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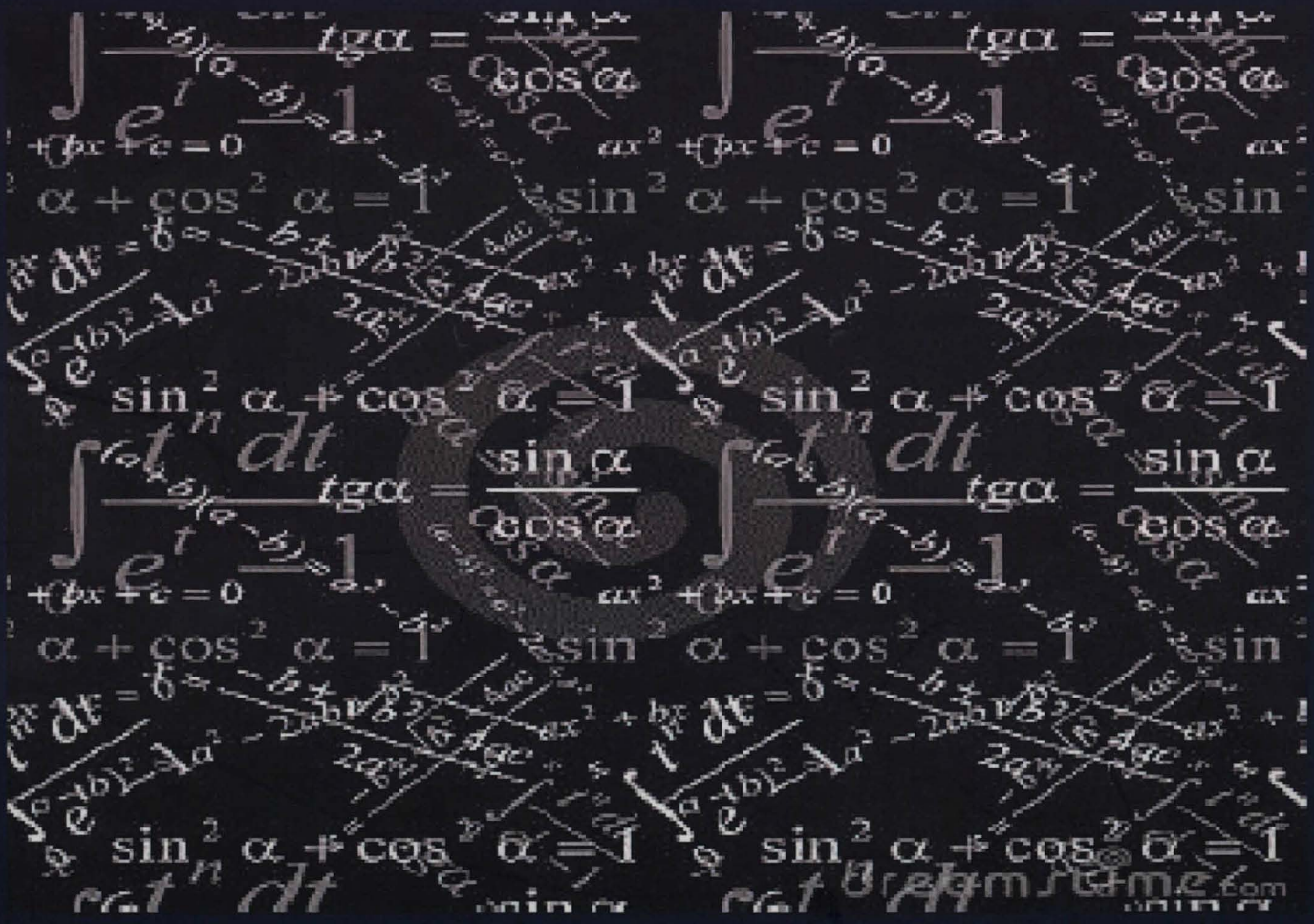

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CONTINUAL FAMILY OF ERGODIC NON-HOMOGENEOUS MARKOV CHAINS

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Abstract

Inspired by the recent work of Jiahong Shen on a geometric approach to ergodic non-homogeneous Markov chains, we construct a continual family of ergodic non-homogeneous Markov chains on finite states.

Quadratic Stochastic Operator

Let

$$S^{m-1} = \{ \mathbf{x} = (x_1, \cdots, x_m) \in R^m : x_i \geq 0, \sum_{i=1}^{m} x_i = 1 \}$$

is the $(m-1)$ -dimensional canonical simplex in R^m. The transformation $V : S^{m-1} \to S^{m-1}$ is called a quadratic stochastic operator(q.s.o.) if

$$V : (V\mathbf{x})_k = \sum_{i,j=1}^{m} p_{ij,k} x_i x_j, \quad (k = 1, \cdots, m) \tag{1}$$

where

$$a) p_{ij,k} \geq 0; \quad b) p_{ij,k} = p_{ji,k}; \quad \text{and} \quad c) \sum_{k=1}^{m} p_{ij,k} = 1$$

for arbitrary $i, j, k = 1, \cdots, m$. Quadratic stochastic operator was first introduced in [1].Such operator frequently arises in many models of mathematical genetics[1-3].Consider a biological population,that is a community of organisms closed with respect to reproduction.Assume that each individual in this population belongs to precisely one species $1, \cdots, m$. The scale of species is such that the species of the parents i and j unambiguously determines the probability of every species k for the first generation of direct descendants.Denote this probability,that is to be called the heredity coefficient, by $p_{ij,k}$. It is then obvious that $p_{ij,k} \geq 0$ for all i, j, k and that

$$\sum_{k=1}^{m} p_{ij,k} = 1 (i, j, k = 1, \cdots, m).$$