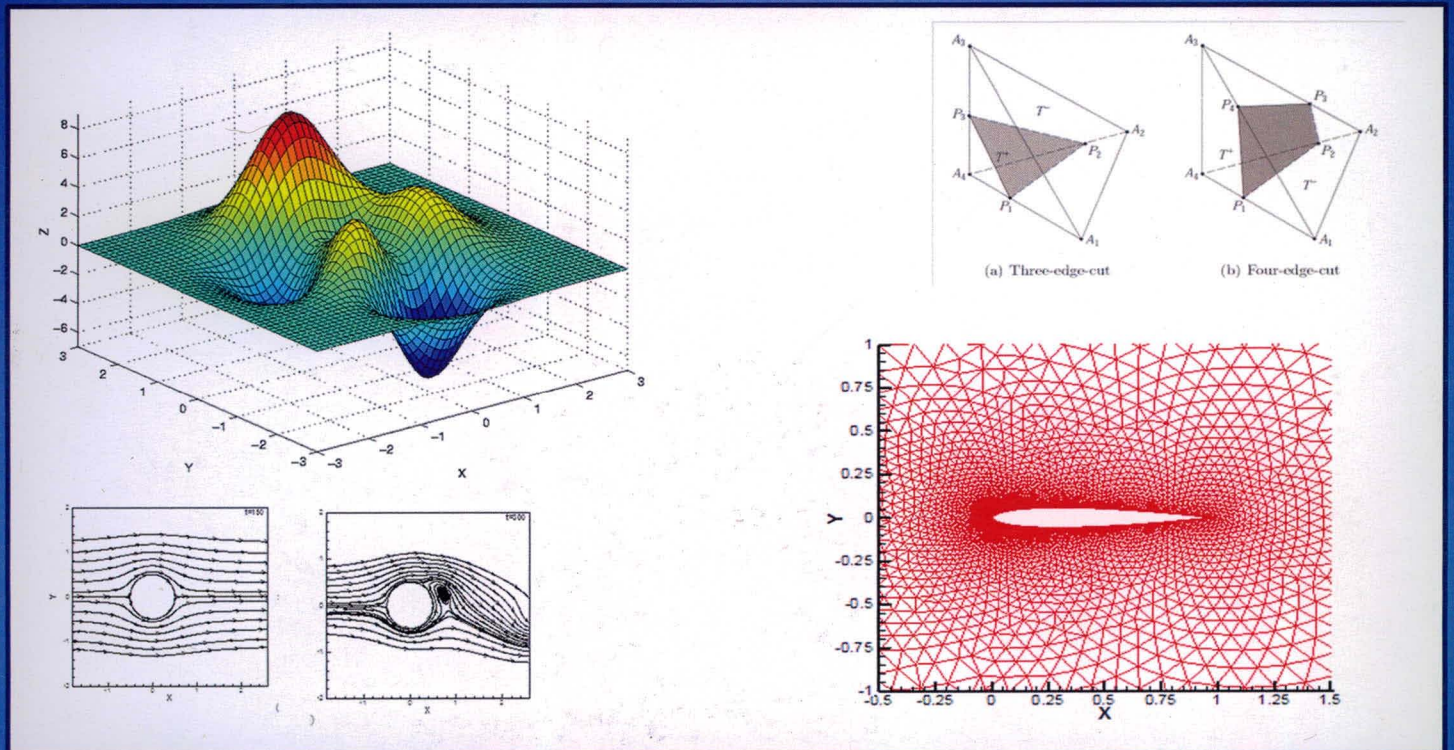
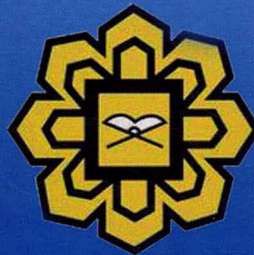


ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING and SCIENCE



Editors
AHMAD TARIQ JAMEEL
WAQAR ASRAR



IIUM Press

International Islamic University Malaysia

2011

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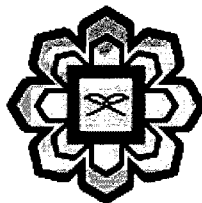
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IIUM Press
International Islamic University Malaysia
2011

Published by:
IIUM Press
International Islamic University Malaysia

First Edition, 2011
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Perpustakaan Negara Malaysia

Cataloguing-in-Publication Data

Ahmad Tariq Jameel & Waqar Asrar: Advanced Numerical Techniques in Engineering & Science

ISBN: 978-967-418-020-1

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by :
IIUM PRINTING SDN. BHD.
No. 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

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CHAPTER 5

Higher-Order Compact Finite Difference Schemes

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Authors' Note

The purpose of this chapter is to introduce fresh graduate students and researchers in CFD to the topic of higher-order compact schemes. Full details of the procedure have been provided with detailed examples for a quick grasp of the subject. The literature review is not exhaustive but sufficient. It is expected that more recent additions to the literature will be added by the researchers. It is our wish that the chapter is of benefit to the scientific community.

ABSTRACT

This chapter discusses higher-order compact finite difference schemes. A literature review is presented. Attention is focused on two major approaches (Hermitian and governing equation based) for discretization of the governing equations. Both schemes are applied to an example problem of viscous Burgers' equations. Detailed derivations are made with applications of boundary conditions. The methods are extended to clustered grids. Results of the two approaches are compared.