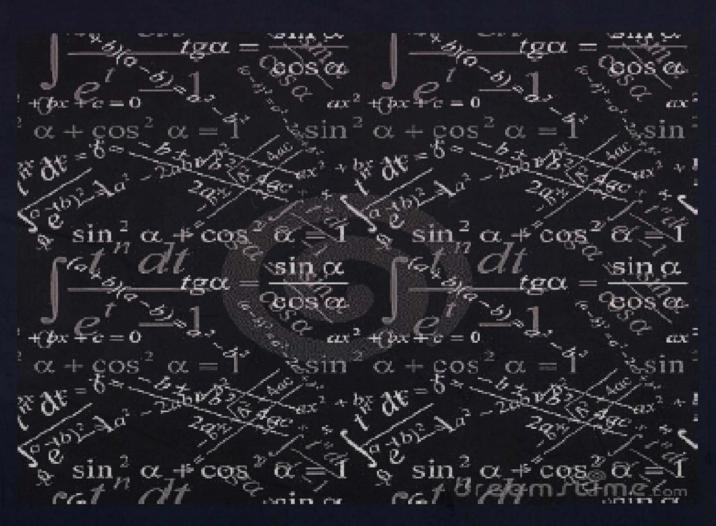
**VOLUME 2** 

# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of Computational and Theoretical Sciences, Faculty of Science, IIUM



Chief Editor: Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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Vol. 2



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## ISING MODEL WITH COMPETING INTERACTIONS ON CAYLEY TREE OF ORDER FOUR

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#### **Abstract**

Using techniques in dynamical system, we study the phenomenon of phase transition on Ising model with Competing Interactions on Cayley tree of order 3 and order 4. Nearest neighbor interactions and one level neighbor interactions were considered in this model. Recurrent equation for the two cases with respect to ratio of partition function with positive boundary and negative boundary are derived. Result of order 2 is also reproduced for comparison and it is in strong agreement. Result for order 3 disagrees with analytic result. A phase diagram for Cayley tree of order 2,3, and 4 is then constructed numerically. From the formula of order 2 and order 3, one can conjecture the formula for order k, the general case.

**Keywords:** component: Ising model, Cayley tree, Phase transition, Competing interactions

#### Introduction

The Ising model is a mathematical model of ferromagnetism in statistical mechanics. It was suggested by the physicist, Wilhelm Lenz who then gave the problem to his student, Ernst Ising [1]. Ising model, is a highly simplified model, has greatest application not only in physics but in a very wide range of fields such as sociology, economics, genetics, network and etc. It still make agreeable to mathematical behavior at least in one and two dimensions of Ising model. Phase transition is the transformation of a thermodynamic system from one phase or state of matter to another. It is commonly used to describe the transition between solid, liquid, and gaseous states of matter. For the case of ferromagnetism in the absence of magnetic field h = 0, the magnetization is either positive or negative which subject to is the initial condition, when ever it is below the Curie temperature  $T < T_c$ . When  $T > T_c$  in the absence of magnetic field h = 0, the magnetization is zero, which characterize absence of phase transition.

In one dimension, Ising claimed that the solution admits no phase transition. On the basis of this result, he incorrectly concluded that this kind of model does not exhibit phase behavior in any dimension. Physicists like Lars Onsager [2], Yang, Kramers and Wannier brought the development on Ising model and important results were obtained especially in the study of phase