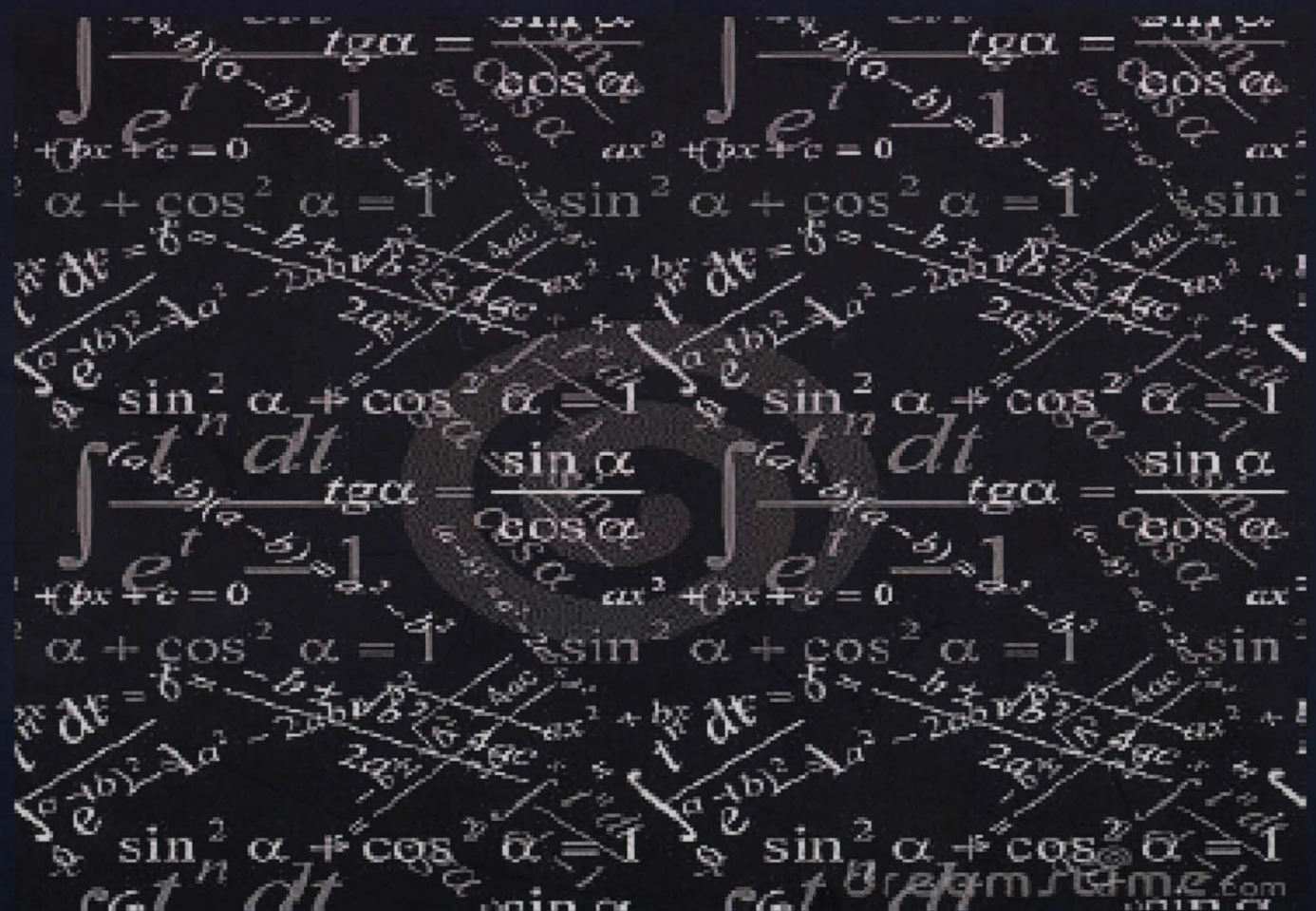




RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of
Computational and Theoretical
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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DYNAMICAL SYSTEMS OF XY-MODELS ON A CAYLEY TREE OF ORDER TWO

Farrukh Mukhamedov ¹ and Mansoor Saburov ²

^{1,2} *Department of Computational & Theoretical Sciences,
Faculty of Science, International Islamic University Malaysia,
P.O. Box, 141, 25710, Kuantan, Pahang, Malaysia
E-mail: ¹farrukh_m@iiu.edu.my, ²msaburov@gmail.com*

Abstract

In the present paper by means of a construction of Quantum Markov chain on a Cayley tree, we consider XY-models of such chains and we study dynamical systems of such models.

Keywords: Cayley tree, quantum Markov chain, a toy model.

Introduction

Markov fields play an important role in classical probability, in physics, in biological and neurological models and in an increasing number of technological problems such as image recognition. It is quite natural to forecast that the quantum analogue of these models will also play a relevant role. One of the basic open problems in quantum probability is the construction of a theory of quantum Markov fields, that is quantum Markov processes with a (possibly) multi-dimensional index set. In the papers [1], [2], [7] a first attempts to construct a quantum analogue of classical Markov fields have been done. These papers extend to fields the notion of *quantum Markov state* introduced in [4] as a sub-class of the *quantum Markov chains* introduced in [7]. Note that in such papers quantum Markov fields were considered over multidimensional integer lattice Z^d . This lattice has so called amenability condition. Therefore, it is natural to investigate quantum Markov fields over non-amenable lattices. One of the simplest non-amenable lattices is a Cayley tree. First attempts to investigate Quantum Markov chains over such trees was done in [9], such studies were related to investigate thermodynamic limit of valence-bond-solid models on a Cayley tree [6]. The mentioned considerations naturally suggest the study of the following problem: the extension to fields of the notion of generalized Markov chain. In [10,11] a construction of a Quantum Markov chains on the Cayley tree has been provided. The present paper, using that construction for a toy model we consider associated Quantum Markov chain, and show it is unique for the given model.

The point is that, as we know from Dobrushin's seminal work [8], the natural localization for fields on a discrete set L is given by the finite subsets