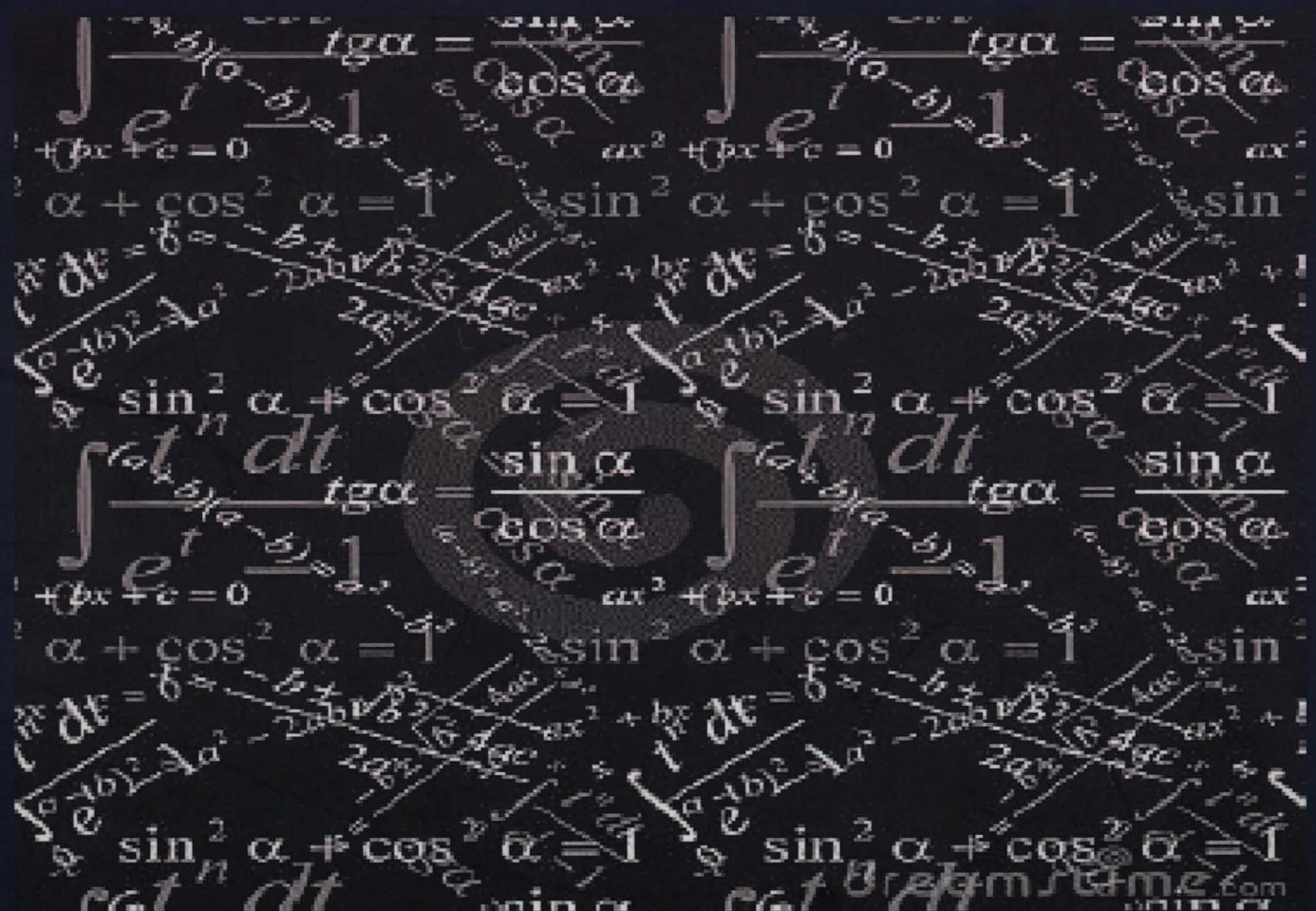




# RECENT ACHIEVEMENTS IN DYNAMICAL SYSTEMS

Proceedings of Department of  
Computational and Theoretical  
Sciences, Faculty of Science, IIUM



Chief Editor : Farrukh Mukhamedov

Editors : Nasir Ganikhodjaev

: Mansoor Saburov

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## SOME EXAMPLES OF LOTKA-VOLTERRA TYPE MODELS

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### Abstract

In this paper we study nonlinear Lotka-Volterra (in short LV) type models. LV type models can exhibit any asymptotical behavior such as equilibrium states, periodic cycles, and Attractors.

**Keywords:** *LV type model; equilibrium state; periodic cycle*

### Look at the Background

Predator-prey models have been studied for a long time. Many biologists believe that if the unique positive equilibrium point of a predator-prey system is local asymptotically stable, then it is global asymptotically stable. However, this is not always true. It is found that a unique positive local asymptotically stable equilibrium point has at least one limit cycle surrounding the equilibrium point under suitable condition. Therefore, many mathematicians try to use some well-known methods to find conditions for global stability for the equilibrium point of predator-prey systems.

The Lotka-Volterra (in short LV) model is the simplest model of predator-prey interactions. It is based on linear per capita growth rates and written as follows

$$\begin{cases} \dot{x} = x(b - py) \\ \dot{y} = y(-d + rx) \end{cases} \quad (1)$$

where

- (i)  $y$  is the number of some predators (for example, wolves);
- (ii)  $x$  is the number of its preys (for example, rabbits);
- (iii)  $\dot{y}$  and  $\dot{x}$  represent the growth of the two populations against the time;
- (iv)  $t$  represents the time;

$b, p, d, r \geq 0$  are parameters representing the interaction of the two species

The equation (1) has periodic solutions which do not have a simple expression in terms of the usual trigonometric functions. At any given time in the phase plane, the system is in a limit cycle and lies somewhere on the inside of these elliptical solutions