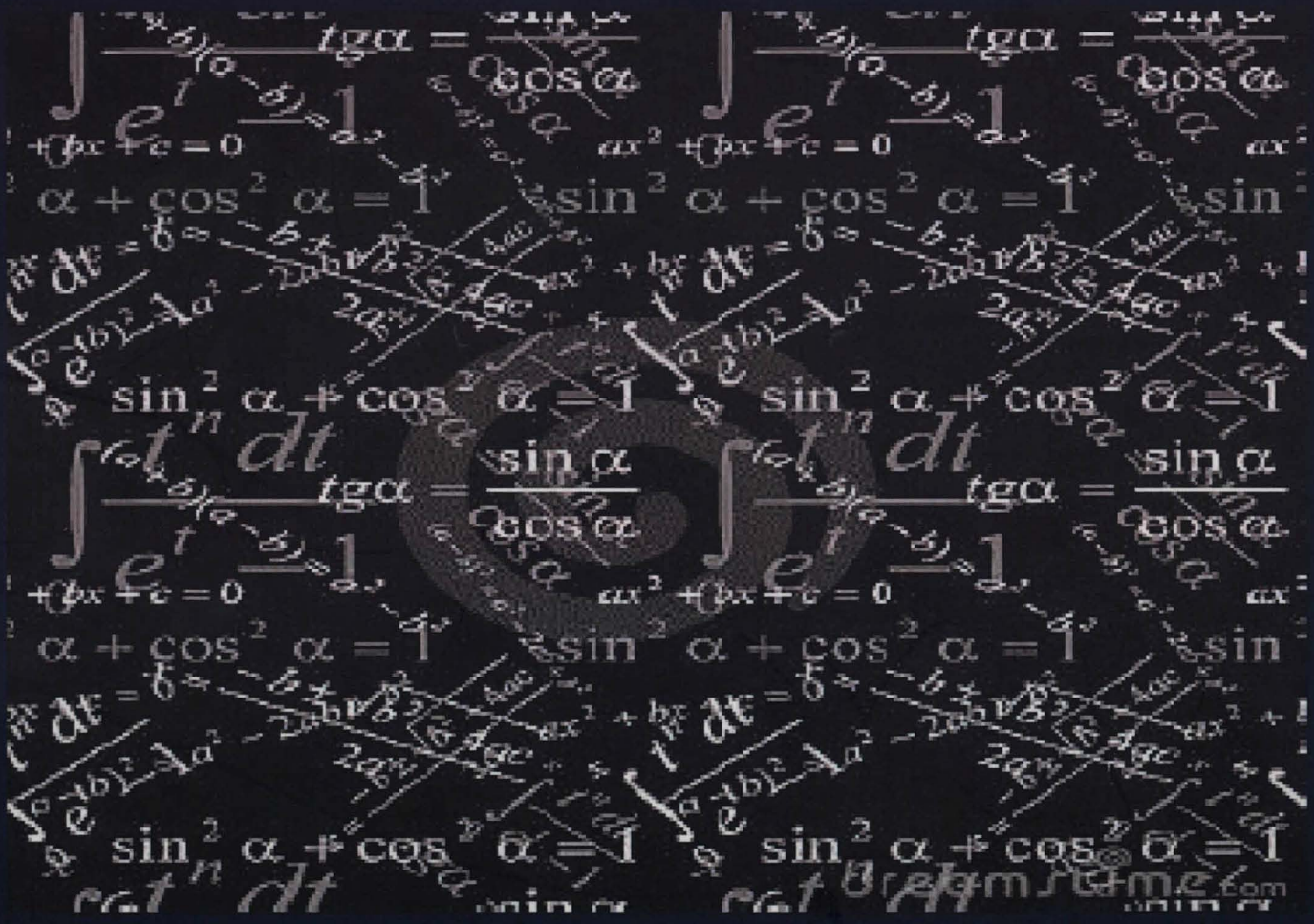
الجامعة الإسلامية العالمية ماليزيا
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA
يونيرسيتي إسلام انتارابغسا ملسيا

VOLUME 2

RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of Computational and Theoretical Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov
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Vol. 2

IIUM Press

Published by:
IIUM Press
International Islamic University Malaysia

First Edition, 2011

Perpustakaan Negara Malaysia Cataloguing-in-Publication Data

Farrukh Mukhamedov, Nasir Ganikhodjaev & Mansoor Saburov
Recent Achievements in Dynamical Systems
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ISBN: 978-967-418-201-4

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by :
IIUM PRINTING SDN. BHD.
No. 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

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Part I. Quadratic Operators and Their Dynamics

Part II. Dynamical Systems Arising From Physical Models

ON G-DECOMPOSITIONS OF MATRICES

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Abstract

G.D Birkhoff characterized the set of extreme doubly stochastic matrices. Namely his result states as follows: the set of extreme points of the set of doubly stochastic matrices coincides with the set of all permutations matrices. One can consider a generalization of Birkhoff's result for nonlinear doubly stochastic operators. Among all nonlinear operators, the simplest one is a quadratic operator. In this work we introduce a notion of G-decomposition of matrices which enables to study Birkhoff's problem for quadratic G-doubly stochastic operators. We find necessary and sufficient conditions for the matrices having G-decomposition in the class of stochastic and substochastic matrices. We study geometrical structures of the set of those matrices.

Keywords: *G-decomposition; doubly stochastic quadratic operator; stochastic matrix; substochastic matrix.*

Introduction

Let us recall that a matrix $A_m = (a_{ij})_{i,j=1}^m$ is said to be

i) *stochastic* if its elements are non-negative and each row sum is equal to one;
ii) *substochastic* if its elements are non-negative and each row sum is less or equal to one;
iii) *doubly stochastic* if its elements are non-negative and each row and column sums are equal to one.

In [1] G.D Birkhoff characterized the set of extreme doubly stochastic matrices. Namely his result states as follows: the set of extreme points of the set of $m \times m$ doubly stochastic matrices coincides with the set of all permutations matrices. One can consider a generalization of Birkhoff's result. One may consider the description of all extreme points of the set of *nonlinear doubly stochastic operators*.

The present paper is related to the Birkhoff's problem for nonlinear doubly stochastic operators. In this case, we will face with a few contretemps. In fact, first of all, we should define a conception of stochasticity for nonlinear