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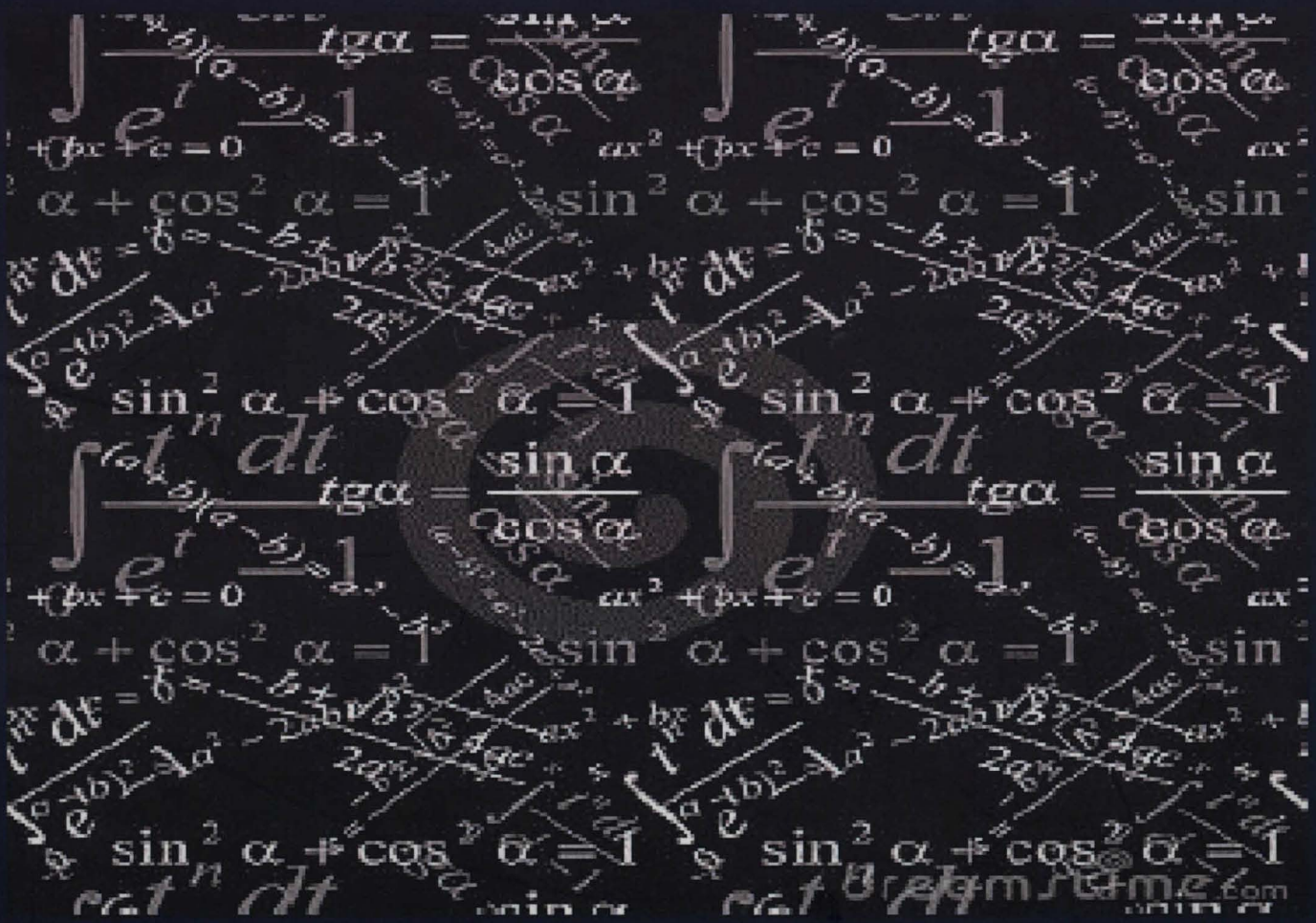

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: Mansoor Saburov

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Recent Achievements in Dynamical Systems

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Nasir Ganikhodjaev
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ON LINEARIZATION OF QUADRATIC STOCHASTIC OPERATORS

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Abstract

Quadratic stochastic operators frequently arise in many models of mathematical genetics, namely theory of heredity. In this paper we study the linearization of quadratic stochastic operators, state some properties of the its Jacobian and consider some application in medicine.

Keywords: *Quadratic stochastic operator; linear operator; Jacobian;*

Quadratic Stochastic Operator

Let

$$S^{m-1} = \{x = (x_1,...,x_m) \in R^m : x_i \geq 0 \text{ for any } i = 1,...,m \text{ and } \sum_{i=1}^{m} x_i = 1\} \quad (1)$$

be the (m-1) -dimensional canonical simplex in R^m. The transformation $V : S^{m-1} \to S^{m-1}$ is called a quadratic stochastic operator if

$$V : (Vx)_k = \sum_{i,j=1}^{m} p_{ij,k} x_i x_j \text{ ,} k=1,\ldots,m \quad (2)$$

where

$$a)\ p_{ij,k} \geq 0 \text{ ; } b)\ p_{ij,k} = p_{ji,k} \text{ ; and } c)\ \sum_{k=1}^{m} p_{ij,k} = 1 \quad (3)$$

for arbitrary i ,j, k=1,..., m. Quadratic stochastic operator was first introduced in [1]. Such operator frequently arises in many models of mathematical genetics [2,3].Consider a biological population, that is a community of organisms closed with respect to reproduction. Assume that each individual in this population belongs to precisely one species 1,..., m. The scale of species is such that the species of the parents i and j unambiguously determines the probability of every species k for the first generation of direct descendants. Denote this probability, that is to be called the heredity coefficient, by $p_{ij,k}$. It is then obvious that

$$p_{ij,k} \geq 0 \text{ and } \sum_{k=1}^{m} p_{ij,k} = 1 \quad (i, j=1,\ldots,m)$$

Assume that the population is so large that frequency fluctuations can be neglected. Then the state of the population can be described by the m- tuple x