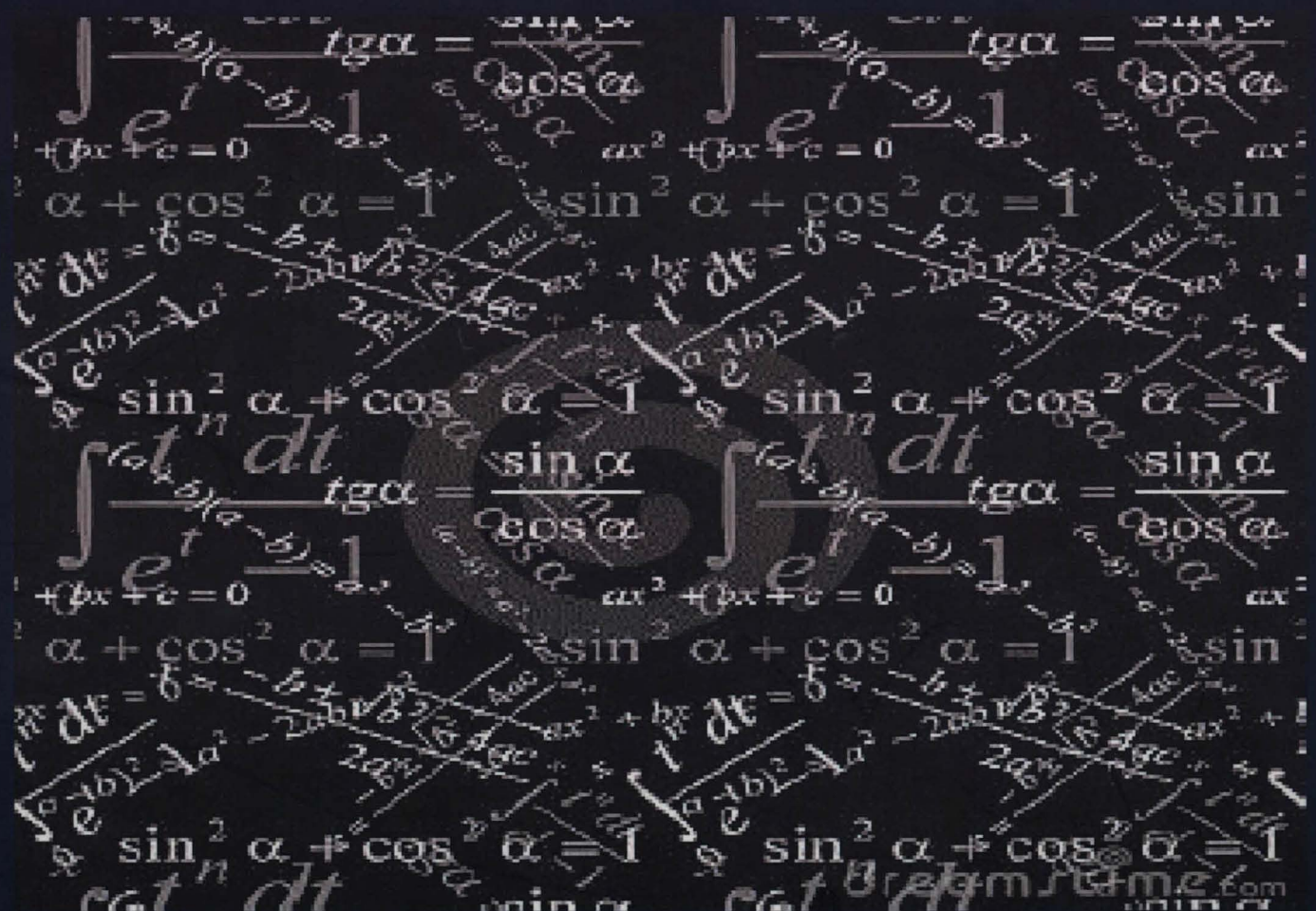




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

Proceedings of Department of  
Computational and Theoretical Sciences,  
Faculty of Science, IIUM

# Recent Achievements in Dynamical Systems

Chief Editor: Farrukh Mukhamedov

Editors:  
Nasir Ganikhodjaev  
Mansoor Saburov

Vol. 2



IIUM Press

Published by:  
IIUM Press  
International Islamic University Malaysia

First Edition, 2011  
©IIUM Press, IIUM

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without any prior written permission of the publisher.

