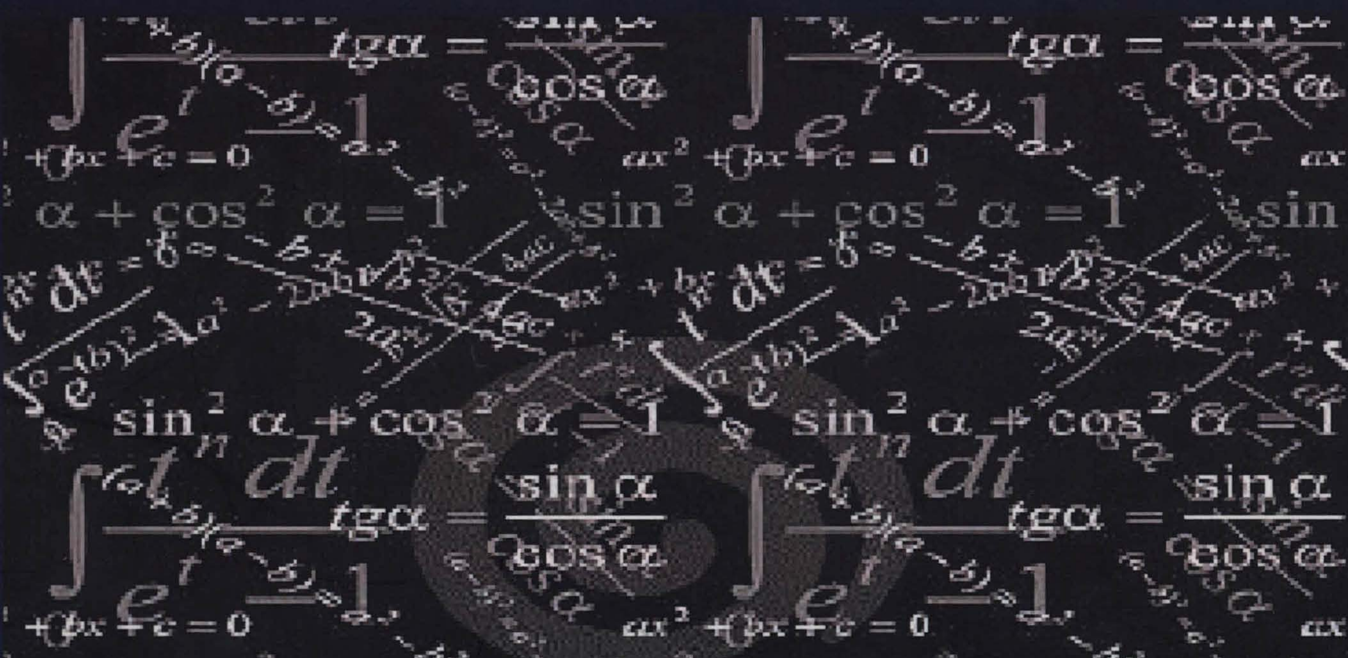
الجامعة الاسلامية العالمية ماليزيا
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA
يونيۏرسيتي اسلام انتارابڠسا مليسيا

VOLUME 2

RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of Computational and Theoretical Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov
Editors : Nasir Ganikhodjaev
: Mansoor Saburov

Proceedings of Department of Computational and Theoretical Sciences, Faculty of Science, IIUM

Recent Achievements in Dynamical Systems

Chief Editor: Farrukh Mukhamedov

Editors:
Nasir Ganikhodjaev
Mansoor Saburov

Vol. 2

IIUM Press

Published by:
IIUM Press
International Islamic University Malaysia

First Edition, 2011

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov
Recent Achievements in Dynamical Systems
Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov

ISBN: 978-967-418-201-4

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by :
IIUM PRINTING SDN. BHD.
No. 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

Contents

Part II. Dynamical Systems Arising From Physical Models

DYNAMICAL SYSTEMS OF ISING MODEL ON A CAYLEY TREE

Farrukh Mukhamedov [1] and Mansoor Saburov [2]

[1,2] Department of Computational & Theoretical Sciences,
Faculty of Science, International Islamic University Malaysia,
P.O. Box, 141, 25710, Kuantan, Pahang, Malaysia
E-mail: [1]farrukh_m@iium.edu.my, [2]msaburov@gmail.com

Abstract

In the present paper, we study stability of the dynamical system corresponding quantum Markov chain (QMC) associated with the Ising model on Cayley tree of order two. To study certain properties of QMC we reduce our investigation to the study of dynamics of a nonlinear dynamical system. For such a dynamical system it is proved existence of exactly three fixed points and absence of periodic points. Moreover, it is established finiteness and infiniteness of the trajectory of the system.
Keywords: Cayley tree, quantum Markov chain, Ising model, stability of the dynamical system.

Introduction

It is know that Markov fields play an important role in classical probability, in physics, in biological and neurological models and in an increasing number of technological problems such as image recognition. Therefore, it is quite natural to forecast that the quantum analogue of these models will also play a relevant role. The quantum analogues of Markov processes were first constructed in [1], where the notion of quantum Markov chain on infinite tensor product algebras was introduced. Nowadays, quantum Markov chains have become a standard computational tool in solid state physics, and several natural applications have emerged in quantum statistical mechanics and quantum information theory. The reader is referred to [4, 5, 15, 22] and the references cited therein, for recent developments of the theory and the applications.

A first attempts to construct a quantum analogue of classical Markov fields has been done in [19], [3], [5], [8]. These papers extend to fields the notion of quantum Markov state introduced in [7] as a sub–class of the quantum Markov chains introduced in [1]. Typically a system is identified to a point in a graph: if this graph is not isomorphic to an interval in (1–dimensional case). The crucial role of the localization is at the root of the difficulties to construct nontrivial examples of Markov fields. Gaussian states (quasi-free, in the physics terminology) also have a simple structure, but they do not describe physically interesting interactions. Note that in mentioned papers quantum