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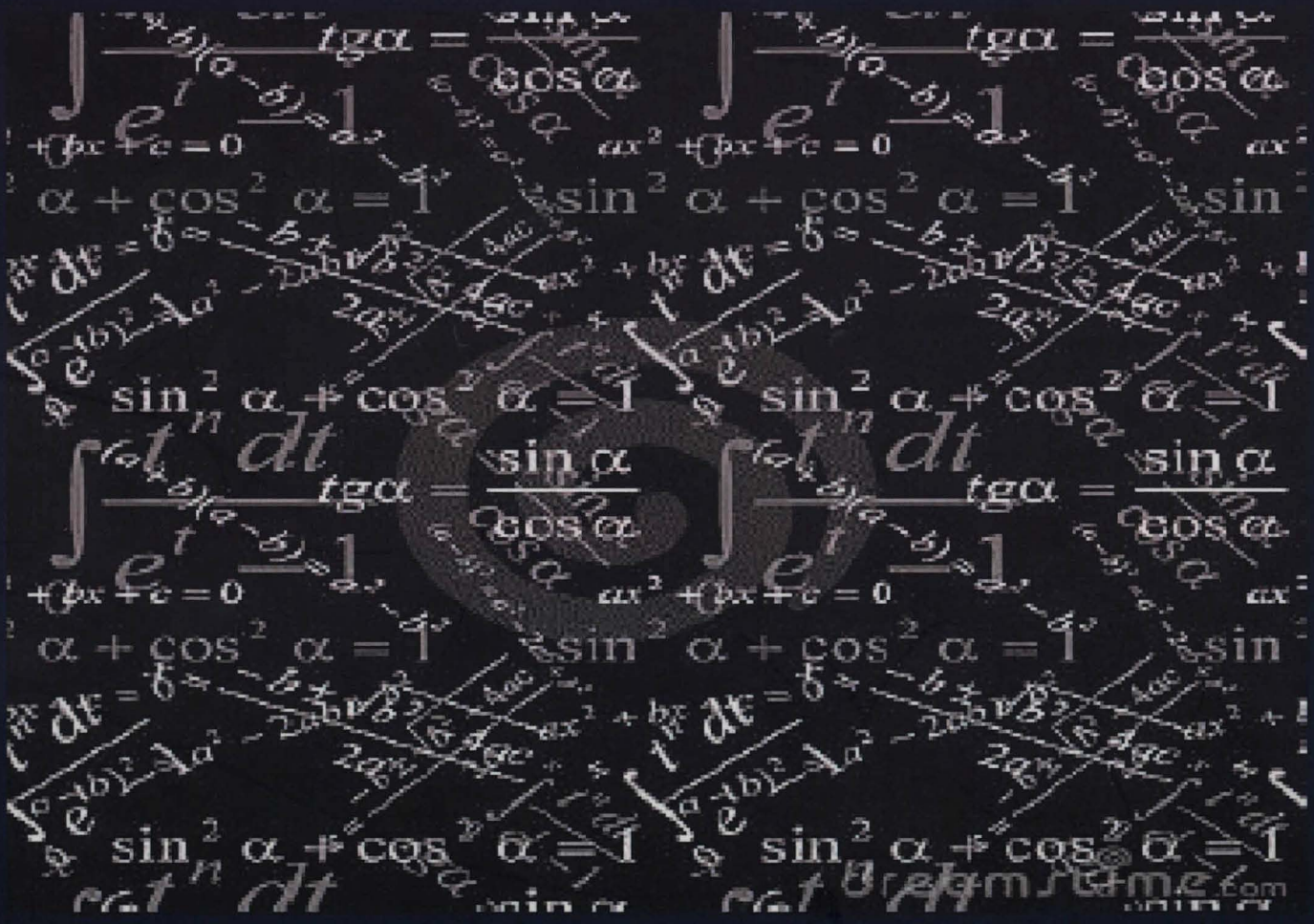

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Contents

Part I. Quadratic Operators and Their Dynamics

Part II. Dynamical Systems Arising From Physical Models

ON DYNAMICS OF A CLASS OF QUANTUM QUADRATIC OPERATORS ON $M_2(C)$

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Abstract

In the present paper we study nonlinear dynamics of a class of quantum quadratic operators (q.q.o) acting on the algebra of 2×2 matrices $M_2(C)$.

Keywords: *component; quantum quadratic operators; Kadison-Schwartz operator.*

Introduction

It is known that the theory of Markov processes is a rapidly developing field with numerous applications to many branches of mathematics and physics. However, there are physical systems that cannot be described by Markov processes. One of such systems is given by quadratic stochastic operators (see [1]), which relates to population genetics. The limit behavior and ergodic properties of trajectories of quadratic stochastic operators were studied in [6,8,9]. However, such kind of operators does not cover the case of quantum systems. Therefore, in [3,4] quantum quadratic operators acting on a von Neumann algebra were defined and studied. However, with a given quadratic operator one can define also a non-linear operator whose dynamics (in non-commutative setting) is not studied yet. Very recently, in [5] convergence of ergodic averages associated with mentioned non-linear operator are studied by means of absolute contractions of von Neumann algebras. Actually, it is not investigated a nonlinear dynamics of convolution operators. Therefore, a complete analysis of dynamics of quantum quadratic operator is not well studied.

In the present paper we are going to study nonlinear dynamics of quantum quadratic operators acting on the algebra of 2×2 matrices $M_2(C)$. Since positive, trace-preserving maps arise naturally in quantum information theory and other situations in which one wishes to restrict attention to a quantum system that should properly be considered a subsystem of a larger system with which it interacts. We describe quadratic operators with Haar state (invariant with respect to trace), and using it we shall provide an example