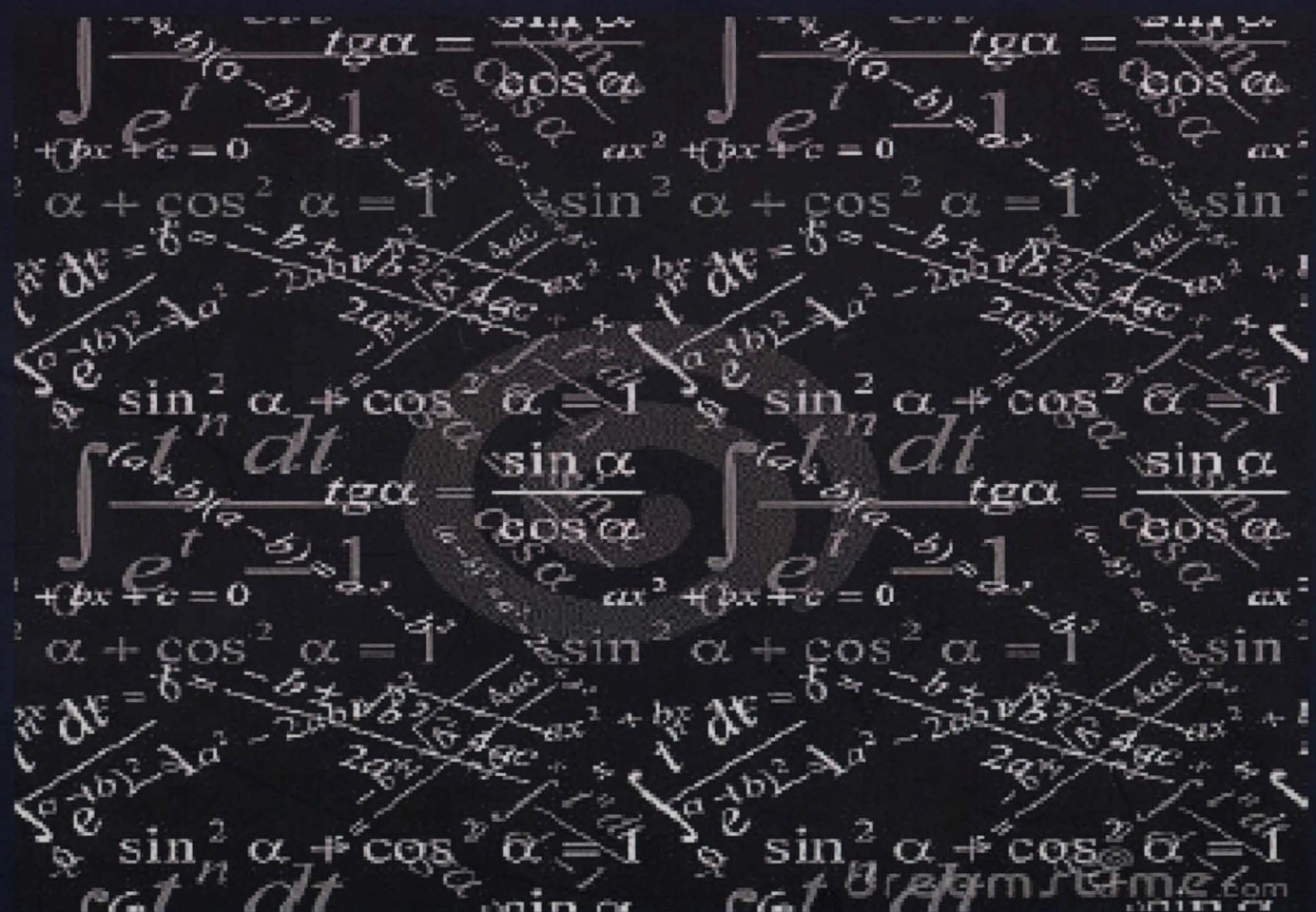




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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## DYNAMICAL SYSTEMS OF ISING MODEL ON A CAYLEY TREE

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### Abstract

In the present paper, we study stability of the dynamical system corresponding quantum Markov chain (QMC) associated with the Ising model on Cayley tree of order two. To study certain properties of QMC we reduce our investigation to the study of dynamics of a nonlinear dynamical system. For such a dynamical system it is proved existence of exactly three fixed points and absence of periodic points. Moreover, it is established finiteness and infiniteness of the trajectory of the system.

*Keywords:* Cayley tree, quantum Markov chain, Ising model, stability of the dynamical system.

### Introduction

It is known that Markov fields play an important role in classical probability, in physics, in biological and neurological models and in an increasing number of technological problems such as image recognition. Therefore, it is quite natural to forecast that the quantum analogue of these models will also play a relevant role. The quantum analogues of Markov processes were first constructed in [1], where the notion of quantum Markov chain on infinite tensor product algebras was introduced. Nowadays, quantum Markov chains have become a standard computational tool in solid state physics, and several natural applications have emerged in quantum statistical mechanics and quantum information theory. The reader is referred to [4, 5, 15, 22] and the references cited therein, for recent developments of the theory and the applications.

A first attempt to construct a quantum analogue of classical Markov fields has been done in [19], [3], [5], [8]. These papers extend to fields the notion of quantum Markov state introduced in [7] as a sub-class of the quantum Markov chains introduced in [1]. Typically a system is identified to a point in a graph: if this graph is not isomorphic to an interval in  $(1\text{-dimensional case})$ . The crucial role of the localization is at the root of the difficulties to construct nontrivial examples of Markov fields. Gaussian states (quasi-free, in the physics terminology) also have a simple structure, but they do not describe physically interesting interactions. Note that in mentioned papers quantum