Perpustakaan Negara Malaysia

Cataloguing-in-Publication Data

Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov  
Recent Achievements in Dynamical Systems  
Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov

ISBN: 978-967-418-201-4

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM  
(Malaysian Scholarly Publishing Council)

Printed by :  
**IIUM PRINTING SDN. BHD.**  
No. 1, Jalan Industri Batu Caves 1/3  
Taman Perindustrian Batu Caves  
Batu Caves Centre Point  
68100 Batu Caves  
Selangor Darul Ehsan

## Contents

### Part I. Quadratic Operators and Their Dynamics

Farrukh Mukhamedov, Abduaziz Abduganiev, Maksut Mukhamedov, On Dynamics of a Class of Quantum Quadratic Operators on $M_2(\mathbb{C})$ .	2
Mansoor Saburov, On Ergodic Principle for Quadratic Volterra Operators.	9
Mansoor Saburov, Fixed Point of Compositions of Volterra Operators.	15
Farrukh Mukhamedov, Afifah Hanum Bt Mohd. Jamal, Classification of $\xi^s$ - Quadratic Stochastic Operators in 2D-Simplex.	21
Farrukh Mukhamedov, Mansoor Saburov, Afifah Hanum Bt Mohd. Jamal, Dynamics of $\xi^s$ - Quadratic Stochastic Operators in 2D-Simplex.	29
Farrukh Mukhamedov, Mansoor Saburov, Some Examples of Lotka-Volterra Type Models.	34
Nasir Ganikhodjaev, Makhsuma Usmanova, On Linearization of Quadratic Stochastic Operators.	40
Nasir Ganikhodjaev, Continual Family of Ergodic Non-Homogeneous Markov Chains.	47
Rasul Ganikhodjaev, Farrukh Mukhamedov, Mansoor Saburov, On G-Decomposition of Matrices.	53
Farrukh Mukhamedov, On $L_1$ -Weak Ergodicity of Nonhomogeneous Discrete Markov Processes	59
Inomjon Ganiev, Farrukh Mukhamedov, On Measurable Bundles of $C^*$ -Dynamical Systems.	65
Inomjon Ganiev, Farrukh Mukhamedov, A Weighted Ergodic Theorem for Contractions Defined on Banach-Kantorovich Lattice.	71

## **Part II. Dynamical Systems Arising From Physical Models**

Farrukh Mukhamedov, Mansoor Saburov, Dynamical Systems of XY-Models On A Cayley Tree Of Order Two.	78
Farrukh Mukhamedov, Mansoor Saburov, Dynamical Systems of XY-Models On A Cayley Tree Of Order Three.	85
Farrukh Mukhamedov, Mansoor Saburov, Dynamical Systems of Ising Model on a Cayley Tree.	91
Nasir Ganikhodjaev, Siti Fatimah Zakaria, Phase Diagram of The Ising Model with Nearest-Neighbor Interactions.	98
Nasir Ganikhodjaev, Siti Fatimah Zakaria, Ising Model on a General Cayley Tree with Competing Next-Nearest-Neighbour Interactions.	107
Pah Chin Hee, Rukiah Ali, Ising Model with Competing Interactions on Cayley Tree of Order Four	118
Massimo Ostilli, Langevin Dynamics for a New Class of Mean-Field Ising Models.	125
Farrukh Mukhamedov, Utkir Rozikov, Free Energy of The Ising Model with Competing Interactions on a Cayley Tree.	133
A. Benseghir, B.A. Umarov, A. Messikh, Modulational Instability In Salerno Model.	141
Nasir Ganikhodjaev, Seyit Temir, On Potts Model with Triple Interactions.	146
Nasir Ganikhodjaev, Ashraf Mohamed Nawi, Mohd Hirzie Mohd Rodzhan, Phase Diagram Of The Potts Model with External Magnetic Field.	152
Nasir Ganikhodjaev, Fatimah Abdul Razak, A Correlation Inequality for Potts Model.	160
Nasir Ganikhodjaev, Ashraf Mohamed Nawi, A Nonlinear Dynamic System Arising in Potts Model.	167

