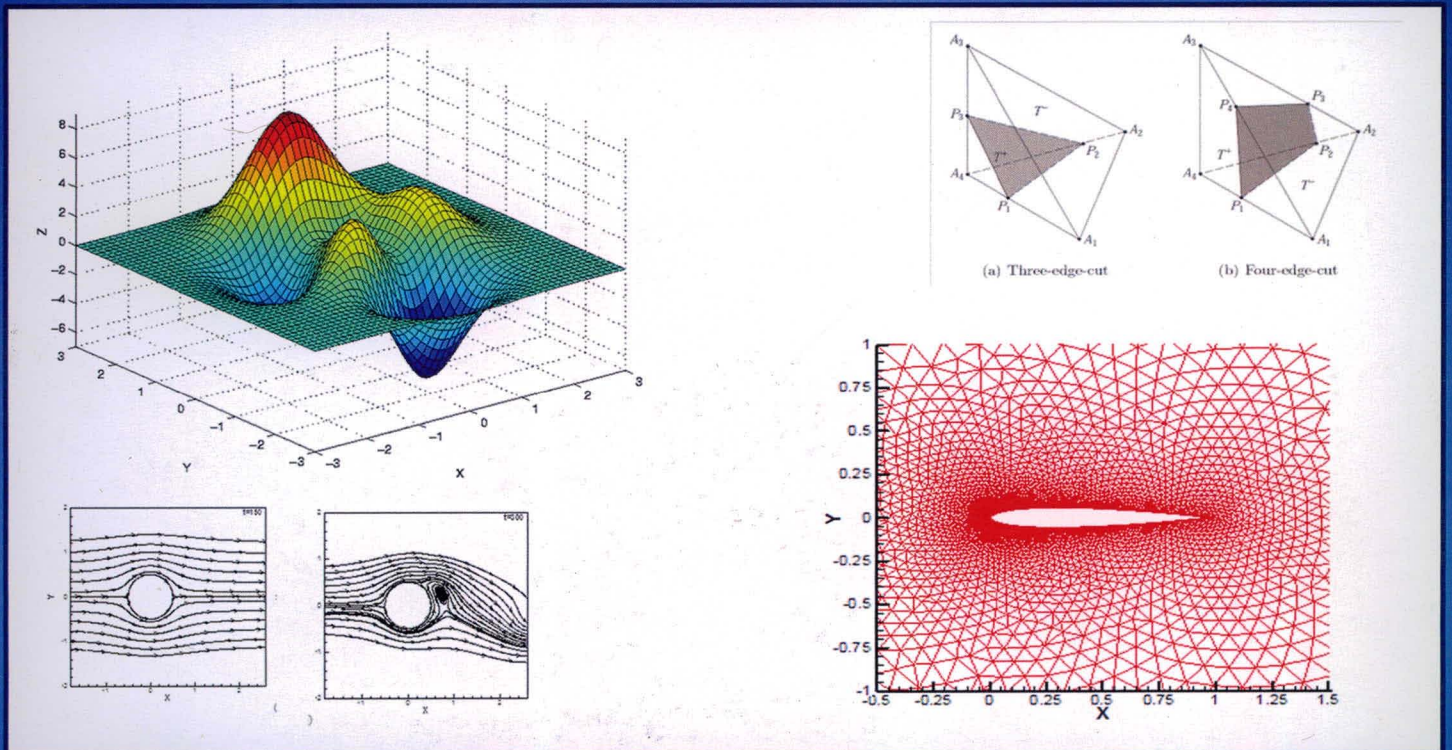


# ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING and SCIENCE



Editors

AHMAD TARIQ JAMEEL

WAQAR ASRAR



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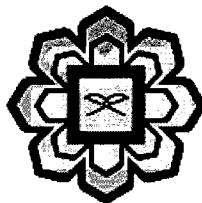
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## CHAPTER 7

### Finite Element Modeling of the Powder Compaction Process

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#### ABSTRACT

Finite element simulation of the rigid die compaction of metal powders can significantly help the designers to avoid some of the typical problems of this forming process such as the high density gradients within the compacted parts and the related cracking. A material constitutive model of the Cap type has been developed, experimentally calibrated for the 316L stainless steel powder and implemented into the finite element code ABAQUS. Simulations are run by an integrated module that uses the Ideas-Master Series software for the geometrical definition of the problem, ABAQUS as the non linear finite element solver, an in-house developed software (Idequs) for the prescription of all the boundary conditions and material properties and finally, the ABAQUS-Post software for the results post processing. This chapter addresses the formulation of the rigid die compaction process as a finite element problem. It also presents some theoretical aspects of the material model as well as its experimental characterization for the 316L stainless steel powder. In addition, the finite element simulation module that has been developed to facilitate the application of the global approach is presented. Finally, the predictive capabilities of the model are illustrated through the simulation of the rigid die compaction of two industrial parts.

**Keywords:** powder metallurgy, compaction process, finite element, density, Cap model, ABAQUS, material parameters, stainless steel powder