Farrukh Mukhamedov, On Existence of Phase Transition for One Dimensional P-Adic Countable State Potts Model.	177
B.A. Umarov, A. Bouketir, Strongly Localized Models In Two-Component Discrete Media With Cubic-Quintic Nonlinearity.	184
<b>Part III. Nonlinear Dynamical Systems</b>	
Farrukh Mukhamedov , Wan Nur Fairuz Alwani Wan Rozali, On P-Adic Generalized Logistic Dynamical System.	196
Farrukh Mukhamedov, Mansoor Saburov, On Equation $x^q = a$ over $\mathcal{Q}_p$ .	201
Farrukh Mukhamedov, Mansoor Saburov, On Unification of The Strong Convergence Theorems for a Finite Family Of TAN Mappings in Banach Spaces.	207
<b>Part IV. Graphs And Networks</b>	
Pah Chin Hee, Single Polygon Counting for Two Fixed Nodes on a Cayley Tree of Order 2.	214
Khikmat Saburov, Mansoor Saburov, Every 3-Connected $K_{1,3}Z_6$ -Free Graph is Hamiltonian.	219
Khikmat Saburov, Mansoor Saburov, Relation Between $K_{1,3}P_7$ -Free and $K_{1,3}N_{1,1,1}$ -Free Graphs.	224
Khikmat Saburov, Mansoor Saburov, Hamiltonicity Of $K_{1,3}B_{i,7-i}$ -Free Graphs.	232
Saadi Bin Ahmad Kamtuddin , Nor Azura Md Ghani, Choong-Yeun Liong And Abdul Aziz Jemain, Artificial Neural Network Implementation on Firearm Recognition System via Ring Firing Pin Impression Image.	242
Pah Chin Hee, Dirichlet's Theorem And Prime Gap Statistics.	256

## ON MEASURABLE BUNDLES OF $C^*$ -DYNAMICAL SYSTEMS

Inomjon Ganiev<sup>1</sup> and Farrukh Mukhamedov<sup>2</sup>

<sup>1</sup>*Faculty of Engineering, International Islamic University Malaysia, Gombak, Selangor;*

<sup>2</sup>*Faculty of Science, International Islamic University Malaysia, Kauntan, 25200 Pahang;  
E-mail: <sup>2</sup>farrukh\_m@iiu.edu.my*

### Abstract

In the present paper we investigate  $L_0$ -valued states and Markov operators on  $C^*$ -algebras over  $L_0$ . In particular, we give representations for  $L_0$ -valued state and Markov operators on  $C^*$  algebras over  $L_0$ , respectively, as measurable bundles of states and Markov operators. Moreover, we apply the obtained representations to study certain ergodic properties of  $C^*$ -dynamical systems over  $L_0$ .

*Keywords:* measurable bundle;  $L_0$ -valued norms;  $C^*$ -dynamical systems

### Introduction

In the present paper we are going to deal with  $C^*$ -algebras over  $L_0$ , such algebras are considered over modulus of measurable functions  $L_0$ . Such  $C^*$ -algebras over  $L_0$  allow to construct meaningful examples of Banach-Kantorovich spaces. Note that the theory of such spaces already well developed (see Gutman, 1993). It is known that a structural theory of  $C^*$ -modules started with papers of Kaplanski (Kaplanski, 1953) who used such objects for an algebraic approach to the theory of  $W^*$ -algebras. Consideration of  $C^*$ -algebras,  $AW^*$ -algebras and  $W^*$ -algebras as modulus over their centers allowed to use methods of Boolean-valued analysis to describe several properties of the mentioned algebras (see Kusraev, 2000, and Takeuti, G. (1983). In (Ganiev et al, 2003) it was introduced the notion  $C^*$  algebra over ring of all measurable functions  $L_0$ . In that paper, by means of the methods of a general theory of Banach measurable bundles (see Kusraev, 1996), it has been shown that any  $C^*$ -algebras over  $L_0$  can be represented as a measurable bundle of  $C^*$ -algebras. In (Chilin et al, 2007) GNS- representation was obtained for such  $C^*$ -algebras. In the present paper we investigate  $L_0$ -valued states and Markov operators on  $C^*$ -algebras over  $L_0$ .